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A Meta Goal Programming Model with Weighted Achievement Function and Mixed-Integer Variables for Multi-Product Manufacturing Systems

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Abstract

In this paper, we present a Meta-Goal Programming (M-GP) model with mixed-integer variables for multi-product manufacturing systems. The M-GP model presented was formulated from a weighted Goal Programming structure. The meta goals considered in the model relate to the percentage sum of unwanted deviations, maximum percentage deviation and percentage of unachieved goals. The M-GP model presented was illustrated with data collected from a multi-product company. The objective was the attainment of three meta-goals - percentage maximum deviation from all goals to be at most 50%, maximum percentage deviation from profit and production target goals to be at most 20% and at most two of the production target goals should unsatisfied, from an initial set of goals which includes the profit goal, goals for each of six products of the company (Star, Gulder, Maltina, Fayrouz, Goldsberg, 33export) and distribution cost goal. The LINGO optimization software was used to obtain the solution in which five out of the production targets eight initial goals were achieved while all the three meta-goals were fully achieved.

Keywords: Goal Programming, Meta-GP model, LINGO Software, Mixed-integer variables, Optimal solutions.

1.0 Introduction

Goal Programming (GP) technique is one of the oldest and widely used multi-criterion decision-making models. The GP technique involves minimizing deviations from estimated targets of certain goals set by the management of a given system. There are many factors that can be constrained; the maximization of profit and minimization of cost in production industries in multi-criterion decision-making models, Acha and Nduaguibe (2012). Therefore, in the GP technique a certain function of the unwanted deviation variables is minimized, Ignizio (1983). The G.P technique has many variants. Among the many G.P variants, the Lexicographic variant and the weighted variant are the most widely used in the literature, Jones and El-Darzi (1995).

In the solution of a problem using the two earlier mentioned widely used GP variants, sensitivity analysis is carried out to take into account some feedback from the decision-maker if the initial solution is not entirely accepted. In many cases, in carrying out the sensitivity analysis the same GP variant is used while some of

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the parameters of the model are changed. In this case, the entire solution of the problem is based on the use of a single variant which of course has its limitations and also has a particular situation in which it is applied. The Meta-GP (M-GP) variant is a goal programming framework that provides a model not based upon a single variant but a mix of variants. It is seen as a GP model of an initial GP model. It enables the decision-makers to establish target values not only for the goals but for another criterion function.

GP presents a technique that can be used by the management of multi-product manufacturing systems to obtain near-optimal solutions for problems with multidimensional objectives in the attainment of estimated targets of their set goals. The M-GP model, when applied in such systems, enables management to be more flexible in expressing their actual preferences of set goals by means of setting meta-goals. These can be thought of as secondary goals derived from an original set of goals.

The M-GP variant has been applied in various areas of life like in industry, agriculture among others. The M-GP variant with three meta-goals was proposed by Uria, Caballero, Ruiz and Romero (2002). The M-GP model was formulated to establish target values for the weighted sum of unwanted deviation variables, the number of goals fully achieved, and for maximum deviation.

Many works have been carried out by researchers on the M-GP variant. Caballero, Ruiz, Uria and Romero (2006), formulated an M-GP approach within the interactive framework. The satisficing targets of the model were allocated to each attribute and the meta-achievement function was selected in accordance with the decision-makers' actual preferences. Also, an interactive Meta-goal programming model that overcomes the distributive decision making challenges that are faced by small and medium manufacturing enterprises engaged in collaborative manufacturing was proposed by Lin, Nagalingan, and Lin, (2007).

The model introduced the concept of the Meta-goal and interactive process to further enrich the performance of conventional GP models. Sakhdari and Sabuohi (2012), designed an optimal cropping pattern using Goal and Meta GP for agriculture in Neyshabour district. The objective was to meet the estimated target area for some crops for certain cropping patterns. Jones and Jimenez (2013), introduced two new meta-objectives into the extended GP framework. The number of unmet goals was the first meta-objective added while the measure of close to the pairwise comparison given by the decision-maker was the second meta-objective. These complemented the original two meta-objectives of the weighted sum of deviation and the maximal weighted deviation to provide a flexible four meta-objective framework.

The lexicographic and non-lexicographic representation of the framework was developed. The potential use of the meta-goal programming approach for solving multi-criteria de novo programming problems was explored by Zheng and Hoocine (2017). The objective of the De Novo programming was converted into meta-goals during the formulation following the Meta-GP technique to arrive at a satisfactory decision in the multi-objective decision making context. Sayed, Hamed, Ramadam, and Hosny, (2015) proposed a meta-goal programming benefit of doubt (MGP-BoD) methodology for setting weights of composite indicators. The methodology comprised two sets of goals and two meta-goals. It enhanced BoD discriminating power by eliminating all ties in confidence interval values and hence, country ranks.

In this paper, a mixed-integer variable Meta-goal programming model that can be used by multi-product manufacturing systems is formulated The model presented can be used by the management of multi-product systems in obtaining a satisficing solution for target values of their different set goals in a given period while considering the addition of meta-goals when the initial GP variant used could not give desirable solutions.

2.0 Research Methodology

2.1 The Meta Goal Programming with continuous variables.

The Meta-GP framework was proposed by Uria *et al.,* (2002). It is regarded as a GP model of an initial GP model. It is presented as follows;

Considering the general setting of a scenario with *s* goal and *m* additional constraints given as

$f_i(x) + n_i + p_i = t_i\ ,\ \ i = 1,2, \dots, s$ (1)

$g_i(x) \leq b_j,\ j = 1,2, \dots, m$ (2)

$x \in R^n$ (3)

where the functions $f_i(x)$ are concave and the function $g_i(x)$ are convex with x as a continuous variable. After minimizing the unwanted deviation variables with say the lexicographic or weighted variant and the result obtained are not considered acceptable by the decision-maker, we can incorporate to the GP model, new aspirations levels.

If the decision-maker is using the weighted option with the achievement function of the GP model taking the form

$h(n) = \sum_{i=1}^{s} w_i \frac{n_i}{t_i}$ (4)

where w_i represents a preferential weight and the deviation variables have been normalized by adding them among their corresponding target values. If the final solution presents values that are not desired by the decision-maker, then the decision-maker may want to give aspiration levels for the final values of the achievement function. This originates new sets of goals that are, in some sense, goals of the original goals which are called Meta-Goals (MGs). The following type of meta-goals was proposed by Uria *et al.,* (2002).

Type 1: The percentage sum of unwanted deviation variables cannot surpass a certain bound Q_1. Hence the constraint $\sum_{i=1}^{s} w_i \frac{n_i}{t_i} \leq Q_1$ (5)

is imposed in the model.

Type 2: The maximum percentage cannot surpass a certain bound Q_2. Hence the constraint

$Max_{\ i=1,\dots,s} \left\{w_i \frac{n_i}{t_i}\right\} \leq Q_2 \quad \Leftrightarrow \{w_i \frac{n_i}{t_i} - D \leq 0,\ \ D \leq\ Q_2 \quad ,\quad i = 1,2, \dots, s.$ (6)

Type 3: The percentage of unachieved goals cannot surpass a certain bound Q_3. This can be modelled by adding the following constraints;

$n_i - m_i y_i \leq 0 \qquad i = 1,2, \dots, s$ (7)

$\frac{\sum_{i=1}^{s} y_i}{s}\ \leq\ Q_3$ (8)

where y_i is a binary variable and M represents arbitrarily large values that the corresponding attributes cannot achieve. Consequently the value of $\sum_{i=1}^{s} y_i$ in the optimum solution measures the number of goals that have been fully achieved.

2.2 Formulation of the Meta-Goal Programming model with mixed-integer variables.

The M-GP model proposed by Uria *et al.,* (2002) is hereby presented with weighted achievement function and mixed-integer decision variables for multi-product systems.

Considering a weighted percentage model for a multi-product system with products given as $\bar{x}\ (x_1, x_2, \dots, x_n)$. Given that we have q original goals and 3 meta-goals (MGs) are considered which includes;

MG 1: Meta-goal relating to the percentage sum of unwanted variables for some/all goals.

MG 2 : Meta-goal relating to the maximum percentage deviation for some/all goals

MG 3 : Meta-goal relating to the maximum number of unachieved goals allowed.

The maximum bound of meta-goals one, two and three are given as $B_1, B_2 and\ B_3$ while the negative and positive deviations from the *i*th meta-goal are given as α_i and β_i respectively.

The Meta-goal model is presented as follows;

$$Minimize\ d = w_1\left(\frac{\beta_1}{Q_1}+\frac{\alpha_1}{Q_1}\right) + w_2\left(\frac{\beta_1}{Q_1}+\frac{\alpha_1}{Q_1}\right) + w_3\left(\frac{\beta_1}{Q_1}+\frac{\alpha_1}{Q_1}\right) \quad (9)$$

Subject to

$\sum a_{kq}x_k \ + n_q - p_q = b_q$ ….. Goal constraints, $q = 1,2,\ldots,Q,\ k = 1,2,\ldots,n.$ (10)

$\sum_{j=1}^{m} c_{jk}\, x_k \ \ (\leq, =, \geq)\ g_j$ ……….. System constraints, $j = 1,2,\ldots,m$ (11)

$\sum_{q=1}^{Q} w_q\left[\frac{n_q}{k_q}+\frac{p_q}{k_q}\right] + \alpha_1 - \beta_1 \leq B_1$, for some/all q ……… Meta-goal 1 constraint (12)

$$w_q\frac{n_q}{k_q} - \lambda \leq 0 \quad (13)$$

$w_q\frac{p_q}{k_q} - \lambda \leq 0$ …… for either n_q or p_q for each goal q, …….Meta-goal 2 constraints (14)

$$\lambda + \alpha_2 - \beta_2 \leq Q_2 \quad , \quad \lambda \leq B_2 \quad (15)$$

$$n_q - My_q \leq 0 \quad (16)$$

$p_q - My_q \leq 0$ …… for either n_q or p_q in each goal q …….Meta-goal 3 constraints (17)

$$\sum_{q=1}^{Q} y_q + \alpha_3 - \beta_3 \leq B_3 \quad (18)$$

$x_1, x_2, \ldots, x_n\ \geq 0$ and integer,$n_q, p_q, k_q \geq 0 \quad \lambda,\ \alpha_i, \beta_i \geq 0 \quad , i = 1,2,3.$ (19)

$$y_q = \ 1\ ,\ If\ goal\ q\ is\ not\ satisfied \quad , for\ some\ some\ /all\ q \quad (20)$$

$$0\ ,\ otherwise \quad (21)$$

$Either\ \alpha_i\ or\ \beta_i = 0$, in the achievement function when not considered for a particular meta-goal.

with

k_q = Normalization constant associated with the *q*th goal.

a_{kq} = amount of contribution of qth goal on product x_k , $k = 1,2,\ldots,n.$

c_{jk} = amount of resource j necessary to manufacture one unit of product x_k .

g_j = total availability of the jth resource for product k.

n_q = negative deviational variable of the qth goal in the goal constraints.

p_q = positive deviational variable of the qth goal in the goal constraints.

b_q = estimated target level for qth goal.

λ = maximum deviation from all/some of the goals.

y_i = Binary variable associated with the number of unsatisfied goals allowed.

M = arbitrarily large values that the corresponding attributes cannot achieve.

3.0 Data illustration of the formulated Meta-goal programming model.

The Meta-goal programming model formulated above will be illustrated using data collected from a production factory of a multi-product manufacturing company, Nigeria Breweries PLC. The data collected are the average monthly estimated values for April 2017.

Table 1 presents the average expected monthly production quantity of the drinks- Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export while Table 2 presents the quantity of each raw material used in producing per unit of each drink. Table 3 on the other hand presents the estimated monthly profit per truckload(700 crate capacity) for each drink and estimated monthly distribution cost (fueling and drivers' allowance) per truckload for each of the considered drinks.

Table 1. Expected truckload per drink produced per month

Star	Gulder	Maltina	Fayrouz	Goldsberg	33 Export
506	250	72	220	397	265

Table 2. Quantity of each raw material used (in Kg) used per unit of drinks

Product	Malted sorghum	Malted barley	White sorghum	Sugar
Star	116	178	312	6
Gulder	59	91	159	0
Maltina	10	6	38	43
Fayrouz	0	38	0	138
Goldberg	0	114	282	0
33 export	0	92	221	0
Total available quantity	**129780**	**362556**	**707540**	**130950**

Table 3. Estimated Profit and distribution cost per truckload of drinks

Product	**Star**	**Gulder**	**Maltina**	**Fayrouz**	**Goldsberg**	**33 Export**	**TARGET VALUE**
Profit per truckload (in Naira)	159288.77	172518.3	165519.34	125128.6	127718.94	136818.7	250,000,000
Distribution cost per truckload (in Naira)	18144.09	18144.09	18144.09	18144.09	18144.09	18144.09	31,008,250

The objective is to minimize deviation from the estimated targets of the considered goals and meta-goals stated below subject to the raw materials' constraints.

The following goals are considered as the original goals of the company;

a. Minimization of the underachievement of the estimated monthly profit target in Naira
b. Minimization of the underachievement of the estimated monthly production target levels in truckload (700-crate capacity) for each of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export.

Minimization of the overachievement of the estimated monthly distribution cost in Naira.
This gives us 8 original goals set to be achieved. A weighted GP model as presented in the appendix was initially used in obtaining a solution that minimizes deviations from the estimated targets of the original goals. Since the results obtained with the weighted GP model were unsatisfactory, meta-goals were established in order to obtain a much more satisficing solution. The following meta-goals were considered;

i. MG 1: The percentage maximum deviation from all goals should be at most 50%.
ii. MG 2: The maximum percentage deviation from any among the profit and production targets of each drink's goals should be at most 30%.
iii. MG 3: In the production target for goals of each of the six different drinks, at most two should be unsatisfied.

Also in the weighted structure of the achievement function, the achievement of meta-goal two is considered to be twice as important as those of the other meta-goals. The system constraints considered include malted sorghum constraint, malted barley constraint, white sorghum constraint, and sugar constraint.

4.0 Results and Interpretation.

The result of the Goal programming problem of Nigeria Breweries solved with a weighted GP model was obtained using the optimization software LINGO as presented in table 4.

Table 4. Summary of LINGO solution of the weighted GP model

Goals	Target Level	Achieved value	% deviation	Goal achievement
Profit goal	250,000,000.00 Naira	249,998,454.7 Naira	0.0006%	Not achieved
Star Production target goal	506 trucks	931 trucks	83.6%	Achieved (Not realistic)
Gulder Production target goal	350 trucks	350 trucks	0%	Achieved
Maltina Production target goal	102 trucks	102 trucks	0%	Achieved
Fayrouz Production target goal	420 trucks	876 trucks	108.6%	Achieved (Not realistic)
Goldsberg Production target goal	70 trucks	114 trucks	62.9%	Achieved (Not realistic)
33 export Production target goal	264 trucks	264 trucks	0%	Achieved
Distribution cost goal	31,008,250 Naira	30,984,750 Naira	0.07579%	Achieved

Due to the unsatisfactory nature of the result evident in the large deviations from some goals' target values with as high as 108.6% deviation which is considered unacceptable for reasons among which includes production capacity of the factory, the M-GP model is therefore considered.
The result of the mixed-integer M-GP model with the weighted achievement function formulated in section 2 above was also solved using the LINGO optimization software solved and is presented in table 5.

From the results, which is for a case in which the second meta-goal is considered to be twice as important as the other two meta-goals, the solution is seen to be of a more balanced Chebyshev nature which is expected as more importance was placed on the second meta-goal which is a Chebyshev type goal.

The values for $\beta_1 = \beta_2 = \beta_3 = 0$ shows there was no positive deviation for any of the meta-goals targets which means the maximum target levels set for each of the meta-goals were not exceeded. The first meta-goal of maximum percentage deviation from all the goals not been more than 50% was also achieved as the value $\alpha_1 = 0.194$ signifies a 19.4% negative deviation from the maximum deviation of 50% which is we see as the sum of the deviations of the goals is 31.6%.

The value of $\lambda = 0.30$, from the second meta-goal, showed that the maximum percentage deviation from any of the profit and production target' goals that was desired not to be at most 30% was achieved as we can see in table 5.

Table 5. Summary of LINGO solution of M-GP model

	Goals	Target Level	Achieved value	% Deviation	Goal achievement
Original goals	Profit goal	200,000,000 Naira	198,947,420 Naira	0.5% (-ve deviation)	Not achieved
	Star Production target goal	506 trucks	506 trucks	0% (No deviation)	Achieved
	Gulder Production target goal	350 trucks	350 trucks	0 % (No deviation)	Achieved
	Maltina Production target goal	102 trucks	102 trucks	0 % (No deviation)	Achieved
	Fayrouz Production target goal	420 trucks	420 trucks	0 % (No deviation)	Achieved
	Goldsberg Production target goal	70 trucks	49 trucks	30% (-ve deviation)	Not achieved
	33 export Production target goal	264 trucks	264 trucks	0 % (No deviation)	Achieved
	Distribution cost goal	20,080,750 Naira	19,869,250 Naira	1.1% (-ve deviation)	Achieved
Meta goals	(1) Deviation for all goals ≤ 50%	≤ 50%	31.6% $\alpha_1 = 0.194$ $\beta_1 = 0$		Achieved
	(2) Deviation from profit and production target for each drink ≤ 20%	≤ 30%	30% $(Max.)$ $\alpha_2 = 0$ $\beta_1 = 0$ $\lambda = 0.3$		Achieved
	(3) At most 1 unsatisfied in production target goal.	≤ 1	1 $\alpha_3 = 0$ $\beta_1 = 0$		Achieved

Also the third meta-goal which is that the production target of at most one out of the six drinks should be unsatisfied was achieved as the production target of only one out of the five drinks was unsatisfied. We had $\alpha_3 = \beta_3 = 0$ meaning there was no deviation from this meta-goal.

In summary, six out of the eight original goals were met while all three meta-goals were satisfied. Also if we compare the solution of the meta GP model in table 5 with that of a weighted GP model before the meta-goals were considered as seen in the appendix and whose result is shown in table 4, we can see that the addition of the meta-goals made the solution more satisficing as the deviations in the weighted GP model are exceedingly large and probably unrealistic and undesirable and hence the importance of setting the meta-goals. Table 6 shows the quantity of each raw material used in the process

Table 6. Quantity of each raw materials used during the production process.

Item	Total Available Monthly Quantity	Slack Value	The quantity used in the process
Malted sorghum	129,780 kg	49414	80,366kg
Malted barley	362,556kg	189860	172,696kg
White sorghum	707,540kg	476324	231,216kg
Sugar	130,950kg	65568	65,382kg

5.0 Conclusions

The meta goal programming model is applied to problems in which the solutions obtained using the weighted options are not considered acceptable. Hence the addition of meta-goals to the initial model. This paper presents an M-GP model that can be used by decision-makers of multi-product systems as an algorithm for exploring, in a more comfortable way, a more satisficing solution using several GP variants at the same time in obtaining a solution for the set goals in a given period.

The M-GP approach was shown to be more flexible than the usual weighted GP model, as it allowed the decision-maker to establish target values not only for the original set of goals but for another set of goals of the initial goals i.e. meta-goals. This was evident in the results obtained from the illustration using data from Nigeria Breweries PLC.

The solution obtained using the weighted GP model gave results that were considered unsatisfactory and undesirable as there were very large deviations from the set targets even up to 108.6% for a particular production target goal. However when the meta-goals were added to the model we had a maximum deviation of 30% from a target value.

Hence the results obtained when the meta-goals were added gave a more satisficing solution as meta-objectives were all achieved with the percentage maximum deviation from all goals being 34.8% while the maximum percentage deviation from any among the profit and production target goals was 30%. This of course can be said to mean that the M-GP model helps let decision-makers have a means of obtaining acceptable solutions if the initially gotten solutions are unrealistic.

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