Interval Type-2 Fuzzy Control Chart for Non-Conformity Per Unit

A. A. Adepoju[1]; U. Mohammed[2]; K. Adamu[3]; N. S. Agog[4]; A. M. Isah[5]; W. A. Yekini[6]

[1]Department of Statistics,
Kano University of Science and Technology, Wudil, Nigeria.

[2]Department of Statistics,
Ahmadu Bello University, Zaria, Nigeria.

[3]Department of Statistics,
Nuhu Bamali Polytechnic, Zaria, Nigeria.

[4]Department of Mathematical Sciences,
Kaduna State University, Kaduna, Nigeria.

[5]Department of Mathematics,
Kashim Ibrahim College of Education, Maiduguri, Borno State. Nigeria.

[6]Department of Computer Science,
School of Technology, Lagos State Polytechnic, Lagos State. Nigeria.
E-mail: akeebola@gmail.com[1]

Abstract **— In this paper, the theoretical framework and application of interval type-2 fuzzy u-control chart was developed via extending the control limits of the traditional u-control chart with the aids of interval type-2 fuzzy set theory. The control chart for nonconformities per unit popularly known as u-control chart is used to monitor average number of nonconformities per unit. Fuzzy set is adopted in this research to handle imprecision or vagueness presumed from the collection of quality characteristics data. The proposed interval type-2 fuzzy control chart for non-conformity per unit deals with vagueness by providing more flexibility to the control limits of the u-control chart.**

Keywords- *Nonconformities, Interval Type-2 Fuzzy Sets, Fuzzy Sets, Average Nonconformities, Fuzzy Control Chart, interval type-2 fuzzy u control chart.*

I. Introduction

Control chart scheme is made of variable and attribute control charts, the former which constitute observations with continuous random variable while the later constitute observations from discrete random variable.

There are various types of attribute control charts which include; p, np, c as well as the u control chart. The p, np, c and u represent the fractional nonconforming units, number of nonconforming units, number of nonconformities and average nonconformities respectively.

The distribution of the p and np control charts are said to follow binomial distribution while c and u control charts follow Poisson distribution, Montgomery(2009). The classical Shewhart control chart data are obtained in crisp form (crisp values). However, this data sometime are collected under some conditions that which may disrupt the crisp values, then becomes vague or uncertain.

The uncertainty which may arise due to discretion of the one saddled with the responsibility measuring the observation usually lead to fuzzy system. Production process characteristics that are measured from such uncertain cannot be monitored by the classical control chart tools without necessary adjustment to the control limits.

II. Literature Review

In a real life situation, vagueness and uncertainty do occur as a result of human uncertainty usually due to judgmental decision based on qualitative measures, which might not be presented with exact (crisp) values. In some cases, we tends to describe distance in qualitative term such as the distance is far, very far etc, but with no certainty about the measurement of the distance.

Zadeh (1965) proposed the conceptual idea of the fuzzy sets theory. He proposed the type-1 fuzzy sets, with a degree of membership called crisp membership value, whose values are over the range 0 to 1. He also proposed the Type-2 fuzzy sets, which is an extension of the type-1 fuzzy sets, this type-2 fuzzy set is three dimensional, that is, it comprises of two membership function, the upper membership function and lower membership function and the representative value, Zadeh (1975).

Considerate number of authors made various contributions to the extensional development in this area of research which includes but not limited to Kanagawa *et al* (1993), Wang and Raz(1990), El-shal and Moris (2000), Rowlands and Wang (2000), Gulbay *et al* (2004), Karnik and Mendel (2001), Cheng (2005),Gulbay and Kahraman (2006a), Mendel (2007), Erginel (2008), Senturk and Erginel (2009), Senturk (2010), Senturk *et al* (2010), Kaya and Kahraman (2011), Erginel *et al* (2011), Senturk *et al* (2014) Erginel (2014), Poongodi and Muthulakshmi (2015), Wang and Hyrniewicz (2015), Edmundas *et al* (2015), Chen and Huang (2016), Hou *et al* (2016), Kaya *et al* (2017), Senturk and Antucheviciene (2017), Erginel *et al* (2018), Adepoju (2018). Adepoju *et al* (2019). Akeem *et al* (2019).

III. METHODOLOGY

Type-2 fuzzy sets

Definition 1 A type-2 fuzzy set (T2 FS) denoted by $\tilde{\tilde{A}}$ in a universe of discourse is characterized by a type-2 membership function given as $\mu_{\tilde{\tilde{A}}} = (x, \mu)$, where $x \in X$ and $\mu \in J_x \subseteq [0,1]$. Mathematically, this can be expressed as

$$\tilde{\tilde{A}} = \left\{\left((x,\mu), \mu_{\tilde{\tilde{A}}} = (x,\mu)\right) \middle| \forall x \in X, \forall \mu \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}} = (x,\mu) \leq 1\right\} \quad (1)$$

Where J_x denotes an interval[0,1]. This type-2 fuzzy set can be expressed as

$$\tilde{\tilde{A}} = \int_{x\in X}\int_{\mu\in J_x} \mu_{\tilde{\tilde{A}}}(x,\mu)/(x,\mu)$$

where $\int\int$ denotes union over all admissible x and u as given by mendel *et al* (2006) as well as Kahraman (2014)

Interval type-2 fuzzy sets

An interval type-2 fuzzy sets (IT2 FS) also known as closed interval type-2 fuzzy set (CIT2 FS) can be defined as a special case of type-2 fuzzy set $\tilde{\tilde{A}}$represented by the type-2 membership function $\mu \approx (x, \mu)$. If all $\mu \approx (x, \mu)$=1, it follows that the interval type-2 fuzzy set is expressed

$$\tilde{\tilde{A}} = \int_{x\in X}\int_{\mu\in J_x} 1/(x,\mu) \quad (2)$$

where $J_x \subseteq [0,1]$, Keshavarz Ghorabaee *et al* (2017),

3.2 Traditional u-control chart

Considering the presence of nonconformities of a product in an inspectional unit whose distribution follows Poisson distribution. These kinds of defects are monitored using attribute control chart. A u control chart, involves establishing a control chart using the average number of nonconformities per inspection unit. The expression is given below

$$u = \frac{c}{n} \qquad \bar{u} = \frac{\sum c}{\sum n} \quad (3)$$

And the control limits are as expressed as

$$LCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$CL = \bar{u} \quad (4)$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

3.3 Trapezoid Interval type-2 fuzzy u-control chart

$$\tilde{\tilde{u}}_n = \left[\left(a_{k1}^U\ a_{k2}^U\ a_{k3}^U\ a_{k4}^U; H_1(\tilde{A}_n^U)H_2(\tilde{A}_n^U)\right), \left(a_{k1}^L\ a_{k2}^L\ a_{k3}^L\ a_{k4}^L; H_1(\tilde{A}_n^L)H_2(\tilde{A}_n^L)\right)\right] \quad (5)$$

Then the fuzzy trapezoid nonconformities averages are as expressed below

$$\bar{u}a_{.1}^U = \frac{\sum_{i=1}^k a_{i1}^U}{k}, \bar{u}a_{.2}^U = \frac{\sum_{i=1}^k a_{i2}^U}{k}, \bar{u}a_{.3}^U = \frac{\sum_{i=1}^k a_{i3}^U}{k}, \bar{u}a_{.4}^U = \frac{\sum_{i=1}^k a_{i4}^U}{k}$$

$$\bar{u}a_{.1}^L = \frac{\sum_{i=1}^k a_{i1}^L}{k}, \bar{u}a_{.2}^L = \frac{\sum_{i=1}^k a_{i2}^L}{k}, \bar{u}a_{.3}^L = \frac{\sum_{i=1}^k a_{i3}^L}{k}, \bar{u}a_{.4}^U = \frac{\sum_{i=1}^k a_{i4}^L}{k}$$

where $a_{k1}^U\ a_{k2}^U\ a_{k3}^U\ a_{k4}^U$ are the lowest, second, third and biggest possible values of the upper membership function respectively, while $a_{k1}^L\ a_{k2}^L\ a_{k3}^L\ a_{k4}^L$ are the lowest, second, third and biggest possible values of the lower membership function respectively.

$H_1(\tilde{A}_n^U)H_2(\tilde{A}_n^U)$ and $H_1(\tilde{A}_n^L)H_2(\tilde{A}_n^L)$ are the maximum membership degrees of the upper membership
Function and maximum membership degrees of the lower membership
Function respectively.

Incorporating this interval type-2 model to fit the traditional control limits for u-control chart. Then upper, center line and the lower control limits for the interval type-2 fuzzy u control chart can be obtained as given below

$$UCL_{\bar{u}}\begin{bmatrix} \bar{u}a_{.1}^U+3\sqrt{\frac{\bar{u}a_{.1}^U}{n}}, \bar{u}a_{.2}^U+3\sqrt{\frac{\bar{u}a_{.2}^U}{n}}, \bar{u}a_{.3}^U+3\sqrt{\frac{\bar{u}a_{.3}^U}{n}}, \bar{u}a_{.4}^U+3\sqrt{\frac{\bar{u}a_{.4}^U}{n}}; min\left(H_1(A_i^U), H_2(A_i^U)\right), \\ \bar{u}a_{.1}^L+3\sqrt{\frac{\bar{u}a_{.1}^L}{n}}, \bar{u}a_{.2}^L+3\sqrt{\frac{\bar{u}a_{.2}^L}{n}}, \bar{u}a_{.3}^L+3\sqrt{\frac{\bar{u}a_{.3}^L}{n}}, \bar{u}a_{.4}^L+3\sqrt{\frac{\bar{u}a_{.4}^L}{n}}; min\left(H_1(A_i^L), H_2(A_i^L)\right) \end{bmatrix}$$

$$CL_{\bar{u}}\begin{bmatrix} \bar{u}a_{.1}^U, \bar{u}a_{.2}^U, \bar{u}a_{.3}^U, \bar{u}a_{.4}^U; min\left(H_1(A_i^U), H_2(A_i^U)\right), \\ \bar{u}a_{.1}^L, \bar{u}a_{.2}^L, \bar{u}a_{.3}^L, \bar{u}a_{.4}^L; min\left(H_1(A_i^L), H_2(A_i^L)\right) \end{bmatrix} \quad (6)$$

$$LCL_{\bar{u}}\begin{bmatrix} \bar{u}a_{.1}^U-3\sqrt{\frac{\bar{u}a_{.1}^U}{n}}, \bar{u}a_{.2}^U-3\sqrt{\frac{\bar{u}a_{.2}^U}{n}}, \bar{u}a_{.3}^U-3\sqrt{\frac{\bar{u}a_{.3}^U}{n}}, \bar{u}a_{.4}^U-3\sqrt{\frac{\bar{u}a_{.4}^U}{n}}; min\left(H_1(A_i^U), H_2(A_i^U)\right), \\ \bar{u}a_{.1}^L-3\sqrt{\frac{\bar{u}a_{.1}^L}{n}}, \bar{u}a_{.2}^L-3\sqrt{\frac{\bar{u}a_{.2}^L}{n}}, \bar{u}a_{.3}^L-3\sqrt{\frac{\bar{u}a_{.3}^L}{n}}, \bar{u}a_{.4}^L-3\sqrt{\frac{\bar{u}a_{.4}^L}{n}}; min\left(H_1(A_i^L), H_2(A_i^L)\right) \end{bmatrix}$$

Defuzzification of trapezoid interval type-2 fuzzy u-control chart

In this research, the modified BNP transformation method is used.
The BNP transformation as modified by Kahraman *et al* (2014) is given below.

$$DIT2_{trap(i)}^U = \frac{(a_{i4}^U-a_{i1}^U)+(H_2(\tilde{A}_1^U)a_{i2}^U-a_{i1}^U)+(H_1(\tilde{A}_1^U)a_{i3}^U-a_{i1}^U)}{4} + a_{i1}^U$$

$$DIT2_{trap(i)}^L = \frac{(a_{i4}^L-a_{i1}^L)+(H_2(\tilde{A}_1^L)a_{i2}^L-a_{i1}^L)+(H_1(\tilde{A}_1^L)a_{i3}^L-a_{i1}^L)}{4} + a_{i1}^L \quad (7)$$

$$DIT2_{trap(i)} = \frac{DIT2_{trap(i)}^U+DIT2_{trap(i)}^L}{2}$$

$i = 1,2,3,\dots,n$

The modified BNP technique can be transformed to the interval type-2 fuzzy u-control chart limits as expressed below.

$$uDIT2_{trap(i)}^U = \frac{(\bar{u}a_{i4}^U-\bar{u}a_{i1}^U)+(H_2(\tilde{A}_1^U)\bar{u}a_{i2}^U-\bar{u}a_{i1}^U)+(H_1(\tilde{A}_1^U)\bar{u}a_{i3}^U-\bar{u}a_{i1}^U)}{4} + \bar{u}a_{i1}^U$$

$$uDIT2_{trap(i)}^L = \frac{(\bar{u}a_{i4}^L-\bar{u}a_{i1}^L)+(H_2(\tilde{A}_1^L)\bar{u}a_{i2}^L-\bar{u}a_{i1}^L)+(H_1(\tilde{A}_1^L)\bar{u}a_{i3}^L-\bar{u}a_{i1}^L)}{4} + \bar{u}a_{i1}^L \quad (8)$$

$$uDIT2_{trap(i)} = \frac{uDIT2_{trap(i)}^U+uDIT2_{trap(i)}^L}{2}$$

For the upper control limit:

$$UCL\left(uDIT2_{trap(i)}^U\right) = \frac{(\bar{u}a_{.4}^U-\bar{u}a_{.1}^U)+(H_2(\tilde{A}_1^U)\bar{u}a_{.2}^U-\bar{u}a_{.1}^U)+(H_1(\tilde{A}_1^U)\bar{u}a_{.3}^U-\bar{u}a_{.1}^U)}{4} + \bar{u}a_{.1}^U$$

$$UCL\left(uDIT2_{trap(i)}^L\right) = \frac{(\bar{u}a_{.4}^L-\bar{u}a_{.1}^L)+(H_2(\tilde{A}_1^L)\bar{u}a_{.2}^L-\bar{u}a_{.1}^L)+(H_1(\tilde{A}_1^L)\bar{u}a_{.3}^L-\bar{u}a_{.1}^L)}{4} + \bar{u}a_{.1}^L \quad (9)$$

$$UCL\left(uDIT2_{trap(i)}\right) = \frac{UCL\left(uDIT2_{trap(i)}^U\right)+UCL\left(uDIT2_{trap(i)}^L\right)}{2}$$

For the lower control limit:

$$LCL\left(uDIT2_{trap(i)}^U\right) = \frac{(\bar{u}a_{.4}^U-\bar{u}a_{.1}^U)+(H_2(\tilde{A}_1^U)\bar{u}a_{.2}^U-\bar{u}a_{.1}^U)+(H_1(\tilde{A}_1^U)\bar{u}a_{.3}^U-\bar{u}a_{.1}^U)}{4} + \bar{u}a_{.1}^U$$

$$LCL\left(uDIT2_{trap(i)}^L\right) = \frac{(\bar{u}a_{.4}^L-\bar{u}a_{.1}^L)+(H_2(\tilde{A}_1^L)\bar{u}a_{.2}^L-\bar{u}a_{.1}^L)+(H_1(\tilde{A}_1^L)\bar{u}a_{.3}^L-\bar{u}a_{.1}^L)}{4} + \bar{u}a_{.1}^L \quad (10)$$

$$LCL\left(uDIT2_{trap(i)}\right) = \frac{LCL\left(uDIT2_{trap(i)}^U\right)+LCL\left(uDIT2_{trap(i)}^L\right)}{2}$$

For the center line:

$$CL\left(uDIT2_{trap(i)}^U\right) = \frac{(\bar{u}a_{.4}^U-\bar{u}a_{.1}^U)+(H_2(\tilde{A}_1^U)\bar{u}a_{.2}^U-\bar{u}a_{.1}^U)+(H_1(\tilde{A}_1^U)\bar{u}a_{.3}^U-\bar{u}a_{.1}^U)}{4} + \bar{u}a_{.1}^U$$

$$CL\left(uDIT2^{L}_{trap(i)}\right) = \frac{(\bar{u}a^{L}_{.4}-\bar{u}a^{L}_{.1})+(H_2(\tilde{A}^{L}_{1})\bar{u}a^{L}_{.2}-\bar{u}a^{L}_{.1})+(H_1(\tilde{A}^{L}_{1})\bar{u}a^{L}_{.3}-\bar{u}a^{L}_{.1})}{4} + \bar{u}a^{L}_{.1} \quad (11)$$

$$CL\left(uDIT2_{trap(i)}\right) = \frac{CL\left(uDIT2^{U}_{trap(i)}\right)+CL\left(uDIT2^{L}_{trap(i)}\right)}{2}$$

This transformation technique above can also be used to obtained the defuzzify each sample, such that every defuzzified sample point is monitored with the defuzzified control limits. And the criterion for in control is given as

$$UCL\left(uDIT2_{trap(i)}\right) < uDIT2_{trap(i)} < UCL\left(uDIT2_{trap(i)}\right) \quad (12)$$

otherwise, the process is out of control.

Application of the interval type-2 fuzzy u- control chart on production line

The method is applied on real time production process. A bakery confectionary, where some target nonconformities were observed, the features that were checked include level of burnt, shape, bread size and weight. The inspection was via physical visualization by the operators.

IV. Results

Table 1: Table of results of the study.

Sample no.	$uDIT2_{trap(i)}$	$0 < uDIT2_{trap(i)} < 0.181634$
1	0.116	in-control
2	0.116	in-control
3	0.096	in-control
4	0.136	in-control
5	0.083	in-control
6	0.096	in-control
7	0.116	in-control
8	0.119	in-control
9	0.096	in-control
10	0.136	in-control
11	0.096	in-control
12	0.096	in-control
13	0.076	in-control
14	0.116	in-control
15	0.116	in-control

The table 1 above is the result of the study which indicates that the product from the bakery confectionary is in-control. By implication, there is no need for production process adjustment. Likewise, the result can be considered as the phase one, whose parameters could be used to monitored subsequent production. The interval type-2 approach is adopted in this research due to the fact that there is existence of vagueness in the mode of the data collection.

V. Conclusion

This study of interval type-2 fuzzy u-control chart enables modeling the vagueness in the average nonconformity in production process. It is more flexible in handling uncertainty than the type-1 fuzzy set. However, data collected in the form of interval type 2 may not be applicable to the classical u-control chart without the modification of the control limits.

Future research: type 2 fuzzy control chart for nonconformities per unit. Comparison of the interval type 2 fuzzy and the type 2 fuzzy control chart for nonconformities per unit, intuitionistic fuzzy for nonconformities per unit, performance metrics of the methods can be analyzed.

References

1. Adepoju A. A. (2018). "Performance of fuzzy control chart over the traditional control chart" *Benin Journal of Statistics* Vol. 1, pp. 90-100
2. Adepoju, A. A., Usman Mohammed, Safiya Sada Sani, Kabiru Adamu, Kabiru Tukur , Aliyu Ismail Ishaq. (2019a). Statistical properties of negative binomial distribution under imprecise observation. *Journal of the Nigerian Statistical Association,* Vol. 31. Page 289-294.
3. Adepoju, A. A., A. M. Isah, S. Ahmed, Y. A. Wasiu, A. N. Samuel, A. Ibrahim. (2019b). Fuzzy-Shewhart Control Chart Based on α -Level Mid-Range Transformation and its Senstivity Measures. *Professional Statisticians Society of Nigeria. Edited Proceedings of 3rd International Conference.* **Vol. 3,** p. 488- 492.
4. Cheng C. B. (2005) Fuzzy process control: construction of control charts with fuzzy numbers. Fuzzy sets Syst 154:287-303.
5. Chen, L.-H., Huang, C.-H. (2016). Design of a fuzzy zone control chart for improving the process variation
6. Edmundas *K. Z., Mehdi K. G., Maghsoud A., Jamshid S. S.* (2015),Multi-Criteria Project Selection Using an Extended VIKOR Method with Interval Type-2 Fussy Sets. *International Journal of Information Technology and Decision Making.* Vol 14
7. El-Shal, S.M., Morris, A.S. (2000). A fuzzy rule-based algorithm to improve the performance of

statistical process control in quality systems. *Journal of Intelligent & Fuzzy Systems*, 9, 207–223.

8. Erginel N. Senturk S. Yildiz G. (2018). Modeling Attribute Control Charts by Interval type-2 Fuzzy sets. *Soft computing.* 22: 5033
9. Erginel N, Senturk S, Kahraman C, Kaya I (2011) Evaluating the packing process in food industry using Fuzzy $\tilde{X}$ and S control charts. Int JComput Intell Syst 4(4):509-520
10. Erginel, N. (2014). Fuzzy rule based p-np control charts. *Journal of Intelligent & Fuzzy Systems*, 27, 159–171. Monitoring capability. *Journal of Applied Sciences*, 16, 201–208.
11. Erginel, N. (2008). Fuzzy individual and moving range control charts with _-cuts. *Journal of Intelligent & Fuzzy Systems*, 19, 373–383.
12. Gulbay, M., Kahraman, C. (2006a). Development of fuzzy process control charts and fuzzy unnatural pattern analyses. *Computational Statistics and Data Analysis*, 51, 434–451.
13. Gulbay, M., Kahraman, C., Ruan, D. (2004). _-cut fuzzy control charts for linguistic data. *International Journal of Intelligent Systems*, 19, 1173–1196.
14. Hou, S.,Wang, H., Feng, S. (2016). Attribute control chart construction based on fuzzy score number. *Symmetry*, 8, 3–13.
15. J. M. Mendel, Type-2 fuzzy sets and systems: An overview [corrected reprint], IEEE Computational Intelligence Magazine 2(2) (2007) 20–29.
16. Kanagawa, A., Tamaki, F., Ohta, H. (1993). Control charts for process average and variability based on linguistic data. *Intelligent Journal of Production Research*, 31(4), 913–922.
17. Karnik, N.N., Mendel, J.M. (2001). Operations on type-2 fuzzy sets. *Fuzzy Sets and Systems*, 122, 327–348.
18. Kaya, İ.,Kahraman, C. (2011). Process capability analyses based on fuzzy measurement and fuzzy control charts. *Expert Systems with Applications*, 38, 3172–3184.
19. Kaya, İ., Erdoğan, M., Yıldız, C. (2017). Analysis and control of variability by using fuzzy individual control charts. *Applied Soft Computing*, 51, 370–381.
20. Kahraman, C., Oztayşi, B., Sarı, İ.U., Turanoˇglu, E. (2014). Fuzzy analytic hierarchy process with interval type 2 fuzzy sets. *Knowledge – Based Systems,* 59, 48–57.
21. Keshavarz Ghorabaee, M., Zavadskas, E.K., Amiri, M., Antucheviciene, J. (2016a). A new method of Assessment based on fuzzy ranking and aggregated weights (AFRAW) for MCDM problems under type-2 fuzzy environment. *Economic Computation and Economic Cybernetics Studies and Research,* 50(1), 39–68.
22. L. A. Zadeh, (1965) Fuzzy sets, Information and Control 8(3) 338–353.
23. L. A. Zadeh, (1975). The concept of a linguistic variable and its application to approximate reasoning_I, Information Sciences 8(3) 199–249.
24. Montgomery, Douglas, C.(2009) Introduction to Statistical Quality Control, Sixth Edition. John Wiley & Sons, Inc.
25. Mendel, J.M., John, R.I., Liu, F. (2006). Interval type-2 fuzzy logic systems made simple. *IEE Transactions on Fuzzy Systems*, 14(6), 808–821.
26. Poongodi, T.,Muthulakshmi, S. (2015). Fuzzy control chart for number of customer of Ek /M/1 queueing model. *International Journal of Advanced Scientific and Technical Research*, 3(5), 9–22.
27. Rowlands, H.,Wang, L.R. (2000). An approach of fuzzy logic evaluation and control in SPC. *Quality Reliability Engineering Intelligent*, 16, 91–98.
28. Şenturk, S. (2010). Fuzzy regression control chart based on _-cut approximation. *International Journal of Computational Intelligence Systems*, 3(1), 123–140.
29. Senturk S., Erginel N., Kaya I., Kahraman C. (2014) Fuzzy exponentially weighted moving average control chart for univariate data with a real case application, Applied Soft Computing, Vol 22, 1-10
30. Şenturk, S., Erginel, N. (2009). Development of fuzzy $\tilde{\tilde{X}} - \tilde{R}$ and $\tilde{\tilde{X}} - \tilde{S}$ control charts using α- cuts. *Information Sciences*, 179, 1542–1551.
31. Sevil Senturk, Jurgita Antucheviciene. (2017). Interval Type-2 Fuzzy c-Control Charts: An Application in a Food Company. *INFORMATICA*, Vol. 28, No. 2, 269–283
32. Wang, J.H., Raz, T. (1990). On the construction of control charts using linguistic variables. *Intelligent Journalof Production Research*, 28, 477–487.

1. Wang, D., Hyrnıewıcz, O. (2015). A fuzzy nonparametric shewhart charts based on the bootstrap approach. *International Journal of Applied Mathematics and Computer Science*, 25, 389–401.

APPENDIX

Data collected on the interval type-2 trapezoidal fuzzy number of nonconformities in the production process.

Sample point	Number of nonconformities				maximum membership degrees of the upper membership function		Number of nonconformities				Minimum membership degrees of the lower membership function		Sample size
1	3	4	5	6	1	1	2	3	4	5	0.7	0.5	50
2	2	3	4	6	1	1	1	2	3	5	0.8	0.6	50
3	2	3	3	5	1	1	1	2	3	4	0.7	0.6	50
4	4	5	6	7	1	1	3	4	5	6	0.8	0.6	50
5	3	4	5	6	1	1	2	2	3	4	0.6	0.5	50
6	2	3	4	4	1	1	1	2	3	4	0.8	0.6	50
7	3	4	5	6	1	1	2	3	4	5	0.9	0.8	50
8	3	3	5	6	1	1	1	2	4	5	0.7	0.5	50
9	2	3	4	5	1	1	1	2	3	4	0.8	0.6	50
10	4	5	6	7	1	1	3	4	5	6	0.8	0.7	50
11	2	3	5	6	1	1	1	2	3	4	0.6	0.5	50
12	2	4	5	6	1	1	1	2	3	4	0.7	0.6	50
13	2	3	4	5	1	1	1	2	3	3	0.7	0.5	50
14	4	5	6	7	1	1	3	4	5	5	0.8	0.7	50
15	3	4	5	6	1	1	2	3	4	5	0.9	0.8	50