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Original Research Article

Exponentiated Transmuted Inverse Exponential Distribution with Application

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Abstract

In this article, a two-parameter Transmuted Inverse Exponential Distribution (TIED) was generalized to its three-parameter variant entitled Exponentiated Transmuted Inverse Exponential Distribution (ETIED) which is more flexible than the existing TIED. Mathematical expressions for its Moments, Moment Generating Function (MGF), survival function, hazard function and the Quantile Function of the proposed model were presented. The parameters of the proposed distribution were estimated using the method of maximum likelihood. Real-life datasets were considered for making the comparison and the performance of the competing models were rated based on their Akaike Information Criteria (AIC) value.

Keywords: Exponentiated Transmuted Exponential Distribution, Moments, Moment generating function, Quantile function.

1.0 Introduction

Numerous generalized probability distributions have been proposed and extensively used in statistics for modeling real datasets. This therefore, creates an opportunity for proposing or developing new distributions which could provide greater flexibility and wider acceptability in modeling of real-lifetime datasets.

Oguntunde and Adejumo (2015) studied a two-parameter model called Transmuted Inverse Exponential Distributions by using a quadratic rank transmutation map. The also derived its statistical properties and highlighted its usefulness in modeling breast and bladder cancer data sets. Owoloko et al., (2015) investigated the performance rating of the Transmuted Exponential (TE) distribution concerning some other generalized models. The TE distribution appeared to be better than the Beta Exponential distribution and Generalized Exponential distribution also known as Exponentiated Exponential distribution in terms of flexibility when applied real-life data. Yahaya and Mohammed (2017) studied and derived some basic properties of Transmuted Kumaraswamy inverse Exponential Distribution.

Gupta and Kundu (1999) introduced the exponentiated exponential distribution as a generalization of the standard exponential distribution. Nadarajah and Kotz (2006) proposed, based on the same idea, four more exponentiated types of distributions to extend and derive the structural properties of the gamma, Weibull, Gumbel and Fréchet distributions. Nadarajah (2005) provides a comprehensive treatment of the basic properties of this new distribution and demonstrate its use for modeling rainfall data from Orlando, Florida. Among the mathematical properties, they derive the analytical shapes of the corresponding probability density function and the hazard rate function, calculate expressions for the nth moment and the asymptotic distribution of the extreme order statistics and investigate the variation of the skewness and kurtosis

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measures. Shirke and Kakade (2006) studied and proposed exponentiated log-normal and Nadarajah and Gupta (2007) for exponentiated gamma distributions.

This work aimed at developing Exponentiated Transmuted Inverse Exponential Distribution to increase the flexibility of the Transmuted Inverse Exponential Distribution to make it properly fit for data of various shapes that cannot be appropriately fitted with the existing distributions. Furthermore, the proposed distribution can be used in analyzing skewed datasets as well as in reliability analysis.

2.0 Material and Methods

The Exponentiated family of distributions is derived by raising the cumulative density function (cdf) of an arbitrary parent distribution by a shape parameter say $\theta > 0$; Its cdf is given by;

$$F(x) = (G(x))^{\theta} \tag{1}$$

By differentiating $F(x)$ in (1) with respect to x, the corresponding probability density function (pdf) is given by;

$$f(x) = \theta g(x)(G(x))^{\theta-1} \tag{2}$$

But the pdf and cdf of Transmuted Inverse Exponential distribution are given in (3) and (4) respectively (Oguntunde and Adejumo, 2015) as;

$$g(x) = \frac{\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right) \qquad for\ \alpha > 0\ and -1 \leq \lambda \leq 1 \tag{3}$$

$$G(x) = e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right) \tag{4}$$

Thus, from equations (1) and (2), the cdf and pdf of the Exponentiated Transmuted Inverse Exponential Distribution (ETIED) are obtained respectively as;

$$F(x) = \left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)^{\theta}$$

$$f(x) = \frac{\theta\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)^{\theta-1}$$

$$f(x) = \frac{\theta\alpha}{x^2} e^{-\frac{\alpha\theta}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right)^{\theta-1} \quad for\ \alpha,\theta > 0\ and -1 \leq \lambda \leq 1$$

But,

$$\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right) = \sum_{j=0}^{\infty}\binom{\theta-1}{j}\lambda^j\left(1-e^{-\frac{\alpha}{x}}\right)^j = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j e^{-\frac{\alpha j}{x}}\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)} \tag{5}$$

by substituting (5) into (4) we have,

$$f(x) = \frac{\theta\alpha}{x^2} e^{-\frac{\alpha\theta}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j e^{-\frac{\alpha j}{x}}\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)} \tag{6}$$

$$f(x) = \frac{\theta\alpha}{x^2}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)} e^{-\frac{\alpha(\theta+j)}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right) \tag{7}$$

By choosing some arbitrary values for parameters: we provide some possible shape for the cdf and pdf of the ETIED as shown in Figures 1 and 2:

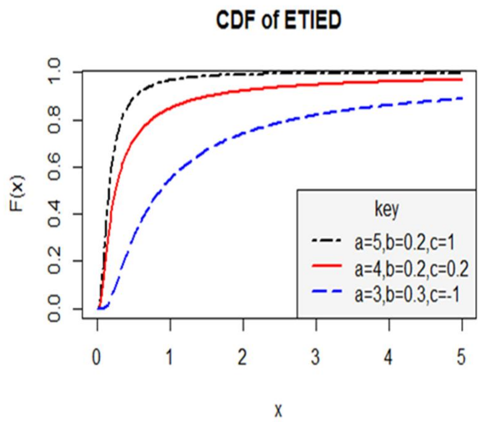

Figure 1: The cdf plot of ETIED

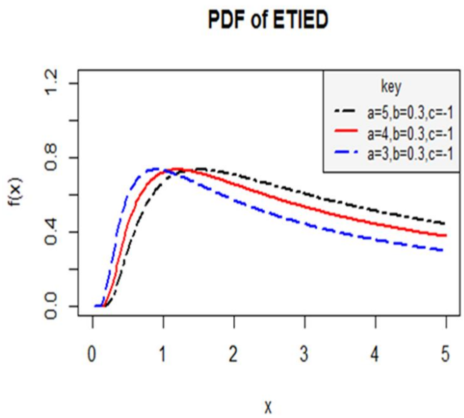

Figure 2: The pdf plot of ETIED

3.0 Moments

Lemma: the r^{th} moment of ETIED is given by;

$$\mu_r' = m\alpha^r\,\Gamma(1-r)[(1+\lambda)\,(\theta+j)^{r-1} - 2\lambda(\theta+j+1)^{r-1}] \quad (8)$$

$$where, m = \theta \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)}$$

Proof:

$$\mu_r' = \int_{-\infty}^{\infty} x^r f(x)dx$$

$$\mu_r' = \int_0^{\infty} x^r \left\{\frac{\theta\alpha}{x^2}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)} e^{-\frac{\alpha\,(\theta+j)}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\right\} dx$$

$$\mu_r' = \theta\alpha\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)} \int_0^{\infty} x^{r-2} e^{-\frac{\alpha\,(\theta+j)}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right) dx$$

$$let, m = \theta \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{\lambda^j\,\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)}$$

$$\mu_r' = m\alpha\left\{(1+\lambda)\int_0^{\infty} x^{r-2} e^{-\frac{\alpha\,(\theta+j)}{x}} dx - 2\lambda\int_0^{\infty} x^{r-2} e^{-\frac{\alpha\,(\theta+j+1)}{x}} dx\right\}$$

$$\mu_r' = m\alpha\left\{(1+\lambda)\int_0^{\infty}\left(\frac{\alpha(\theta+j)}{y}\right)^{r-2}\frac{\alpha(\theta+j)}{y^2} e^{-y}dy - 2\lambda\int_0^{\infty}\left(\frac{\alpha(\theta+j+1)}{y}\right)^{r-2}\left(\frac{\alpha(\theta+j+1)}{y^2}\right) e^{-y}dy\right\}$$

$\mu_r' = m\alpha\left\{(1+\lambda)\alpha^{r-1}(\theta+j)^{r-1}\int_0^\infty y^{(1-r)-1}e^{-y}dy - 2\lambda\alpha^{r-1}(\theta+j+1)^{r-1}\int_0^\infty y^{(1-r)-1}e^{-y}dy\right\}$

$\mu_r' = m\alpha\{(1+\lambda)\alpha^{r-1}(\theta+j)^{r-1}\Gamma(1-r) - 2\lambda\alpha^{r-1}(\theta+j+1)^{r-1}\Gamma(1-r)\}$

$\mu_r' = m\alpha^r\Gamma(1-r)\{(1+\lambda)(\theta+j)^{r-1} - 2\lambda(\theta+j+1)^{r-1}\}$

3.1 Moment Generating Function

Lemma: the moment generating function (mgf) of ETIED is given as:

$$M_x(t) = c\alpha^l\Gamma(1-l)\{(1+\lambda)(\theta+j)^{l-1} - 2\lambda(\theta+j+1)^{l-1}\} \quad (9)$$

$where, c = \Gamma(\theta+1)\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^k\frac{\lambda^j t^l\,\Gamma(j+1)}{j!k!l!\,\Gamma(j-k+1)}$

Proof:

$M_x(t) = \int_0^\infty e^{tx}\left\{\frac{\theta\alpha}{x^2}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k\frac{\lambda^j\Gamma(\theta)\,\Gamma(j+1)}{j!k!\,\Gamma(j-k+1)}e^{-\frac{\alpha(\theta+j)}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\right\}dx$

$M_x(t) = \Gamma(\theta+1)\alpha\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^k\frac{\lambda^j t^l\,\Gamma(j+1)}{j!k!l!\,\Gamma(j-k+1)}\int_0^\infty x^{l-1}\left\{e^{-\frac{\alpha(\theta+j)}{x}}\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)\right\}dx$

$where, c = \Gamma(\theta+1)\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^k\frac{\lambda^j t^l\,\Gamma(j+1)}{j!k!l!\,\Gamma(j-k+1)}$

$M_x(t) = c\alpha\left\{(1+\lambda)\int_0^\infty x^{l-2}e^{-\frac{\alpha(\theta+j)}{x}}dx - 2\lambda\int_0^\infty x^{l-2}e^{-\frac{\alpha(\theta+j+1)}{x}}dx\right\}$

$M_x(t) = c\alpha\left\{(1+\lambda)\int_0^\infty\left(\frac{\alpha(\theta+j)}{y}\right)^{l-2}\frac{\alpha(\theta+j)}{y^2}e^{-y}dy - 2\lambda\int_0^\infty\left(\frac{\alpha(\theta+j+1)}{y}\right)^{l-2}\left(\frac{\alpha(\theta+j+1)}{y^2}\right)e^{-y}dy\right\}$

$M_x(t) = c\alpha\left\{(1+\lambda)\alpha^{l-1}(\theta+j)^{l-1}\int_0^\infty y^{(1-l)-1}e^{-y}dy - 2\lambda\alpha^{l-1}(\theta+j+1)^{l-1}\int_0^\infty y^{(1-l)-1}e^{-y}dy\right\}$

$M_x(t) = c\alpha\{(1+\lambda)\alpha^{l-1}(\theta+j)^{l-1}\Gamma(1-l) - 2\lambda\alpha^{l-1}(\theta+j+1)^{l-1}\Gamma(1-l)\}$

$M_x(t) = c\alpha^l\Gamma(1-l)\{(1+\lambda)(\theta+j)^{l-1} - 2\lambda(\theta+j+1)^{l-1}\}$

3.2 Survival Function

The survival function is the probability that a system or an individual will survive beyond a given time.

Mathematically, the survival function is given by:

$S(x) = 1 - F(x)$

where $F(x)$ is cdf of the proposed distribution.

$$S(x) = 1 - \left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)^\theta \quad (10)$$

By choosing some arbitrary values for parameters: $\theta = a, \alpha = b \; and \; \lambda = c$ we provide some possible shape for the survival function of the ETIED as shown in Figure 3:

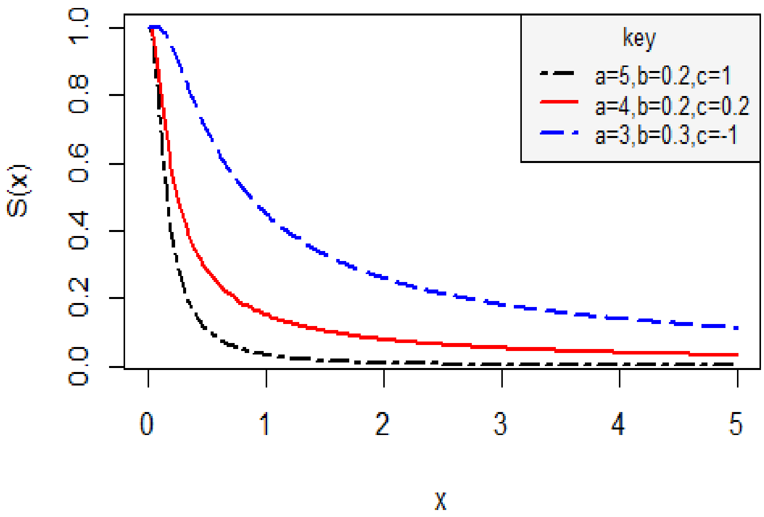

Figure 3: The survival plot of ETIED.

3.3 Hazard Function

The hazard function is also called the failure or risk function and is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x) = \frac{f(x)}{1-F(x)}$$

$$h(x) = \frac{\frac{\theta\alpha}{x^2} e^{-\frac{\alpha\theta}{x}} \left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right) \left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right)^{\theta-1}}{1-\left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)^{\theta}} \qquad (11)$$

By choosing some arbitrary values for parameters: $\theta = a, \alpha = b \; and \; \lambda = c$ we provide some possible shape for the survival function of the ETIED as shown in Figure 4:

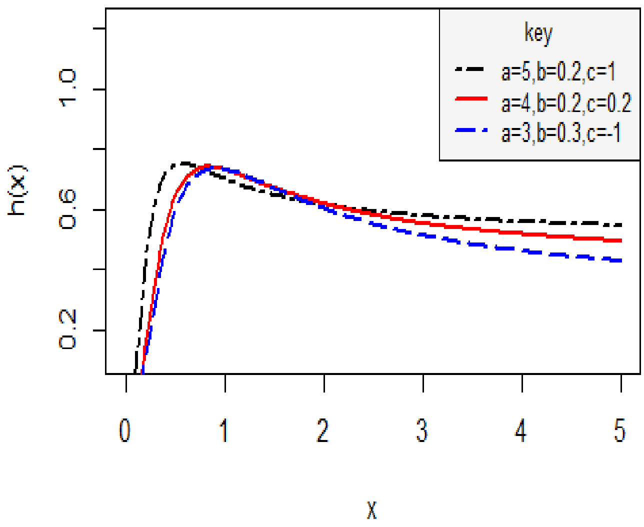

Figure 4: The hazard plot of ETIED.

3.4 Quantile Function

The quantile function of ETIED is given as;

$$x_u = \alpha\left[ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)^{-1}\right]^{-1} \tag{12}$$

Proof:

$$u = \left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)^{\theta}$$

$$u^{1/\theta} = \left(e^{-\frac{\alpha}{x}}\left(1+\lambda-\lambda e^{-\frac{\alpha}{x}}\right)\right)$$

$$u^{1/\theta} = (1+\lambda)e^{-\frac{\alpha}{x}} - \lambda e^{-\frac{2\alpha}{x}}$$

$$\lambda e^{-\frac{2\alpha}{x}} - (1+\lambda)e^{-\frac{\alpha}{x}} + u^{1/\theta} = 0$$

$$e^{-\frac{\alpha}{x}} = \frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}$$

$$-\frac{\alpha}{x}=ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)$$

$$\frac{\alpha}{x}=-ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)$$

$$\frac{\alpha}{x}=ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)^{-1}$$

$$\alpha=x\ ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)^{-1}$$

$$x_u=\frac{\alpha}{ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)^{-1}}$$

$$x_u=\alpha\left[ln\left(\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u^{1/\theta}}}{2\lambda}\right)^{-1}\right]^{-1}$$

4.0 Estimation of Parameters of the ETIED

The estimation of the parameters of the ETIED is done by using the method of maximum likelihood estimation. Let $X_1, X_2, \dots, X_n$ be a random sample from the ETIED with unknown parameter vector $\Omega=(\theta,\alpha,\lambda)^T$. The total log-likelihood function for Ω is obtained from $f(x)$ as follows:

$$lnL(\Omega)=nln\theta+nln\alpha-2\sum_{i=1}^{n}lnx_i-\theta\alpha\sum_{i=1}^{n}\frac{1}{x_i}+\sum_{i=1}^{n}ln\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)+(\theta-1)\sum_{i=1}^{n}ln\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right) \quad (13)$$

So, differentiating $lnL(\Omega)$ partially with respect to each of the parameter (θ,α,λ) and setting the results equal to zero gives the maximum likelihood estimates of the respective parameters. The partial derivatives of $lnL(\Omega)$ with respect to each parameter or the score function is given by:

$$\frac{\delta lnL(\Omega)}{\delta\theta}=\frac{n}{\theta}-\alpha\sum_{i=1}^{n}\frac{1}{x_i}+\sum_{i=1}^{n}ln\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right) \quad (14)$$

$$\frac{\delta lnL(\Omega)}{\delta\alpha}=\frac{n}{\alpha}-\theta\sum_{i=1}^{n}\frac{1}{x_i}+2\lambda\sum_{i=1}^{n}\frac{\frac{1}{x_i}e^{-\frac{\alpha}{x}}}{\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)}+\lambda(\theta-1)\sum_{i=1}^{n}\frac{\frac{1}{x_i}e^{-\frac{\alpha}{x}}}{\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right)}=0 \quad (15)$$

$$\frac{\delta lnL(\Omega)}{\delta\lambda}=\sum_{i=1}^{n}\frac{1-2e^{-\frac{\alpha}{x}}}{\left(1+\lambda-2\lambda e^{-\frac{\alpha}{x}}\right)}+(\theta-1)\sum_{i=1}^{n}\frac{1-e^{-\frac{\alpha}{x}}}{\left(1+\lambda\left(1-e^{-\frac{\alpha}{x}}\right)\right)}=0 \quad (16)$$

Hence, the MLE is obtained by solving this nonlinear system of equations. Solving this system of nonlinear equations is complicated, we can therefore use statistical software to solve the equations numerically.

5.0 Application to real dataset

The flexibility of Exponentiated Transmuted Inverse Exponential Distribution is illustrated in application to a real dataset. The fits of ETIED will be compared with those of the three baseline distributions which include Transmuted Inverse Exponential Distribution (TIED), Inverse Exponential Distribution (IED). The analyses were performed with the aid of R software.

Data Set: The dataset represents the Vinyl Chloride data (in mg/l) that was obtained from clean up gradient monitoring wells. Thess data had been previously used by Bhaumik et al., (2009), Shanker (2015) and Oguntunde (2017). The data are as given below;

5.1, 1.2, 1.3, 0.6, 0.5, 2.4, 0.5, 1.1, 8.0, 0.8, 0.4, 0.6, 0.9, 0.4, 2.0, 0.5, 5.3, 3.2, 2.7, 2.9, 2.5, 2.3, 1.0, 0.2, 0.1, 0.1, 1.8, 0.9, 2.0, 4.0, 6.8, 1.2, 0.4, 0.2.

Table 1: Summary of the data on Vinly Chloride

Min.	Max.	Mean	Variance	Skewness	Kurtosis
0.100	8.000	1.879	3.8126	1.6037	5.005

Table 2: The performances of the various models with standard errors in parenthesis

No.	Distribution	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\lambda}$	Log-Likelihood	AIC
1	ETIE	0.07182 (0.01267)	4.753 (0.09065)	-1.000 (0.000009)	-56.39617	118.7923
2	TIE	0.4138 (0.1089)	-	-0.6301 (0.3078)	-57.9202	119.8000
3	IE	0.5725 (0.0982)	-	-	-59.1930	120.4000

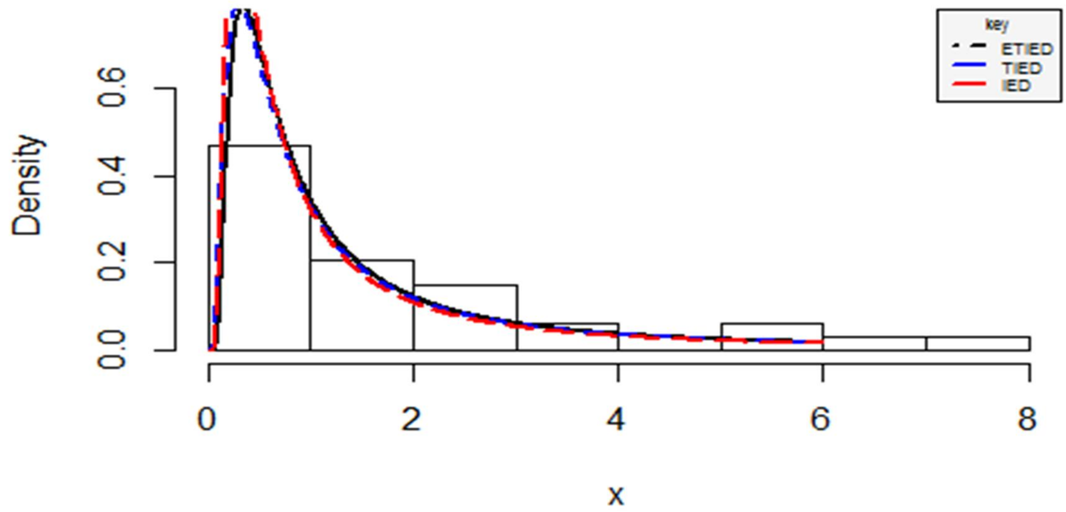

Figure 5: The density plots of the empirical data and the fitted Models

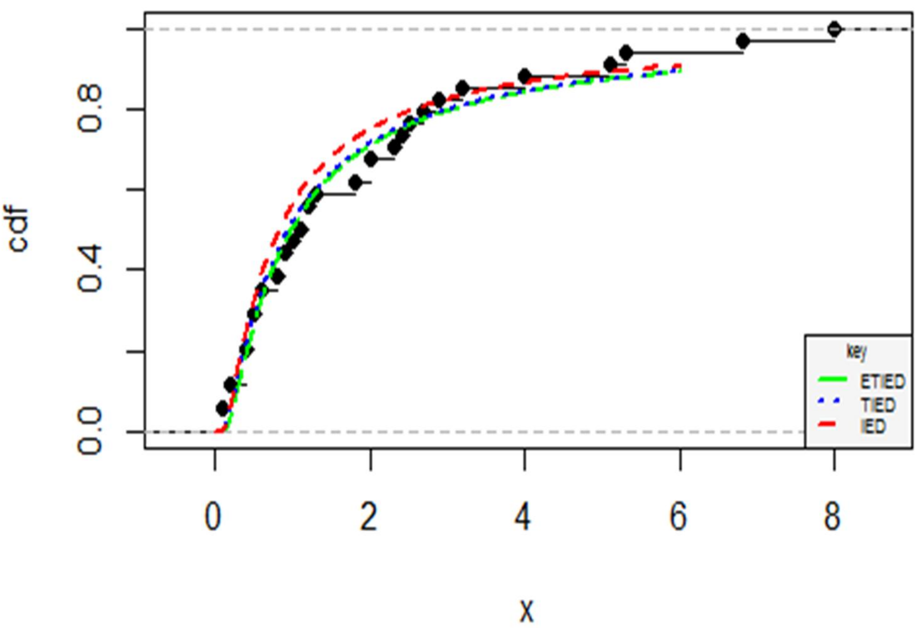

Figure 6: The plot of the distribution functions of the fitted model to real-life data.

5.1 Conclusion

We have successfully defined a new three-parameter model called Exponentiated Transmuted Inverse Exponential Distribution (ETIED). The distribution is positively skewed. An explicit expression for its statistical properties was derived. We observed that the moment for the ETIED only exists for $r<1$.

Exponentiated Transmuted Inverse Exponential Distribution possesses the least values of AIC on its fitting, to real-life data set. Hence ETIED will be considered as a best-fitted distribution to the data sets given in table-2 as compared to other special models.

Acknowledgments

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References

Bhaumik, D. K., Kapur, K. & R. D. Gibbons (2009). Testing Parameters of a Gamma Distribution for Small Samples, Technometrics, 51(3), 326-334.

Gupta, R. D. and Kundu, D. (1999). Generalized exponential distributions. *Australian and New Zealand Journal of Statistic,* 41, 173–188.

Nadarajah, S. (2005). The Exponentiated Gumbel distribution with climate application. Environmetrics 17, 13-23.

Nadarajah, S. and Gupta, A. K. (2007). The exponentiated gamma distribution with application to drought data. Calcutta Statistical Association Bulletin 59, 29-54.

Nadarajah, S. and Kotz, S. (2006). The exponentiated type distributions. *Acta Applicandae Mathematicae,* 92, 97–111.

Oguntunde, P. E. and Adejumo, A. O. (2015). The Transmuted Inverse Exponential Distribution. *International journal of advanced statistics and probability*, 3(1): 1-7..

Oguntunde, P. E., Adejumo, A. O. and Owoloko, E. A. (2017). On flexibility of Transmuted Inverse Exponential Distribution. Proceedings of the World Congress on Engineering, Vol I WCE 2017, July 5-7, 2017, London, U.K.

Owoloko, E. A., Oguntunde, P. E. and Adejumo, A. O. (2015). Performance rating of the Transmuted Exponential distribution: an analytical approach. SpringerPlus 015-159D-6.

Shanker, R., Fesshaye, H. & Selvaraj, S. (2015). On Modeling Lifetimes Data Using Exponential and Lindley Distributions, Biometric and Biostatistics International Journal, 2(5), 00042.

Shirke, D. T. and Kakade, C. S. (2006). On exponentiated lognormal distribution. International Journal of Agricultural and Statistical Sciences 2, 319-326.

Yahaya, A. and Mohammed, A. S. (2017). On the Transmuted Kumaraswamy Inverse Exponential Distribution and its properties. *Proceedings of the 1st International Conference of Nigeria Statistical Society*, Vol. 1: 26-29, edited by Nuamah et al. *www.nss.com.ng*