On the Development of Robust Distributed Lag Model With Non-Normal Error Term

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***Abstract*—Distributed Lag Models (DLM) is a major workhorse in dynamic single-equation regressions. Its popularity in applied time series econometrics has ever be on the increase and it requires stringent assumptions for validity just like classical regression models. One of the most critical assumptions is the normality of the error distribution which are often violated which leads to unreliable inference. Violations of other assumptions has been considered in the literature but limited with non-normal error term. Therefore, Robust Distributed Lag Models (RDLM) was developed aimed at improving inference when the residual terms are not normal. The statistical properties of the proposed model are provided and the method of Maximum Likelihood Estimation was used in estimating its parameters. Simulated data of various sizes were used to establish the validity of the developed model.**

Keywords: *Exponentiated generalised normal distribution, Maximum likelihood, Distributed Lag.*

I. Introduction

The practice of statistical analysis often consists of fitting a model to data, testing for violations of the estimator assumptions, and searching for appropriate solutions when the assumptions are violated [1]. One motivation for every statistician is to produce statistical methods that are not unduly affected by outliers or departures from model assumptions.

Generally robust statistics are referred to as statistics with good performance for data drawn from a wide range of probability distributions, especially for distributions that are not normally distributed. Robust statistical methods have been developed for many common problems, such as estimating location, scale and regression parameters. Strictly speaking, a robust statistic is resistant to errors in the results, produced by deviations from assumptions

A Distributed Lag Model (DLM) is a model for time series data in which a regression equation is used to predict current values of a dependent variable based on both the current values of an explanatory variable and the lagged (past period) values of this explanatory variable. The distributed lag models have been found to be very useful in Economics, Biological and Process control.

Basically, there are four principal assumptions which justify the use of most regression models for purposes of inference or prediction and DLM is not an exemption and this include normality of the normality of the error term. If any of these assumptions is violated, then the forecasts, confidence intervals, and scientific insights yielded by a regression model may be (at best) inefficient or (at worst) seriously biased or misleading. Sometimes the error distribution is skewed by the presence of a few large outliers [2].

Looking at some of the works on DLM, [3] estimated parameters in Autoregressive models in non-normal error situations and two distributions, Gamma and Generalised logistic were used and it was shown to be efficient and robust compared to Least Square Estimation. [4]. Reference [5] worked on specification of DLM with outlier infested time series, which was on specification of DLM in the presence of auto-correlated residuals**.** Reference[6] investigated the relationship between expenditure and economic growth in Nigeria using a two-stage robust autoregressive DLM approach.

From literature review, DLM with errors assumption violation has not been thoroughly studied and this work intent to fill this gap.

II. Research Methodology

In this section, parameters of DLM with normal error term and exponentiated generalised normal distributions are derived so also the statistical properties of the new proposed distribution.

Derivation of the parameter of DLM with Normal error term

The Distributed Lag model of the general form is given as

$$Y_t = \beta_0 + \sum_{i=1}^{p} \beta_i X_{t-i} + \sum_{j=1}^{q} \alpha_j Y_{t-j} + e \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where Y_t is the value at time period t of the dependent variable , $\beta_{i,i=1\ldots P}$ and α_j,j=1,2….q are the lag weights to be estimated placed on the value i periods previously of the explanatory variable x and ℓ is the error term and assumed to be normally distributed.

Now, if Y_t is assumed to follow a normal distribution i.e.$Y_t \sim N(\mu, \sigma^2)$,

then the probability distribution function of Y_t given as

$$f(y_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{y_t-\mu}{\sigma}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

III. Analysis

In other to show the performance of the Robust Distributed Lag Model (RDLM) over the existing DLM, simulated data set of sample sizes, 10, 20, 50, 100, 500, 1000, 5000 and 10,000 were made use of. Also, in other to compare the distributions, some criteria were considered which include; Log likelihood (LL), Akaike Information criterion (AIC), Hannan Quinine Information Criterion (HQIC) and the Bayesian Information Criterion (BIC) values. It should be noted that AIC = 2k − 2l; BIC = k log (n) − 2l; CAI_c= AIC + {2k (k+1)/ n−k−l}; and HQIC= 2klog [log (n)] −2l, where k is the number of parameters in the statistical model, n is the sample size and l is the maximized value of the likelihood function under the considered model. From Table 1 below, the Exponentiated Generalised Normal Distribution perform better than the Normal Distribution at the various sample sizes considered in terms of having lower AIC, BIC, AICc and HQIC values consistently.

Also at different lags, the Exponentiated Generalised Normal Distribution has lower criteria values compared to Normal distribution which makes Exponentiated Generalised Normal Distribution better.

IV. Results

Without doubt, it is clearly indicated that the Exponentiated Generalised Normal Distribution has lower criteria values compared to Normal distribution which makes Exponentiated Generalised Normal Distribution better.

A. Equations

Putting (1) into (2) we have

$$f(y_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}} \left[\left(y_t - \left(\beta_0 + \sum_{i=1}^{p} \beta_i X_{t-i} + \sum_{j=1}^{q} \alpha_i Y_{t-j}\right)\right)\right]^2 \ldots\ldots(3)$$

Taking the likelihood of (3), we have

$$\prod_{t=1}^{n} f(y_t) = (2\pi\sigma^2)^{-\frac{n}{2}} e^{-\frac{1}{2\sigma^2}} \left[\left[Y_t - \left(\beta_0 + \sum_{i=1}^{p} \beta_i X_{t-i} + \sum_{j=1}^{q} \alpha_i Y_{t-j}\right)\right]\right]^2 \ldots\ldots\ldots\ldots(4)$$

Denoting the log likelihood of (4) by m as:

$$m = log \prod_{t=1}^{n} f(y_t) = -\frac{n}{2}\log 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_{t-1}^{n}\left[Y_t - \beta_0 - \sum_{i=1}^{q} \beta_i X_{t-i} \sum_{j=1}^{p} \alpha_j Y_{t-j}\right]^2 \ldots\ldots\ldots(5)$$

Now differentiating (5) with respect to: $\beta_0, \beta_i,\ \alpha_j$ and σ respectively gives

$$\frac{\partial m}{\partial \beta_0} = \frac{1}{\sigma^2}\sum_{t-1}^{n}\left[Y_t - \beta_0 - \sum_{i=1}^{q} \beta_i X_{t-i} \sum_{j=1}^{p} \alpha_j Y_{t-j}\right] \quad .(6)$$

$$\frac{\partial m}{\partial \beta_i} = \frac{1}{\sigma^2}\sum_{t-1}^{n} X_{t-i}\left[Y_t - \beta_0 - \sum_{i=1}^{q} \beta_i X_{t-i} \sum_{j=1}^{p} \alpha_j Y_{t-j}\right] \quad (7)$$

$$\frac{\partial m}{\partial \alpha_j} = \frac{1}{\sigma^2}\sum_{t-1}^{n} Y_{t-j}\left[Y_t - \beta_0 - \sum_{i=1}^{q} \beta_i X_{t-i} \sum_{j=1}^{p} \alpha_j Y_{t-j}\right] \quad .(8)$$

$$\frac{\partial m}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma^3}\sum_{t-1}^{n}\left[Y_t - \beta_0 - \sum_{i=1}^{q} \beta_i X_{t-i} \sum_{j=1}^{p} \alpha_j Y_{t-j}\right]^2 \quad .(9)$$

Fisher Information Matrix DLM with Normal error term

The Fisher's information matrix is a very useful tool for calculating the interval estimates, asymptotic variances, covariance and tests of hypothesis of β_0, βi, *and* α_j are now derived.

Now, taking the second derivatives of (6), (7), (8), and (9) respectively we have:

$$\frac{\partial^2 m}{\partial \beta_0^2} = \frac{\partial m}{\partial \beta_0}\left[{}^1/_{\sigma^2}\sum_{t=1}^{n}\left[Y_t - \beta_0 - \sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j Y_{t-j}\right.\right. \quad (10)$$

$$= \frac{\partial m}{\partial \beta_0}\left[\frac{1}{\sigma^2}\sum_{t=1}^{n}Y_t - \sum_{t=1}^{n}\beta_0 - \sum_{t=1}^{n}\sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j Y_{t-j}\right]$$

$$\frac{\partial^2 m}{\partial \beta_0^2} = \frac{-n}{\sigma^2} \ldots.. (11)$$

$$\frac{\partial^2 m}{\partial \beta_i^2} = \frac{\partial m}{\partial \beta_i}\left[{}^1/_{\sigma^2}\sum_{t=1}^{n}X_{t-i}\left[Y_t - \beta_0 - \sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j Y_{t-j}\right]\right] \ldots\ldots (12)$$

$$= \frac{\partial m}{\partial \beta_i}\left[\frac{1}{\sigma^2}\sum_{t=1}^{n}X_{t-i}Y_t - \beta_0\sum_{t=1}^{n}X_{t-i} - \sum_{t=1}^{n}X_{t-i}\sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j Y_{t-j}\right]\Bigg]$$

$$\frac{\partial^2 m}{\partial \beta_i^2} = \frac{-1}{\sigma^2}\sum_{t=i}^{n}X_{t-i}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\frac{\partial^2 m}{\partial \alpha_j^2} = \frac{\partial m}{\alpha_j}\left[\frac{1}{\sigma^2}\sum_{t=1}^{n}Y_{t-j}\left(Y_t - \beta_0\sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j Y_{t-j}\right)\right] \ldots (14)$$

$$\frac{\partial m}{\partial \alpha_j}\left[\frac{1}{\sigma^2}\left(\sum_{t=1}^{n}Y_{t-j}^2 - \beta_0\sum_{t=1}^{n}Y_{t-j} - \sum_{i=1}^{q}\beta_i X_{t-i}\sum_{j=1}^{p}\alpha_j\sum Y_{t-j}^2\right)\right]$$

$$= \frac{-1}{\sigma^2}\sum_{t=1}^{n}Y_{t-i}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Cramer-Rao variance is the inverse of the element of Fisher Information Matrix, hence the variance of the estimate of the parameters are:

$$Var(\hat{\beta}_0) = \frac{\sigma^2}{n} \quad \ldots\ldots\ldots\ldots (16)$$

$$Var(\hat{\beta}_i) = \frac{\sigma^2}{\sum_{t=1}^{n}X_{t-i}^2} \quad i = 1, 2, \ldots p \quad \ldots (17)$$

$$Var(\hat{\alpha}_j) = \frac{\sigma^2}{\sum_{t=1}^{n}Y_{t-j}^2} \quad j = 1, 2, \ldots q \quad \ldots (18)$$

Interval Estimation of the Parameter for DLM with Normal error term

From the estimates of DLM parameters and their variances obtained above, the interval estimates of the parameter can be obtained as:

(i) $\hat{\beta}_0 \pm t_{n-1,\frac{(\alpha)}{2}}\sqrt{Var(\hat{\beta}_0)}$ .. (19)

(ii) $\hat{\beta}_i \pm t_{n-1,}{}^{\alpha}/_2\sqrt{Var(\hat{\beta}_i)}$ $i, 1, 2, \ldots, p \ldots$ (20)

(iii) $\hat{\alpha}_j \pm t_{n-1,}{}^{\alpha}/_2\sqrt{Var(\hat{\alpha}_j)}$ $j = 1, 2, \ \ldots q$ … (21)

Derivation of the parameters of DLM with Exponentiated Generalized Normal Distribution

For the derivation of the parameters of DLM with exponentiated generalised normal distribution, its link function is given as:

$$g(y) = \alpha[F_{(y)}]^{\alpha-1}f_{(y)} \quad \ldots\ldots\ldots.. (22)$$

where $f_{(y)}$ is the conventional normal distribution i.e.

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} \quad \ldots\ldots\ldots (23)$$

And *F(y)* is the corresponding CDF giving as

$$F(y) = \phi\left(\frac{y-\mu}{\sigma}\right) \quad \ldots\ldots\ldots\ldots (24)$$

By substituting (23) and (24) into (23) we have,

$$g(y) = \alpha\left[\phi\left(\frac{y-\mu}{\sigma}\right)\right]^{\alpha-1}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} \quad \ldots\ldots (25)$$

Given a distributed lag model

$$Y_t = \beta_o + \sum_{i=1}^{\rho}\beta_i X_{t-i} + \sum_{j=1}^{q}\alpha_j Y_{t-j} + e_t \quad \ldots\ldots\ldots..(26)$$

Now, substituting (26) into (25) gives

$$g(y_t) = \alpha\left[\phi\left(\frac{y-\mu}{\sigma}\right)\right]^{\alpha-1}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\frac{\left(y-\beta_o-\sum_{i=1}^{\rho}\beta_i X_{t-i}+\sum_{j=1}^{q}\alpha_j Y_{t-j}\right)}{\sigma^2}} \quad .. (27)$$

The likelihood function of (27) is

$$\prod_{t=1}^{n}g(y_t) = \alpha^n\prod_{t=1}^{n}\left[\phi\left(\frac{y_t-\mu}{\sigma}\right)\right]^{\alpha-1}(2\pi\sigma^2)^{-\frac{n}{2}}e^{-\frac{1}{2\sigma^2}}\sum_{t=1}^{n}\left(y_t - \beta_o - \sum_{i=1}^{\rho}\beta_i X_{t-i} - \sum_{j=1}^{q}\alpha_j Y_{t-j}\right)^2 \quad \ldots\ldots (28)$$

Taking the log of (28), we have the log likelihood function as

$$l = nlog\alpha + (\alpha-1)\sum_{t=1}^{n}log\phi\left(\frac{y_t-\mu}{\sigma}\right) - \frac{n}{2}log2\pi\sigma^2$$

$$-\frac{1}{2\sigma^2}\sum_{t=1}^{n}\left(y_t - \beta_o - \sum_{i=1}^{\rho}\beta_i X_{t-i} + \sum_{j=1}^{q}\alpha_j Y_{t-j}\right)^2 \quad \ldots\ldots (29)$$

Differentiating (29), with respect to $\alpha, \sigma^2, \beta_1, \beta_2 \ \ldots \beta_\rho, \alpha_1, \alpha_2, \ldots \ \alpha_q$, we have

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{t=1}^{n}log\phi\left(\frac{y_t-\mu}{\sigma}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots.. (30)$$

$$\frac{\partial l}{\partial \sigma^2} = (\alpha-1)\frac{\partial}{\partial \sigma}\left[\sum_{t=1}^{n}log\phi\left(\frac{y_t-\mu}{\sigma}\right)\right] + \frac{n}{\sigma} \quad \ldots\ldots. (31)$$

$$\frac{\partial l}{\partial \beta_i} = \frac{1}{2\sigma^2}\sum_{i=1}^{\rho}X_{t-1}\sum_{t=1}^{n}\left(y_t - \beta_o - \sum_{i=1}^{\rho}\beta_i X_{t-i} + \sum_{j=1}^{q}\alpha_j Y_{t-j}\right) \quad \ldots (32)$$

$$\frac{\partial l}{\partial \alpha_j} = \frac{1}{2\sigma^2}\sum_{j=1}^{q}Y_{t-j}\sum_{t=1}^{n}\left(y_t - \beta_o - \sum_{i=1}^{\rho}\beta_i X_{t-i} + \sum_{j=1}^{q}\alpha_j Y_{t-j}\right) \quad \ldots (33)$$

The estimates have no close form solution and has to be obtained by numerical analysis.

Fisher Information with non-Normal error term for DLM

In other to obtain the elements of Fisher Information, equations (30), (31), (32), and (33) are differentiated again to have:

$$\frac{\partial^2 l}{\partial \alpha^2} = -\frac{n}{\alpha^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(34)$$

$$\frac{\partial^2 l}{\partial (\sigma^2)^2} = (\alpha - 1)\frac{\partial^2}{\partial \sigma^2}\left[\sum_{t=1}^{n} log\phi\left(\frac{y_t - \mu}{\sigma}\right)\right] - \frac{n}{\sigma^2} \qquad (35)$$

$$\frac{\partial^2 l}{\partial \beta_i^2} = \frac{-1}{2\sigma^2}\left(\sum_{i=1}^{\rho} X_{t-i}\right)\left(\sum_{i=1}^{\rho} X_{t-i}\right) \qquad \ldots\ldots\ldots\ldots (36)$$

$$\frac{\partial^2 l}{\partial \alpha_j^2} = \frac{-1}{2\sigma^2}\left(\sum_{i=1}^{q} Y_{t-j}\right)\left(\sum_{i=1}^{q} Y_{t-j}\right) \qquad \ldots\ldots\ldots \quad (37)$$

The reciprocal of the Fisher information is the Cramer-Rao variance. The Cramer-Rao variance can be used in statistical inferences in obtaining interval estimation and conducting hypothesis testing on the parameter under consideration.

Variances for DLM with non-normal error term.

The respective variances of the estimated parameters for the distributed lag model with non-normal error term are:

$$V(\alpha_j) = \frac{1}{\frac{\partial^2 l}{\partial \alpha_j^2}} = \frac{2\sigma^2}{(\sum_{i=1}^{q} Y_{t-j})(\sum_{i=1}^{q} Y_{t-j})}$$

$$V(\beta_i) = \frac{1}{\frac{\partial^2 l}{\partial \beta_i^2}} = \frac{2\sigma^2}{(\sum_{i=1}^{\rho} X_{t-i})(\sum_{i=1}^{\rho} X_{t-i})}$$

$$V(\alpha) = \frac{1}{\frac{\partial^2 l}{\partial \alpha^2}} = \frac{\alpha^2}{n^2}$$

$$V(\sigma) = \frac{1}{\frac{\partial^2 l}{\partial \sigma^4}} = \frac{1}{(\alpha-1)\frac{\partial^2}{\partial \sigma^2}\left[\sum_{t=1}^{n} log\phi\left(\frac{y_t-\mu}{\sigma}\right)\right] - \frac{n}{\sigma^2}}$$

3.3 Internal Estimation for the parameters of the DLM with non-normal error term.

The confidence intervals for each of the parameters of the model are:

(1) $\alpha_j \pm Z_{1-\alpha/2}\sqrt{Var\ (\alpha_j)}$

(2) $\beta_i \pm Z_{1-\alpha/2}\sqrt{Var\ (\beta_i)}$

(3) $\alpha \pm Z_{1-\alpha/2}\sqrt{Var\ (\alpha)}$

(4) $\sigma \pm Z_{1-\alpha/2}\sqrt{Var\ (\sigma)}$

Hypothesis Testing

In other to test for all possible hypotheses, the appropriate test statistic for the parameters of the DLM with non-normal error term are defined as follows:

(1) $H_0: \alpha_j = 0 \quad vs \quad H_1: \alpha_j \neq 0$

$$t = \frac{\hat{\alpha}_j}{\sqrt{Var(\hat{\alpha}_j)}} \qquad j = 1, 2, \ldots q$$

(2) $H_0: \beta_i = 0 \quad vs \quad H_1: \beta_i \neq 0$

$$t = \frac{\hat{\beta}_i}{\sqrt{Var(\hat{\beta}_i)}} \qquad i = 1, 2, \ldots p$$

(3) $H_0: \alpha = 1 \quad vs \quad H_1: \alpha \neq 1$

$$t = \frac{\hat{\alpha}}{\sqrt{Var(\hat{\alpha})}}$$

B. Tables

TABLE 1: RESULTS BASED ON SIMULATED DATA

		Exponentiated Generalised Normal				Normal Distribution			
Sample size (n)	Criterion	*Lag1*	*Lag2*	*Lag3*	*Lag4*	*Lag1*	*Lag2*	*Lag3*	*Lag4*
10	AIC	-8.23097	-1.568717	-22.3686	-29.4275	46.0918	46.79099	46.80223	46.6487
	BIC	-7.32344	-0.660962	-21.4609	-28.5197	46.8807	47.57989	47.59113	47.4376
	AICc	-9.83097	-3.168717	-23.9686	-31.0275	44.2918	45.19099	45.20223	45.0487
	HQIC	-9.22677	-2.564522	-23.3644	-30.4233	45.4280	45.79518	45.80643	45.6529
20	AIC	3.96968	31.12805	-31.8213	35.5399	102.233	102.5028	102.8179	101.6981
	BIC	6.95688	34.11525	-28.8341	38.5271	106.010	106.2805	106.5957	105.4759
	AICc	1.16968	28.32805	-34.6213	32.7399	99.8326	99.70278	100.0179	98.89811
	HQIC	4.55281	31.71118	-31.2382	36.1231	102.621	103.0859	103.401	102.2812
50	AIC	32.64179	-81.77733	64.21296	-43.8192	233.229	233.2128	231.7465	233.109
	BIC	38.37786	-76.04126	69.94902	-38.0831	240.796	240.7800	239.3138	240.676
	AICc	29.12179	-85.29733	60.69296	-47.3392	230.468	229.6928	228.2265	229.589
	HQIC	34.82612	-79.59300	66.39728	-41.6349	234.685	235.3971	233.9308	235.293

		Exponentiated Generalised Normal				Normal Distribution			
100	AIC	81.21891	119.7556	152.1133	197.308	469.231	468.8535	467.7976	466.040
	BIC	89.03442	127.5711	159.9288	205.124	479.611	465.0935	478.1781	474.420
	AICc	77.45891	115.9956	148.3533	193.548	466.351	472.0165	464.0376	462.280
	HQIC	84.38199	122.9187	155.2763	200.471	471.340	479.2339	470.9607	469.203
500	AIC	1401.524	36.68676	-336.603	144.666	2265.49	2243.465	2264.043	2264.284
	BIC	1414.168	49.33058	-323.960	157.310	2282.34	2260.316	2280.894	2281.134
	AICc	1397.572	32.73476	-340.555	140.714	2262.51	2239.513	2260.091	2260.332
	HQIC	1406.458	41.64818	-331.642	149.628	2268.80	2248.427	2269.005	2269.245
	Criterion	***Lag1***	***Lag2***	***Lag3***	***Lag4***	***Lag1***	***Lag2***	***Lag3***	***Lag4***
1000	AIC	3254.550	2969.376	2817.916	3397.19	4641.91	4641.429	4641.614	4642.241
	BIC	3269.274	2984.100	2832.640	3411.91	4661.54	4661.056	4661.241	4661.868
	AICc	3250.574	2965.400	2813.940	3393.21	4638.92	4637.453	4637.638	4638.265
	HQIC	3260.146	2974.972	2823.512	3402.79	4645.64	4647.024	4647.210	4647.837
5000	AIC	8415.669	14388.58	8625.896	14555.3	22968.1	22967.28	22965.08	22968.49
	BIC	8435.221	14408.13	8645.447	14574.8	22994.2	22993.34	22991.15	22994.56
	AICc	8411.674	14384.58	8621.901	14551.3	22965.1	22963.28	22961.09	22964.49
	HQIC	8422.522	14395.43	8632.748	14562.1	22972.7	22974.13	22971.94	22975.34
10000	AIC	-24668.31	7789.841	18523.68	19904.9	46460.8	46460.71	46454.1	46459.47
	BIC	-24646.68	7811.472	18545.31	19926.5	46489.6	46489.55	46482.94	46488.31
	AICc	-24672.31	7785.843	18519.68	19900.9	46457.8	46456.71	46450.10	46455.48
	HQIC	-24660.99	7797.163	18531.00	19912.2	46465.7	46468.03	46461.42	46466.80

V. DISCUSSIONS

From the results in Table 1, using simulated perturbed data of sizes (10, 20, 50, 100, 500, 1000, 5000 and 10,000) the Exponentiated Generalised Normal Distribution perform better than the Normal Distribution at the various sample sizes considered in terms of having lower AIC, BIC, AICc and HQIC values consistently.

Also at different lags, the Exponentiated Generalised Normal Distribution has lower criteria values compared to Normal distribution which makes Exponentiated Generalised Normal Distribution better.

VI. CONCLUSION

Robust Distributed Lag Models (RDLM) was developed aimed at improving inference when the residual terms are not normal. The statistical properties of the proposed model were provided and the method of Maximum Likelihood Estimation was used in estimating its parameters which actually paves way for the derivation of fisher information matrix.

Simulated data application of varying sizes at different lags with different comparison criteria indicates that Exponentiated generalised normal distribution apart from its flexibility has better representation of DLM with non-normal error term compared with normal distribution.

ACKNOWLEDGMENT

We thank God for His wisdom, knowledge and understanding He gave to us in carrying out this study.

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