A Bayesian Multiple Change-point Analysis: an Application to Air Temperature and Rainfall Data

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***Abstract* — A Bayesian model was developed to detect multiple abrupt shift in the time series of annual mean of air temperature and rainfall for Ibadan metropolis. Air temperature and rainfall data for the period from 1990 to 2016 were collected from Department of Economics and Statistics, Cocoa Research Institute of Nigeria, Ibadan. Variations and trends of annual mean air temperature, annual mean rainfall were examined. The air temperature and rainfall datasets were modelled by Normal probability distribution using Bayesian hypothesis testing. Gaussian prior was assumed for the mean before and after the change-point and while Inverse Gamma prior was assumed for the variances of both the air temperature and rainfall to obtain the posterior distribution. Here, a triple hypothesis space concerning the annual mean of air temperature and annual mean of rainfall was considered: "a no change in the mean", "a single change in the mean" and "a double change in the mean". A Bayesian approach was formulated to demonstrate the posterior probability of each hypothesis and its relevant model parameters through Markov Chain Monte Carlo (MCMC) method which was updated through the use of Gibbs sampler. Thus, the marginal posteriors for means, variance and change-point were derived using Gibbs sampling approach. The method adopted detected a major change-point in mean annual around the year 1994 and for the mean annual air temperature, the change points occurred in the year 2000 and 2002. Before the change, the mean annual rainfall and mean annual air temperature were 26mm and 20^{0}C respectively, while after the change, the mean annual rainfall and air temperature were 104.3mm and 14.5^{0}C respectively.**

Keywords- ; *Air Temperature, Rainfall Time Series, Gibb Sampler, Conjugate Prior, Inverted Gamma Distribution, Posterior Distribution.*

I. Introduction

Climate change and global warming are recognized worldwide as the most crucial environmental dilemma that the world is experiencing today [5]-[7]. Concern in climate change and global warming by the international community, non-government organizations and governments has brought great interest to climate scientists leading to several studies on climate trend detection at global, hemispherical and regional scales [7, 8, and 17]. Nowadays, study of long-term temperature variability has been a topic of particular attention for climate researchers as temperature affects straightaway human activities in all domains Increase in anthropogenic greenhouse gases' concentrations in the atmosphere is mainly due to human activities such as deforestation, burning of fossil fuel and the conversion of the Earth's land to urban uses driven largely by the rapid growth of the human population that are major causes of warming of the climate system and of the process of climate change [8 and 17].

Several studies of long-term time series of temperatures have been done [3, 10 and 18]. Results have shown that the Earth's surface air temperature has increased by 0.6˚C - 0.8˚C during the 20th century, along with changes in the hydrologic cycle. Temperatures in the lower troposphere have augmented between 0.13˚C and 0.22˚C per decade since 1979, according to satellite temperature measurements [16]. In an analysis of a time series combining global land and marine surface temperature records from 1850 to 2010 developed by the Climate Research Unit (CRU), the year 2005 was seen as the second warmest year, behind 1998 with 2003 and 2010 tied for third warmest year [3, 9, 11, 12 and 19]. The two most recent decades were compared with the period 1979-1990. Warming has been observed to be concentrated in the most recent decade, from 2001 to 2010. The results were attributed to natural variability of the climate and/or to human activity but not to the El Niño-Southern Oscillation as previously suggested by other authors [9, 12 and 13]. Generally, there is consent among scientists that most of the observed increase in globally averaged temperatures since the mid-20th century is unequivocal and very likely due to the observed increase in anthropogenic greenhouse gas concentrations. The 10 warmest years of the 20th century all occurred in the last 15 years of the century, 1998 being the warmest. The Intergovernmental Panel on Climate Change [3] projected that the average global surface temperature will continue to increase to between 1.4˚C and 5.8˚C above 1990 levels, by 2100 [6]. To some extent, other factors, such as variations in solar radiation [3] and land use at regional scale, are also considered to be among the causes of the observed global warming [2, 14, and 15].

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II. Materials and Method

A. Data Description

The data used in this study were collected from Department of Economics and Statistics, Cocoa Research Institute of Nigeria (CRIN), Ibadan. They consisted of time series of year wise monthly average of air temperature and rainfall for the period ranging from 1990 to 2016.

B. Methodology

In this research, the change point analysis focuses on the detection of change(s) in the mean level(s) of the time series data. Several classical methods in change point analysis include non-parametric Wilcoxon test, the Student-t test, Maximum likelihood, and the sequential Mann-Kendall test. The alternative for these classical approaches is Bayesian approach which takes into consideration prior information, the model of the shift assumed and observed data into forming posterior distribution to model the data into forming a posterior distribution to model associated analysis. Bayesian approach in change identification problem for mean level in time series data have been used by previous researchers such as [1, 4 and 19]. The Bayesian approach in this study is based on single shifting model distributions on the unknown change point. In contrast to the classical approach, the Bayesian methods take into consideration the parameter of the model as random variables represented by a statistical distribution (prior distribution) rather fixed value.

The Bayesian method allows the integration of statistical analysis through the prior distribution with the most current
information based on the observations into a posterior distribution. In other words, the prior to experimentation; the posterior distribution an updated belief about the parameters after sample data is obtained. The analysis involved changes getting the mean value before and after the change, the amount of the change and the variation in observations. In this study, two related problems will be analyzed that is on the detection of the change point and the estimation of the change point.

Hypothesis Model

In this study, it is assumed that the probability of more than two change points within the desired period is eligible. In principle, the method espoused here can be easily extended to more than two change points, although the complexity of the problem is correspondingly increase.

Suppose that there is a sequence of independent normal random variables, $X_1, X_2 \ldots X_n$. These are observed along with time. The objective of this paper is to test hypothesis of the form

Hypothesis H_0: "A no change in the mean" of the time series dataset

$$x_i \sim N(x|\mu_0, \sigma^2);\ i = 1,2,\cdots,n \qquad (1)$$

where the prior distribution for the two parameters of interest μ_0, σ^2 are given as

$$\begin{aligned} &\mu_0 \sim N(\mu_0|m_0, \sigma^2 v_0) \\ &\sigma^2 \sim IG(\sigma^2|a_0, b_0) \end{aligned} \qquad (2)$$

Hypothesis H_1: "A single change in the mean" of time series dataset

$$H_1: x_i \sim \begin{cases} N(\mu_1, \sigma^2) & i = 1,2,\cdots\cdots\cdots,\tau \\ N(\mu_2, \sigma^2) & i = \tau+1, \tau+2, \cdots, n \end{cases} \qquad (3)$$

where the prior distribution for the three parameters of interest $\mu_{11}, \mu_{12}, \sigma^2$ are given as

$$\begin{aligned} &\mu_{11} \sim N(\mu_1|m_{11}, \sigma^2 v_{11})\ ; \\ &\mu_{12} \sim N(\mu_2|m_{12}, \sigma^2 v_{12}); \\ &\sigma^2 \sim IG(\sigma^2|a_{11}, b_{11}) \\ &\tau \sim U(1, \cdots, n) \end{aligned} \qquad (4)$$

Hypothesis H_2: "A double change in the mean" of the time series dataset

$$H_2: x_i \sim \begin{cases} N(\mu_{21}, \sigma^2)\ ; & i = 1,2,\cdots\cdots\cdots,\tau_1 - 1 \\ N(\mu_{22}, \sigma^2)\ ; & i = \tau_1, \tau_1+1, \cdots, \tau_2 - 1 \\ N(\mu_{23}, \sigma^2); & i = \tau_2+1, \tau_2+2, \cdots, n \end{cases} \qquad (5)$$

where the prior distribution for the three parameters of interest $\mu_{21}, \mu_{22}, \mu_{23}$ *and* σ^2 are given as

$$\begin{aligned} &\mu_{21} \sim N(\mu_{21}|m_{21}, \sigma^2 v_{21}) \\ &\mu_{22} \sim N(\mu_{22}|m_{22}, \sigma^2 v_{22}) \\ &\mu_{23} \sim N(\mu_{23}|m_{23}, \sigma^2 v_{23}) \\ &\sigma^2 \sim IG(\sigma^2|a_{22}, b_{22}) \\ &\tau_1 \sim U(1, \cdots, \tau_2 - 1) \\ &\tau_2 \sim U(\tau_1+1, \cdots, n) \end{aligned} \qquad (6)$$

Where $N(\mu, \sigma^2)$ *represents a Normal distribution with the density function given*

$$f(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(y-\mu_r)^2}{2\sigma^2}\right\}, x \in \mathbb{R} \qquad (7)$$

Bayesian inference under each hypothesis

Bayesian inference under H_0

To obtain the posterior distribution under the null hypothesis we combining (1) and (2)

$$p(\mu,\sigma^2|H_0) = \left(\frac{1}{2\pi}\right)^{\frac{n+1}{2}}\left(\frac{1}{\sigma^2}\right)^{a_0'+1} A e^{-\frac{1}{2\sigma^2}[b_0']} e^{-\frac{1}{2\sigma^2 v_0'}(\mu_0-m_0')^2} \quad (8)$$

where $v_0' = \frac{v_0}{v_0 n+1}$

$a_0' = \frac{n}{2} + a_0$

$A = \frac{{b_0}^{a_0}}{(2\pi)^{\frac{n}{2}}\Gamma(a_0)v_0^{\frac{1}{2}}}$

$m_0' = (1-v_0'n)\bar{x} + v_0' n m_0$

$b_o' = b_0 + \frac{1}{2}(ns^2 + (1-v_0'n)n(\bar{x}-m_0)^2)$

Integrating (8) with respect to μ and σ^2 results to a prior predictive density for the null hypothesis can be expressed as

$$p(x|H_0) = \frac{{b_0}^{a_0} {v_0'}^{\frac{1}{2}}\Gamma(a_0')}{(2\pi)^{\frac{n}{2}}\Gamma(a_0)v_0^{\frac{1}{2}}{b_0'}^{a_0'}} \quad (9)$$

Bayesian inference under H_1

To obtain the posterior distribution under a single change hypothesis we combining (3) and (4)

$$p(\boldsymbol{\mu},\sigma^2|H_1) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}}\left(\frac{1}{\sigma^2}\right)^{a_1'+1} \times A_1 e^{-\frac{1}{2\sigma^2}[b_1']} e^{-\frac{1}{2\sigma^2}\left[\frac{(\mu_{11}-m_{11}')^2}{v_{11}'}+\frac{(\mu_{12}-m_{12}')^2}{v_{12}'}\right]} \quad (10)$$

where $v_{11}' = \frac{v_{11}}{v_{11}\tau+1}$

$v_{12}' = \frac{v_{12}}{v_{12}(n-\tau)+1}$

$m_{11}' = (1-v_{11}'\tau)\bar{x}_{11} + v_{11}'\tau m_{11}$; $a_1' = \frac{n}{2} + a_1$

$m_{12}' = (1-v_{12}'(n-\tau))\bar{x}_{12} + v_1'(n-\tau)m_{12}$

$$A_1 = \frac{{b_1}^{a_1}}{(2\pi)^{\frac{n}{2}}\Gamma(a_1)(2\pi v_{11})^{\frac{1}{2}}(2\pi v_{12})^{\frac{1}{2}}}$$

$$b_1' = b_1 + \frac{1}{2}(\tau {s^2}_{11} + (n-\tau)s_{12}^2 + (1-v_{11}'\tau)\tau(\bar{x}_{11}-m_{11})^2 + (1-v_{12}'\tau)(n-\tau)(\bar{x}_{12}-m_{12})^2)$$

Integrating (10) with respect to μ_{11} *and* μ_{12} and σ^2 results to a prior predictive density for the null hypothesis can be expressed as

$$p(x|H_1) = \frac{{b_1}^{a_1}{v_{11}'}^{\frac{1}{2}}{v_{12}'}^{\frac{1}{2}}\Gamma(a_1')}{(2\pi)^{\frac{n-1}{2}}\Gamma\left(\frac{a_1}{2}\right)v_{11}^{\frac{1}{2}}v_{12}^{\frac{1}{2}}{b_1'}^{a_1'}} \quad (11)$$

Bayesian inference under H_2

To obtain the posterior distribution under a single change hypothesis we combining (5) and (6)

$$p(\boldsymbol{\mu},\sigma^2|H_1) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}}\left(\frac{1}{\sigma^2}\right)^{a_2'+1} A_2 \times e^{-\frac{1}{2\sigma^2}[b_2']} e^{-\frac{1}{2\sigma^2}\left[\frac{(\mu_{21}-m_{21}')^2}{v_{21}'}+\frac{(\mu_{22}-m_{22}')^2}{v_{22}'}+\frac{(\mu_{23}-m_{23}')^2}{v_{23}'}\right]} \quad (12)$$

Where $A_2 = \frac{{b_2}^{a_2}}{(2\pi)^{\frac{1}{2}}\Gamma(a_2)}\frac{1}{\sqrt{v_{21}v_{22}v_{23}}}$

$v_{21}' = \frac{v_{21}}{v_{21}\tau_1+1}$

$v_{22}' = \frac{v_{22}}{v_{22}(\tau_2-\tau_1)+1}$

$v_{23}' = \frac{v_{23}}{v_{23}(n-\tau_2)+1}$

$a_2' = \frac{n}{2} + a_2$

$m_{21}' = (1-v_{21}'\tau_1)\bar{x}_{21} + v_{21}'\tau_1 m_{21}$

$m_{22}' = (1-v_{22}'(\tau_2-\tau_1))\bar{x}_{22} + v_{22}'(\tau_2-\tau_1)m_{22}$

$m_{22}' = (1-v_{23}'(n-\tau_2))\bar{x}_{23} + v_{23}'(n-\tau_2)m_{23}$

$$b_2' = b_2 + \{\tau_1 s_1^2 + (\tau_2-\tau_1)s_2^2 + (n-\tau_2)s_3^2 + (1-v_{21}'\tau_1)\tau_1\bar{x}_{21} + \left(1-v_{22}'(\tau_2-\tau_1)\right)(\tau_2-\tau_1)\bar{x}_{22} + \left(1-v_{23}'(n-\tau_2)\right)(n-\tau_2)\bar{x}_{23}\}$$

Integrating (12) with respect to μ_{21}, μ_{22} *and* μ_{23} and σ^2 results to a prior predictive density for the null hypothesis can be expressed as

$$p(x|H_2) = \frac{{b_2}^{a_2}{v_{21}'}^{\frac{1}{2}}{v_{22}'}^{\frac{1}{2}}{v_{23}'}^{\frac{1}{2}}\Gamma(a_2')}{(2\pi)^{\frac{n-1}{2}}\Gamma(a_1)v_{21}^{\frac{1}{2}}v_{22}^{\frac{1}{2}}v_{23}^{\frac{1}{2}}{b_2'}^{a_2'}} \quad (13)$$

Full conditional distribution for H_0

The collection of full conditionals for model H_0 is proportional to (8)

$p(\mu|\sigma^2, H_0) = N(\mu|m_0, v_0'\sigma^2)$

$p(\sigma^2|\mu, H_0) = IG(\sigma^2|a_0', b_o' + \frac{(\mu_0-m_0')^2}{2v_0'})$

Full conditional distribution for H_1

The collection of full conditionals for model H_1 is proportional to (10)

$$p(\mu_{11}|\mu_{12},\sigma^2,\tau,H_1) = N(\mu_{11}|m_{11},v'_{11}\sigma^2)$$
$$p(\mu_{12}|\mu_{11},\sigma^2,\tau,H_1) = N(\mu_{12}|m_{12},v'_{12}\sigma^2)$$
$$p(\sigma^2|\mu,\tau,H_0) = IG(\sigma^2|a'_1,b'_1 + \frac{(\mu_{11}-m_{11}')^2}{2v'_{11}} + \frac{(\mu_{12}-m_{12}')^2}{2v'_{12}})$$
$$p(\tau|\mu_{11},\sigma^2,\mu_{12},H_1)\alpha\, b'_1\sqrt{v'_{11}v'_{12}}$$

Full conditional distribution for H_2

The collection of full conditionals for model H_2 is proportional to (12)

$$p(\mu_{21}|\mu_{22,},\mu_{23,}\,\sigma^2,\tau_1,H_2) = N(\mu_{21}|m_{21},v'_{21}\sigma^2)$$
$$p(\mu_{22}|\mu_{21,},\mu_{23,}\,\sigma^2,\tau_1,\tau_2,H_2) = N(\mu_{22}|m_{22},v'_{22}\sigma^2)$$
$$p(\mu_{23}|\mu_{22,},\mu_{2,}\,\sigma^2,\tau_2,H_2) = N(\mu_{23}|m_{23},v'_{23}\sigma^2)$$
$$p(\sigma^2|\mu_{21},\mu_{22},\mu_{23,},\tau_1,\tau_2,H_2) = IG(\sigma^2|a'_2 + \frac{(\mu_{21}-m_{21}')^2}{2v'_{21}} + \frac{(\mu_{22}-m_{22}')^2}{2v'_{22}} + \frac{(\mu_{23}-m_{23}')^2}{2v'_{23}})$$
$$p(\tau_1|\mu_{21},\mu_{22},\mu_{23,},\tau_1,\tau_2,H_2) = \frac{P(\tau_1|y,\mu_{21},\mu_{22},\mu_{23},\tau_2,\sigma^2,M_2)\Pr(\tau_1|H_2)}{\sum_{\tau_1=1}^{\tau_2} P(\tau_1|y,\mu_{21},\mu_{22},\mu_{23},\tau_2,\sigma^2,M_2)\Pr(\tau_1|H_2)}$$
$$p(\tau_2|\mu_{21},\mu_{22},\mu_{23,},\tau_1,\tau_2,H_2) = \frac{P(\tau_2|y,\mu_{21},\mu_{22},\mu_{23},\tau_1,\sigma^2,M_2)\Pr(\tau_2|H_2)}{\sum_{\tau_2=(\tau_1+1)}^{n} P(\tau_2|y,\mu_{21},\mu_{22},\mu_{23},\tau_1,\sigma^2,M_2)\Pr(\tau_2|H_2)}$$

III. Results And Discussion

A. Results

Fig 1 clearly shows that a change indeed occurred in the sequence of observation for annual rainfall in the year 1994 and for annual air temperature in the year 2000 and 2002.

The length of the burn-in period was 10,000 and the number of iterations of the Gibbs sampler after the burn-in period is chosen as 135,000. Figures 2 and 3 shows the graphical summary for the iteration of the intensity of change-point variable for the annual rainfall and air temperature respectively and Tables 1 and 2 shows the summary statistics of the annual mean rainfall and annual air temperature before and after the change point was detected respectively.

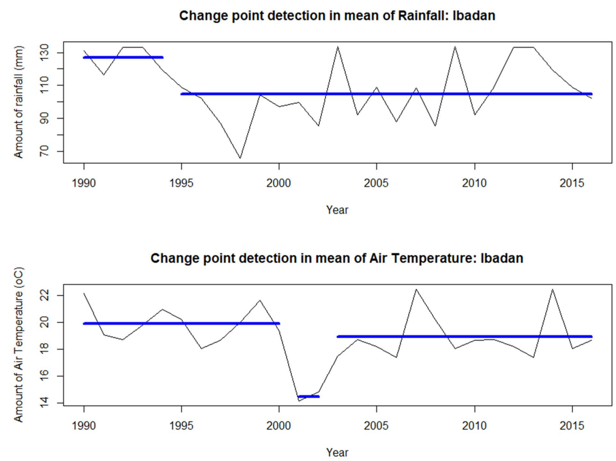

Figure 1: Change point detection in annual mean rainfall and annual mean air temperature in Ibadan.

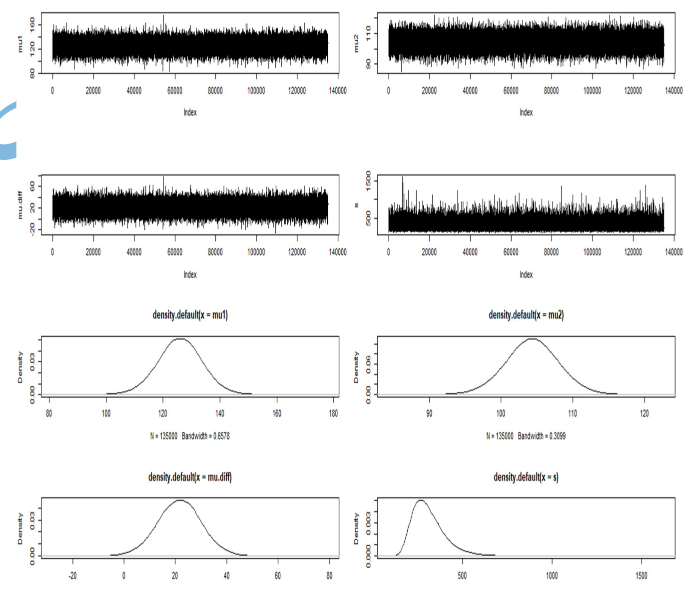

Figure 2: MCMC Plots for Annual Rainfall

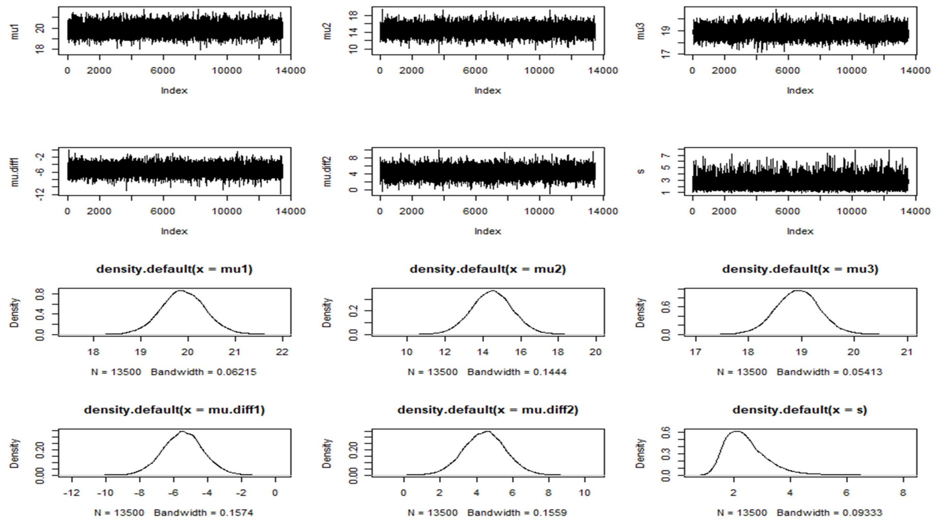

Figure 3: MCMC Plots for Annual Air Temperature

B. Discussions

Figures 1 showed the evidence that a single change point actually occurred in the annual mean in year 1994 and a double change point detection in the annual mean temperature in year 2000 and 2002. Tables 1 and 2 provided the summary of the quantity for each parameters for the annual mean rainfall and annual mean temperature using R programming language. This includes the means, differences in mean, variance and the 95% posterior interval. It was observed that that there was a decrease in the amount of annual mean rainfall after the change occurred and also for the annual air temperature there was a decrease in the amount of air temperature before the first change and an increase in the annual air temperature after the first change point.

Table 1: Summary statistics characterizing the posterior distributions for the single point change point model for annual mean rainfall.

	Mean	sd	25%	50%	97.5%
μ_1	125.957	7.870	110.80	125.00	141.400
μ_2	104.336	3.768	101.90	104.30	111.700
$\mu_2 - \mu_1$	-21.621	8.725	-27.37	-21.65	-4.309
σ^2	310.830	95.891	243.60	293.70	544.500

Table 2: Summary statistics characterizing the posterior distributions for the single point change point model for air annual mean temperature.

	Mean	sd	25%	50%	97.5%
μ_1	19.901	0.468	19.590	19.90	20.820
μ_2	14.491	1.094	13.770	14.49	16.650
μ_3	18.933	0.418	18.660	18.93	19.750
μ_2-μ_1	-5.410	1.191	-6.196	-5.408	-3.041
μ_3 - μ_2	4.442	1.168	3.679	4.444	6.745
σ^2	2.432	0.770	1.893	2.292	4.302

IV. Conclusion

In this research work, the researcher have described a multiple Bayesian change point model aimed to detect the time when there is a shift in the rate of an normal distribution model with application to annual mean rainfall and temperature.

One of the most important motivations for using a Bayesian approach instead of a frequentist one is the fact that the Bayesian estimation of uncertainty (like variances and confidence intervals) is not based on asymptotic sampling arguments that requires the availability of larges samples.

This approach can be extended to other type of an exponential models and event to other types of probability distribution functions.

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