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A New Log-Exponentiated Weighted Weibull Distribution for Modeling Survival Data

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Abstract

We introduce a new distribution called the log-exponentiated weighted Weibull distribution that leads to a log-linear regression model and it includes a special sub-model in the existing literature. Some of its properties like the moments, moment generating function were derived. A regression model based on the new distribution is used to fit and predict a terminal event (survival data) and applied to survival data. The regression model is fitted to a data set of One Hundred and Eighty-Seven Ph.D. students who have completed their program from the University of Ibadan, Nigeria. The number of years spent by each of the students was used as the response variable while five other potential factors that are likely to correlate with the response variable were used as the explanatory variables. The results showed that the proposed model is very flexible and better fitted to the data.

Keywords: Beta weighted Exponential, Moment Generating, Regression, Response Variable, Weibull

1.0 Introduction

Many authors have introduced models and demonstrated how to handle survival data in the literature using generator approach/logit of beta function introduced by Eugene (2002) and Jones (2004); including Famoye et al. (2005), Akinsete et al. (2008), Fisher and Vaughan (2010), Lemonte Artur (2014), Shittu and Adepoju (2013), Badmus and Bamiduro (2014) and Badmus and Bamiduro (2014). All the parametric models mentioned above can only estimate univariate survival function.

Furthermore, in the field of survival analysis, there is still a need to construct robust estimation for flexible parametric models. Recently, different forms of regression models have been proposed in survival analysis, among which we have location-scale regression model studied by several authors/researchers in the existing literature such as Ortega *et al*. (2009), Hashimoto *et al*. (2010), Pescim *et al.* (2010) and Ortega *et al.* (2013). Therefore, in this paper, we propose a location-scale regression model based on exponentiated weighted Weibull distribution when $b = 1$ in beta weighted Weibull distribution and it is referred to as log-exponentiated weighted Weibull (LEWW) regression model which can be used for modeling any type of failure rate function. This model is used to study Ph.D. data of some students who have completed their Ph.D. program from the University of Ibadan, Nigeria The reason for this study is that many applications of parametric models on survival data are based on clinical trial data, censored, uncensored data, etc. For instance, Lawless (2003), Ortega *et al. (2013)*, Ortega *et al.* (2009), Pescim *et al.* (2013) they made use of censored and clinical data, etc.

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The paper is organized as follows: we define the LEWW distribution and the density and distribution function are given in section 2, section 3 contains the properties, moment and generating function of LEWW distribution. In section 4, we propose a LEWW regression model for terminal event data, a Ph.D. data set is analyzed to show the flexibility and applicability of our model in section 5 and section 6, results and concluding remark.

2.0 The Log-Exponentiated Weighted Weibull Distribution

Various generalized weighted Weibull distributions have been proposed in the literature to fit data sets in reliability and other fields to provide a better fit than generalized Weibull distributions. The Beta weighted Weibull (BWW) density function by Badmus and Bamiduro (2014) with four parameters $a, b, \alpha\ and\ \beta > 0$ is given by $(for\ t > 0)$, but for this distribution when $b = 1$ and it gives

$$f(t) = \left[\frac{\alpha+1}{\alpha}\beta t^{\beta-1}exp(-t^{\beta})\left(1-exp(-\alpha t^{\beta})\right)\right] a\left[\frac{\alpha+1}{\alpha}\left\{\left(1-exp(-\alpha t^{\beta})\right)-\frac{1}{\alpha+1}\left(1-exp(-(1+\alpha)t^{\beta})\right)\right\}\right]^{a-1} \quad (1)$$

where α and β are weight and shape parameters. Then, a is the additional shape parameter to the weighted Weibull (WW) distribution to model the skewness and kurtosis of the data. The expression of weighted Weibull distribution can be re-written in another Weibull version for more simplification, we have

$$f(x) = \frac{\gamma+1}{\gamma}\left(\frac{\alpha}{\beta}\right)^{\alpha} x^{\alpha-1}exp\left(-\left(\frac{\alpha}{\beta}\right)^{\alpha}\right)\left(1-exp\left(-\gamma\left(\frac{\alpha}{\beta}\right)^{\alpha}\right)\right) \quad (2)$$

Then, we transformed (2) to have the Log-Weighted Weibull (LWW) distribution by letting $Y = \log(x)\ implies\ x = e^{y}, \alpha = \frac{1}{\sigma}\ and\ \mu = \log(\beta)\ implies\ \beta = e^{\mu}$ and substituting the transformation in (2), we obtain

$$f(y) = \frac{\gamma+1}{\gamma}\exp\left(\frac{y-\mu}{\sigma}\right) exp\left(-exp\left(\frac{y-\mu}{\sigma}\right)\right)\left(1-exp\left(-\gamma exp\left(\frac{y-\mu}{\sigma}\right)\right)\right) \quad (3)$$

(3) is the Log Weighted Weibull (LWW) distribution; and we can equally write the exponentiated weighted Weibull (EWW) distribution by adding the beta function to the existing expression in (3).

$$f(t) = \left[\frac{\gamma+1}{\gamma}\left(\frac{\alpha}{\beta}\right)^{\alpha} t^{\alpha-1}exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right)\left(1-exp\left(-\gamma\left(\frac{t}{\beta}\right)^{\alpha}\right)\right)\right] a\left[\frac{\gamma+1}{\gamma}\left\{\left(1-exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right)\right)-\frac{1}{\gamma+1}\left(1-\right.\right.\right.$$

$$\left.\left.\left.exp\left(-(1+\gamma)\left(\frac{t}{\beta}\right)^{\alpha}\right)\right)\right\}\right]^{a-1} \quad (4)$$

Figure 1 provides the plot of the EWW density curves (in 1) for different values of the parameters $\alpha, \beta, \gamma\ and\ a$.

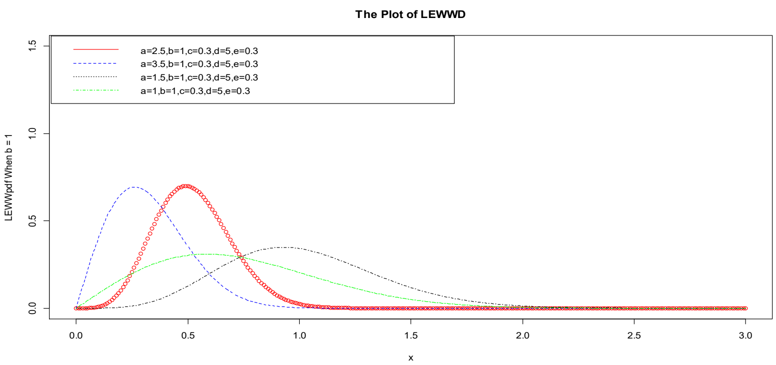

Figure 1: Plots of the EWW density function for some parameter values.

$t \sim EWW\ (a, \alpha, \beta, \gamma)$ distribution: where γ, α, β and a are weighted, shape, scale and additional shape parameters; and (4) becomes four parameters EWW distribution. The characteristic of EWW distribution is contains sub-models, the WW $a = \beta = 1, \gamma = \alpha$ and $\alpha = \beta$ (Shahbaz et al, 2010), WW $a = 1, \alpha = \beta, \gamma = \alpha^{\beta}$ (Ramadan, 2013), $a = \beta = 1$ (Gupta and Kundu, 2009) and various other distributions (weighted extreme $a = 1$ and $\gamma = \alpha^{\beta}$ (see this in Ramadan, 2013. Moreover, the corresponding cumulative distribution function, the survival rate and hazard rate function are as follows

$$F(t) = a\int_0^{F(t)} (M)^{a-1}dm = I_{F(t)}(a)$$

where K and $F(t)$ is the parent cumulative distribution function.

$$S_{(t)} = 1 - a\int_0^{F(t)} (M)^{a-1}dm = 1 - I_{F(t)}(a)$$

and

$$h_{(t)} = \frac{a(M)^{a-1}M'}{1 - [1 - (1 - t)^{a}]}$$

respectively, and where M' is the pdf of the parent distribution. $I_y(a) = a\int_0^y (K)^{a-1}dk$ called the in-complete beta function ratio.

Here, by letting T be a random variable having the exponentiated weighted Weibull density function in (2); and the mathematical properties of the LEWW distribution were investigated and also defined by the random variable $Y = \log(T)$. Now, the density function of Y is parameterized in terms of $\alpha = \frac{1}{\sigma}$ and $\mu = \log(\beta)$. The density function of Y can be expressed as

$$f(y; a, \gamma, \mu, \sigma) = \frac{\gamma+1}{\sigma\gamma} a exp\left(\frac{y-\mu}{\sigma}\right) exp\left(-exp\left(\frac{y-\mu}{\sigma}\right)\right)\left(1 - exp\left(-\gamma exp\left(\frac{y-\mu}{\sigma}\right)\right)\right)$$

$$\left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp\left(\frac{y-\mu}{\sigma}\right)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)\exp\left(\frac{y-\mu}{\sigma}\right)\right)\right)\right\}\right]^{a-1} \quad (5)$$

where $-\infty < y < \infty, \sigma > 0$ and $-\infty < \mu < \infty$ and (5) is the new model as LEWW distribution, say $y \sim LEWW\ (\mu, \sigma, \gamma, a)$ where μ, σ, γ and a are location, dispersion, weighted and additional shape parameters. Therefore, these results hold: If $X \sim EWW\ (\alpha, \beta, \gamma, a)$ then $Y = \log(x) \sim LEWW\ (a, \gamma, \mu, \sigma)$. The corresponding survival rate function of (5) is

$$S_{(y)} = 1 - a\int_0^{F(y)} (K)^{a-1}dk = 1 - I_{a[F(y)]} \quad (6)$$

where $F_{(y)} = \left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp\left(\frac{y-\mu}{\sigma}\right)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)\exp)\left(\frac{y-\mu}{\sigma}\right)\right)\right)\right\}\right]$.

3.0 Properties of the LLWW distribution

Now, we want to study some properties of the standardized LEWW random variable defined by $Z = \frac{Y-\mu}{\sigma}$. The density function of Z becomes

$$\pi(z; a, \gamma) = a\left[\frac{\gamma+1}{\gamma}\left\{\{exp(z)exp(-exp(z))\left(-exp(-\gamma exp(z))\right)\right\}\right]$$

$$\left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp(-exp(z))\right) - \frac{1}{\gamma+1}\left(1 - exp(-(1+\gamma)exp(z))\right)\right\}\right]^{a-1} \quad (7)$$

The corresponding cumulative distribution function (cdf) is

$$F_z{}^{(Z)} = I_{\left[\frac{\gamma+1}{\gamma}\left\{\left(1-exp\left(-exp\left(\frac{y-\mu}{\sigma}\right)\right)\right)-\frac{1}{\gamma+1}\left(1-exp\left(-(1+\gamma)exp\left(\frac{y-\mu}{\sigma}\right)\right)\right)\right\}\right]}(\,a).$$

The basic $a = 1$ associated to the standardized weighted weibull distribution.

3.1 Linear Combination

By expanding the binomial term in (7), we can write

$$\pi(z; a, \gamma) = a\sum_{j=0}^{\infty}(-1)^j\binom{a-1}{j}\left[\frac{\gamma+1}{\gamma}\left\{\{exp(z)exp\left(-exp(z)\right)\left(-exp\left(-\gamma exp(z)\right)\right)\right\}\right]\left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp(z)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)exp(z)\right)\right)\right\}\right]^{(j+1)-1} \quad (8)$$

The density function $h_a = (a-1)\left[\frac{\gamma+1}{\gamma}\left\{\{exp(z)exp\left(-exp(z)\right)\left(-exp\left(-\gamma exp(z)\right)\right)\right\}\right]$ for $a > 0$ gives EWW (Badmus and Bamiduro, 2014) and Ortega *et al.*, 2013) and its corresponding cumulative function is

$$H_a^{(x)} = 1 - \left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp(z)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)exp(z)\right)\right)\right\}\right]^a$$

Then, $\pi(z;\ a,\gamma) = \sum_{j=0}^{\infty} k_j h_{(j+1)}(z)$, where the coefficients are $k_j = \frac{(-1)^j\binom{a-1}{j}}{(j+1)\,(a)}$

3.2 Moments and Generating Function

The sth ordinary moment of the LEWW distribution (7) is

$$\mu_s' = E(Z^s) = a\int_{-\infty}^{\infty} Z^s\left[\frac{\gamma+1}{\gamma}\left\{\{exp(z)exp\left(-exp(z)\right)\left(-exp\left(-\gamma exp(z)\right)\right)\right\}\right]\left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp(z)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)exp(z)\right)\right)\right\}\right]^{a-1} dz$$

We expanding the binomial term and setting $u = e^z$, we get

$$\mu_s' = a\sum_{j=0}^{\infty}(-1)^j\binom{a-1}{j} I_{(r,(j+1))} \quad (9)$$

(9) gives the moments of the LEWW distribution and other measures i.e skewness and kurtosis can be estimated from the ordinary moments using well known relationship and the measures are mainly controlled by the parameter of a.

The moment generating function (mgf) of Z, say $M(t) = E(e^{tz})$, follows from (7) as

$$M(t) = a\sum_{j=0}^{\infty}(-1)^j\binom{a-1}{j}\int_0^{\infty} U^t\left\{a\left[\frac{\gamma+1}{\gamma}\left\{\{exp(u)exp\left(-exp(u)\right)\left(-exp\left(-\gamma exp(u)\right)\right)\right\}\right]\left[\frac{\gamma+1}{\gamma}\left\{\left(1 - exp\left(-exp(u)\right)\right) - \frac{1}{\gamma+1}\left(1 - exp\left(-(1+\gamma)exp(u)\right)\right)\right\}\right]^{(j+1)-1} u\right\} du$$

Then,

$$M(t) = \frac{\Gamma(t+1)}{a}\sum_{j=0}^{\infty}(-1)^j\binom{a-1}{j}\left[\left((j+1)-1\right)\right]^{-(t+1)} \quad (10)$$

Meanwhile, the moment (9) can be derived from (10) using simple differentiation.

4.0 The Log-Exponentiated Weighted Weibull (LEWW) Regression Model.

Based on LEWW distribution function, the linear location-scale regression model linking the dependent y_i and the independent variables vector $X_i^T = \left(x_{i1}, \dots \dots .., x_{ip}\right)$ are as follows:

$$y_i = X_i^T\beta + \sigma z_i, \qquad i = 1, \dots, n \quad (11)$$

Where the residual error z_i has distribution function (7), $\left(\beta_1, \dots \dots .., \beta_p\right)^T$ σ, a and γ are greater than zero and unknown parameters. The location of $y_i\ is\ \mu_i = X_i^T\beta$ parameter. The location parameter vector $\mu = (\mu_1, \dots .., \mu_n)^T$ is represented by a linear model $\mu = X\beta$ where $X = (X_1, \dots \dots, X_n)^{\mathrm{T}}$ is a known model matrix; and the LEWW model (11) gives possibilities for fitting skewed data

4.1 Maximum Likelihood Estimation

The likelihood function for the vector of parameters $\theta = (a, \gamma, \sigma, \beta^T)^T$ from model (11) has the form $l(\theta) = \sum_{i\epsilon F} log[f(y_i)] + \sum_{i\epsilon C} log[s(y_i)]$, $f(y_i)$ is the density function (5) and $S(y_i)$ is the survival function (6) of Y_i.

The log-likelihood function for θ is given as:

$$l(\omega) = -rlog\{log(\sigma) + log[a]\} + \left[\frac{\gamma+1}{\gamma}\left\{\sum_{i\epsilon F}(z_i) + \sum_{i\epsilon F}\left(-exp(z_i)\left(1 - exp\left(-\gamma exp(z_i)\right)\right)\right)\right\}\right]$$
$$+(a-1)\sum_{i\epsilon F} log\left[\frac{\gamma+1}{\gamma}\sum_{i\epsilon F}\left\{\left(1 - exp\left(-exp(z_i)\right)\right) - \frac{1}{\gamma+1}\sum_{i\epsilon F}\left(1 - exp\left(-(1+\gamma)exp(z_i)\right)\right)\right\}\right]$$
$$+\sum_{i\epsilon C} log\left\{1 - I_{\left[\frac{\gamma+1}{\gamma}\sum_{i\epsilon C}\left\{\left(1-exp(-exp(z_i))\right) - \frac{1}{\gamma+1}\sum_{i\epsilon C}\left(1-exp\left(-(1+\gamma)exp(z_i)\right)\right)\right\}\right]}\right\}^{(a)} \quad (12)$$

where, $z_i = \frac{(y - X_i^T\beta)}{\sigma}$.

The MLE $\widehat{\omega}$ of the vector ω can be calculated by maximizing the likelihood (12). The LEWW model survival function of Y for any individual with explanatory vector **x** is given as:

$$S\left(y;\ \hat{a}, \hat{\gamma}, \widehat{\sigma},\ \widehat{\beta}^T\right) = 1 - I_{\left[\frac{\hat{\gamma}+1}{\hat{\gamma}}\left\{\left(1-exp\left(-exp\left(\frac{y-X^T\widehat{\beta}}{\widehat{\sigma}}\right)\right)\right) - \frac{1}{\hat{\gamma}+1}\left(1-exp\left(-(1+\hat{\gamma})exp\left(\frac{y-X^T\widehat{\beta}}{\widehat{\sigma}}\right)\right)\right)\right\}\right]}^{(\hat{a})} \quad (13)$$

The invariance property of the MLEs yields the survival function for T= exp (Y)

$$S\left(y;\ \hat{b}, \hat{\alpha}, \widehat{\beta},\ \hat{\lambda}\right) = 1 - I_{\left[\frac{\hat{\alpha}+1}{\hat{\alpha}}\left\{\left(1-exp\left(-\frac{t}{\hat{\lambda}}\right)^{\widehat{\beta}}\right) - \frac{1}{\hat{\alpha}+1}\left(1-exp\left(-(1+\hat{\alpha})\left(\frac{t}{\hat{\lambda}}\right)^{\widehat{\beta}}\right)\right)\right\}\right]}^{(\hat{a})} \quad (14)$$

where $\widehat{\sigma} = \frac{1}{\hat{\alpha}}$ and $\widehat{\beta} = exp\left(X^T\widehat{\beta}\right)$.

5.0 Data Analysis

Data set on the survival times, the times (in years) it takes to complete Ph.D. program at the University of Ibadan (Olubsoye and Olusoji, 2014) was employed in this study. The data contains 187 candidates who have completed their Ph.D.program at the university. The variables involved in this study are presented in Table 1.

Table 1: Variables and their categories

Variable	Category and their assigned value
y_i - duration	(years each candidate spent on the program)
x_{i1} – Status of the supervisor	(Professor = 3, Reader = 2, Senior Lecturer (SL) = 1, N/A = 0)
x_{i2} – Student's employment status	(full empl = 2, self empl = 1, N/A = 0)
x_{i3} − marital status	(married = 2, single = 1, N/A = 0)
x_{i4} − age	Age of each candidate at the start of the program
x_{i5} − faculty	(Arts = 0, Agric. Forestry = 7, FBMS = 4, Inst. of Edu = 2, Pharmacy = 8, Public health = 6, Science = 5, Social Science = 9, Technology = 3 and Vet. Med.= 1).

This implies that the purpose of the analysis is to investigate the significance of the factors mentioned above using the estimate of the model parameters either positive or negative, the standard error, the p-values, the model with lowest AIC and BIC values and the performance of the model parameters (the best-fitted model).

The LEWW regression model is given as

$$duration_i = \beta_0 + \beta_1 Super + \beta_2 Emp.Stat + \beta_3 Mar.Stat + \beta_4 age + \beta_5 faculty + \sigma z_i, \quad (15)$$

$i = 1, \ldots, 187.$ where the random variable $duration_i$ has the LEWW distribution. Below are some graphs of the original data.

Table 2: Descriptive Statistics of the Duration Data (Time to Completion)

Min	Q_1	Median	Mean	Q_3	Max	Skewness	Kurtosis
4.00	7.000	9.000	9.112	10.500	21.000	0.9373726	5.4713

Table 3: MLEs of the parameters for both proposed and conventional regression models fitted to the Phd data set, the corresponding SEs (given in parentheses), p-value in [.], log-likelihood and the statistics AIC and BIC.

Parameter/Model	LEWWReg	LWWReg	LWeigReg	LWeibReg	WeibReg
a	1.00, (0.29)				
	[<2e-16 ***]	---	---	---	---
b	---	---	---	---	---
γ	2.10, (0.00)	0.13, (0.01)	0.02, (0.00)		
	[<2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	---	---
σ	5.95, (0.07)	5.10, (0.17)	8.09, (8.03)	4.25, (0.00)	-1.28, (0.05)
	[<2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	[0.00]
β_0	-2.83, (0.02)	3.40, (0.62)	3.10, (1.03)	1.50, (0.08)	2.08, (0.14)
	[<2e-16 ***]	[0.00 ***]	[0.00 **]	[< 2e-16 ***]	[0.00]
β_1	-2.83, (0.06)	3.16, (0.21)	-1.40, (0.68)	1.45, (0.25)	0.04, (0.02)
	[<2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	[0.00 ***]	[0.13]
β_2	-2.82, (0.03)	3.21, (0.30)	-1.14, (1.10)	3.10, (0.16)	0.09, (0.03)
	[<2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	[0.00]
β_3	-2.82, (0.06)	4.87, (0.51)	1.06, (0.43)	3.50, (0.34)	0.00, (0.05)
	[<2e-16 ***]	[< 2e-16 ***]	[0.01*]	[< 2e-16 ***]s	[0.98]
β_4	2.85, (0.10)	1.17, (0.05)	1.21, (0.09)	1.02, (0.02)	0.00, (0.00)
	[<2e-16 ***]	[< 2e-16 ***]	[< 2e-16 ***]	{< 2e-16 ***]	[0.23]
β_5	6.05, (0.15)	5.10, (0.30)	-3.56, (1.12)	0.51, (0.22)	-0.02, (0.01)
	[<2e-16 ***]	[< 2e-16 ***]	[0.00 **]	[0.02 *]	[0.03]
$Loglik$	795888.70	55305.57	126411.50	243722.20	448.60
AIC	-1591759	-110595.10	-252807	-487430.40	-911.29
BIC	-1591757	-110593	-252804.80	-487428.50	-933.91

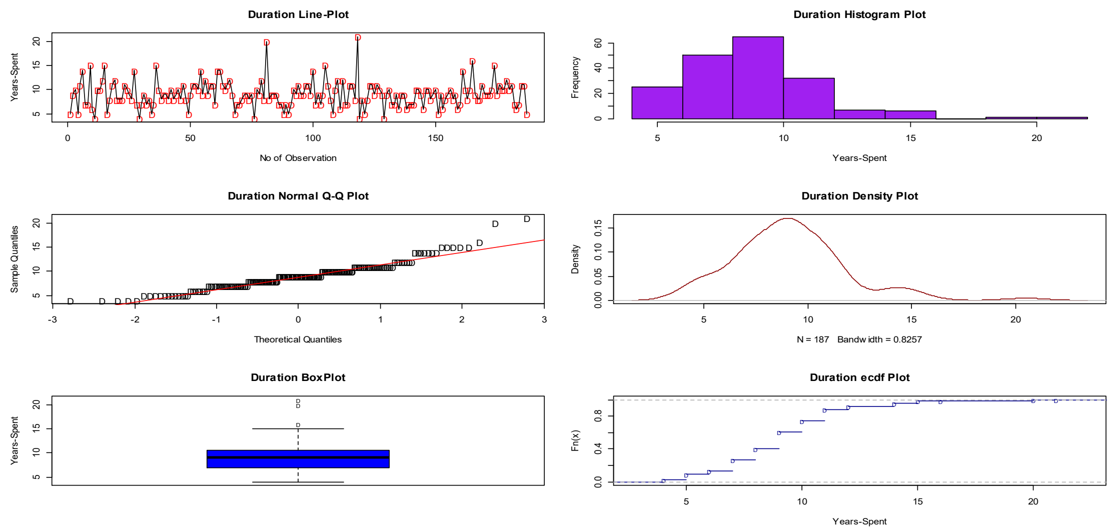

Figure 2: Plots of Time Spent by the Candidates to complete their Ph.D. program

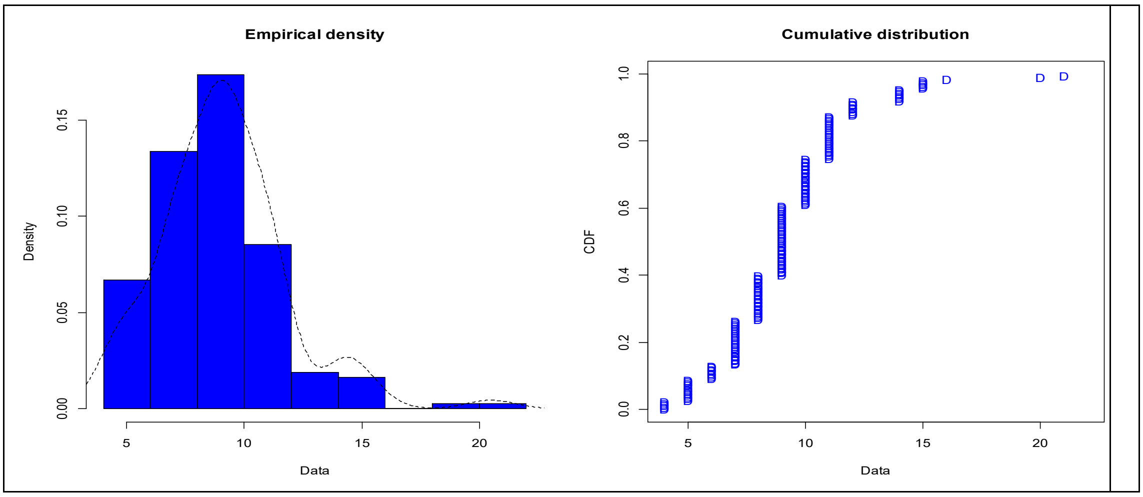

Figure 3: The empirical density and cdf for the data set.

6.0 Results and Concluding Remarks

Figure 2 above consists of some graphs of the data used. i.e the scatter and line plot, QQ plot, plot to detect outlier, the histogram, the density and the ecdf of duration. The empirical density and cumulative distribution of duration (y) are also included, while figure 1 is the probability density function (pdf) plot of LEWW distribution.

Table 1 shows descriptive statistics with some measures of tendency. Thereafter, Table 2 contains the Maximum Likelihood Estimates (MLEs) of the parameters of the proposed model and sub-models. Therefore, LEWWReg is compared with some special sub-models such as LWWReg and LWReg and conventional regression models the LWeibullReg and WeibullReg. Meanwhile, from the estimates and values in Table 2, the p-values of the LEWW regression model show that even at 0% the factors are significant to the completion of the program but not all the p-values in the other models show that the factors significantly tend toward 0% and others show otherwise.

The most preferable model is determined through the log-likelihood and model selection criteria. In essence, that model with a larger value of log-likelihood and a smaller value of AIC and BIC has a better fit. Fortunately, the results in the table above clearly and indicate that the proposed model best fitted the data than all the existing ones. Thus, (15) is stated as follows:

$$duration = -2.83 - 2.83Super - 2.82Emp.Stat - 2.82Mar.stat + 2.85age + 6.01Faculty$$

The estimated parameters of the regression model of the proposed model in Table 2 above are positive and negative values. There will be a decrease of -2.83 in duration for a unit change in supervisor when other variables are held fixed, a decrease of -2.82 in duration for a unit change in employment status when other variables are held fixed. Then, there would be a decrease of -2.82 in duration for a unit change in marital status when other variables are held fixed, also an increase of 2.85 in duration for a unit change in age when other variables are held fixed. Lastly, an increase of 6.01 in duration for a unit change in faculty when other variables are also held fixed

However, the proposed model can be applied to any survival data in any field such as biology, medicine, engineering, credit risk modeling in finance, quality control, economics, etc. since it represents a parametric family of models that includes as special sub-models and representation of data, flexibility and applicability.

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