# Optimal Design of Rectifying Single Sampling Plan with Inspection Errors

**S. T. Iorkegh*; P. A. Osanaiye**

Department of Statistics,
University of Ilorin, Ilorin, Nigeria.
E-mail: simkegh@yahoo.com*

***Abstract* — Two types of errors are inherent in every sampling inspection. They are type I and type II inspection errors. Type I inspection error occurs when a non-defective unit is classified as defective while type II inspection error occurs when a defective unit is classified as non-defective. In this paper, the effect of type I and type II inspection errors on the performance measures of Rectifying Single Sampling (RSS) plan is investigated. A modified Kumar's (2018) economic acceptance single-sampling plan model incorporating inspection error is used to determine an optimal sampling plan that minimizes the total cost and satisfies both the producer's and the consumer's risk requirement is determined. A comparison between the existing model and the modified model is made. Results showed that the Average Outgoing Quality (AOQ) and Average Outgoing Quality Limit (AOQL) decreased but the Average Total Inspection (ATI) increased as the type I inspection error increased and the type II error is at zero. On the other hand, as the type II inspection error increased and the type I inspection error is kept at zero, the AOQ and AOQL increased but the ATI decreased. Also, the optimal sampling plan in the modified model showed minimum total cost with lower producer's and consumer's risk as compared to the optimal sampling plan in the existing model. Therefore the modified model is found to perform better and is more economical than the existing model.**

.

***Keywords:*** *Average Outgoing Quality (AOQ), Average Outgoing Quality Limit, (AOQL) Average Total Inspection (ATI), producer's risk. Consumer's risk.*

## I. Introduction

In quality control, acceptance Sampling is one of the methods used by inspectors for lot sentencing. A sample is randomly taken from a lot and based on the acceptance sampling criteria a decision is taken to either accept or reject the lot. According to Amitava (2016), Acceptance sampling can be performed during the inspection of incoming raw materials, components, and assemblies, in various phases of in-process operations, or during final product or service inspection. It can be used as a form of product inspection between companies and their vendors, and between manufacturers and their customers.

An acceptance sampling plan is classified as either by variable or attributes. In acceptance sampling by variable, quality characteristics is measured using numerical value while in acceptance sampling plan by attributes**,** quality characteristics are expressed on a "go, no-go" basis (Montgomery 2009). A sampling plan may either be for Acceptance-Rejection or Acceptance–Rectification type. In Acceptance –Rejection, the lot is either accepted or rejected based on information obtained from the sample taken from the lot. While in Acceptance-Rectification if the lot is rejected based on the sample disposition, 100% inspection is carried out on the rejected lot and all defective units are replaced with non-defective units.

It is assumed that sampling inspection is with no error neglecting the fact that inspection errors are inherent in every sampling inspection even in Acceptance-Rectification sampling. Type I inspection error occurs if a non-defective unit is classified as defective while type II inspection error occurs if a defective unit is classified as a non-defective unit. The probability of type I and Type II inspection errors and the cost objective functions can be suitable factors in estimating the statistical reliability of a sampling system. Fallahnezhad *et al* (2017).

Duffua (1996) studied the statistical and economic effects of inspector errors on the sampling plan. Markowski and Markowski (2002) showed that it is necessary to consider inspection errors in designing sampling plans. They stated that ignoring the inspection errors led to suboptimal solutions for sampling plans.

Jamkhaneh et al. (2011) presented a single sampling plan with inspection errors, while the fraction of defective items is considered a fuzzy number. Chattinnawat (2013) used numerical methods to analyze the inspection errors in a single-sampling plan with zero acceptance numbers. Muhammad and Chang (2016) developed a model to reduce inspection costs in the acceptance single sampling plan by determining the optimal number of quality inspectors with respect to their skill levels using goal programming. Muhammad and Chang (2016) developed a model to reduce inspection costs in the acceptance single sampling plan by determining the optimal number of quality inspectors concerning their skill levels using goal programming.

Kumar(2018) developed an economic model to determine the optimal design parameters by minimizing total cost while satisfying both the producer's and the consumer's requirements

In this paper, the effect of type I and type II inspection errors on the performance measures of rectifying a single sampling plan is investigated. A modified Kumar (2018) economic single-sampling plan model incorporating inspection error is used to determine the optimal sampling plan in Rectifying Single Sampling (RSS) plan.

## II. Materials And Methods

### 2.1 Rectifying Single Sampling (RSS) plan

A random sample of size $n$ is randomly taken from a lot of size $N$, the number of defective units in the sample $x$ is compared with the acceptance number $c$. If $x \leq c$ the lot is accepted If $x > c$ the lot is rejected and the rejected lot is subjected to 100% inspection where all the defective units are replaced with non-defective units

Probability of acceptance $P_a$ is given as:

$$P_a = p(x \leq c) = \sum_{x=0}^{C} \binom{n}{x} p^x (1-p)^{n-x} \qquad (1)$$

where c =the acceptance number

n = sample size

p = the true fraction defective units

When inspection error is taken into consideration, the value of the true fraction defective p is replaced by the value of the apparent fraction defective ($p_e$ ). The probability of acceptance becomes:

$$P_{a_e} = \sum_{x=0}^{C} \binom{n}{x} [e_1(1-p) + p(1-e_2)]^x . [1 - e_1(1-p) - p(1-e_2)]^{n-x},$$

$$P_{a_e} = \sum_{x=0}^{C} \binom{n}{x} {p_e}^x (1-p_e)^{n-x} \qquad (2)$$

### 2.2 Average Outgoing Quality (AOQ)

Average Outgoing Quality (AOQ) represents the average quality of the stream of lots after rectifying inspection. The average outgoing quality limit (AOQL) is the maximum average quality of the stream of lots that leave the inspection station after rectifying the inspection.

The AOQ for RSS is:

$$AOQ = \frac{npe_2 + p(N-n)P_{a_e} + p(N-n)e_2(1-P_{a_e})}{N} \qquad (3)$$

**The average total inspection (ATI),** is the average number of units inspected per lot.

The ATI for a single sampling plan under inspection with no error assumption is calculated as:

$$\text{ATI} = n + (1-P_a)(N-n) \qquad (4)$$

When inspector error is considered, the amount of inspection is calculated as:

$$ATI_e = n + (1-P_{ae})(N-n) \qquad (5)$$

### 2.3 Proportion of Defective Units in Rectifying Single Sampling (RSS) Plan

According to Kumar (2018) proportion of defective units not detected ($Dn$) and the proportion of defective units detected ($Dd$) during rectifying sampling inspection in RSS under sampling with no error is stated below:

$$Dn = (N-n)pP_a \qquad (6)$$

$$Dd = np - p(1-P_a)(N-n) \qquad (7)$$

When inspection error is considered, a defective unit not detected ($Dn_e$) and the proportion of defective units detected ($Dd_e$) in the inspected sample and a rejected lot is stated below:

$$Dn_e = npe_2 + p(N-n)P_{a_e} + p(N-n)(1-P_{a_e})e_2 \qquad (8)$$

$$Dd_e = np(1-e_2) + p(N-n)(1-e_2)(1-P_{a_e}) \qquad (9)$$

### 2.4 Probability of detecting defective unit in a sample

Let $x$ be the true number of defective units taken from a sample of size $n$ and $x$ is a binomial random variable that ranges from 0 to.

The probability of observing no defective unit in a sample size (n) i.e when $x = 0$ is given as:

$$P(x=0) = (1-p)^n \qquad (10)$$

The probability of observing one or more defective units in the sample is:

$$P(x \geq 1) = 1 - P(x=0) = 1 - (1-p)^n \qquad (11)$$

Lot proportion defective according to Fallahnezhad et al (2018) is then given as:

$$p = 1 - (1-p)^n \qquad (12)$$

### 2.5 Apparent Proportion Defective ($p_e$)

Sampling inspection errors give false results by classifying a non-defective unit as defective leading to producer's risk or classifying a defective unit as non-defective resulting to the consumer's risk.

Let

$$e_1 = P\{unit\ is\ classified\ as\ defective | the\ unit\ is\ non-defective\}$$

$$e_2 = P\{unit\ is\ classified\ as\ non-defective | the\ unit\ is\ defective\}$$

Apparent fraction defective $p_e$ is thus obtained as:

$$p_e = (1-e_2)p + (1-p)e_1 \quad (13)$$

$$p_e = 1-(1-p)^n(1-e_2) + e_1(1-p)^n \quad (14)$$

### 2.6 Formulation of Producer's risk and Consumer's risk in the Design of

**Single Sampling Plan**

In the design of the acceptance single sampling plan, we chose an appropriate sample size (n) and acceptance number (c). Given producer's risk $\alpha$ and its associated quality level $p_1 = AQL$ as well as the consumer's risk β with its associated quality level $p_2 = LTPD$.

We formulate the probability of lot acceptance with inspection with no error at $p_1 = AQL$ as:

$$1-\alpha = P_a(x \le c|n, p_1 = AQL) = \sum_{x=0}^{C} \binom{n}{x} AQL^x (1-AQL)^{n-x} \quad (15)$$

$$1-\alpha = P_a(AQL) \quad (16)$$

The probability of rejection at $p_1 = AQL$ or producer's risk is:

$$1-P_a(AQL) = \alpha \quad (17)$$

When inspection error is considered, the AQL is replaced with the observed acceptable quality level (AQLe). Thus the formulation of the probability of acceptance $1-\alpha$ with inspection error for lot with quality level $p_1 = AQL_e$ is given as: :

$$1-\alpha = P_{ae}(x \le c|n, p_1 = AQL_e) = \sum_{x=0}^{C} \binom{n}{x} AQL_e{}^x (1-AQL_e)^{n-x} \quad (18)$$

$$= \sum_{x=0}^{C} \binom{n}{x} \{1-(1-AQL)^n(1-e_2) + e_1(1-AQL)^n\}^x \{(1-AQL)^n(1-e_2) + e_1(1-AQL)^n\}^{n-x} \quad (19)$$

So the probability of lot acceptance at $p_1 = AQLe$ is:

$$1-\alpha = P_{ae}(AQL_e) \quad (20)$$

The probability of lot rejection at $p_1 = AQLe$ or producer's risk $(\alpha)$ is thus:

$$1-P_{ae}(AQL_e) = \alpha \quad (21)$$

On the other hand, there is consumer's risk if the lot is accepted with quality level $p_2 = LTPD$.Thus the probability of accepting lot with quality level $p_2 = LTPD$ under inspection with no error is stated below:

$$\beta = P(x \le c|n.p_2 = LTPD) = \sum_{x=0}^{C} \binom{n}{x} p_2{}^x (1-p_2)^{n-x} \quad (22)$$

$$\beta = \sum_{x=0}^{c} \binom{n}{x} LTPD^X (1-LTPD)^{n-x} \quad (23)$$

$$\beta = P_a(LTPD) \quad (24)$$

When the inspection error is considered, the Lot Tolerant Percent Defective (LTPD) is replaced with the observed Lot Tolerant percent Defective (LTPDe) then consumer's risk (β) is formulated as shown below:

$$\beta = P_{ae}(x \le c|n, p_1 = LTPD_e) = \sum_{x=0}^{C} \binom{n}{x} LTPD_e{}^x (1-LTPD_e)^{n-x} \quad (25)$$

$$\beta = P_{ae}(x \le c|n, p_2 = LTPD_e) = \sum_{x=0}^{c} \binom{n}{x} [\{1-(1-LTPD)^n\}(1-e_2) + e_1(1-LTPD)^n]^x [\{1-[(1-LTPD)^n]\}(1-e_2) + e_1(1-LTPD)^n]^{n-x} \quad (26)$$

Thus, the probability of lot acceptance at $p_2 = LTPD_e$ or consumer's risk (β) is:

$$\beta = P_{ae}(LTPD_e) \quad (27)$$

### 2.7 Cost Minimization Model,

Kumar (2018) single-sampling cost model as presented below:

Minimize Total cost $(TC) = C_i ATI + C_f Dd + C_o Dn$ (28)

Subject to $1 - P_a(AQL) \le \alpha$

$P_a(LTPD) \le \beta$

where $TC$ is the total cost, $C_i$ is the cost of inspection per unit, $C_f$ is the internal failure cost (which include the cost of repair, scrap or rework of defective unit) and $C_o$ is the external failure cost or post-sales cost (which includes repair or replacement cost). α and β represent producer's and consumer's risk respectively.

### 2.8 Modified Cost Minimization Model

Kumar (2018) acceptance single sampling model is modified to incorporate inspection error with additional objective functions as shown below:

Minimize:

Total Cost $(TC) = C_i ATI_e + C_f Dd_e + C_o Dn_e$ (29)

Maximize $P_{a_e}(AQL_e)$

Minimize $P_{a_e}(LTPD_e)$

Subject to $1 - P_{a_e}(AQL_e) \le \alpha$

$P_{a_e}(LTPD_e) \le \beta$

$AQL_e$ and $LTPD_e$ represent apparent (observed) Acceptable Quality Level and apparent (observed) Lot Tolerant Percent Defective respectively. Other parameters with inspection errors are as stated above.

## III. Results And Discussion

The R- programming software and MS Word –Excel were used for the analysis and the results are presented in the sub-sections below.

### 3.1 The Effect of Inspection error on the Performance Measures of RSS plan

Results of the effects of type I error and type II error on the Average Outgoing Quality (AOQ), Average Outgoing Quality Limit (AOQL) and Average Total Inspection (ATI) of the Rectifying Sampling Plan (RSS) are presented in tables and figures below:

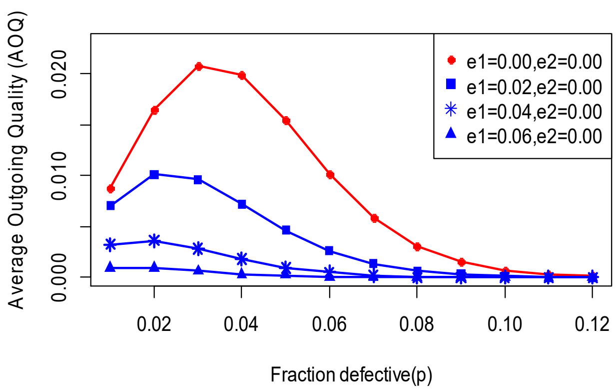


***Fig.3.1:Effect of type I inspection error on AOQ Curve of RSS Plan***

From Fig. 3.1, the AOQ for the situation when inspection with no error is assumed is uniformly higher than those with some form of type I inspection error. The AOQs however get lower as the type I inspection error increases and the type II inspection error is kept at zero. They all converge at p=0.12. The Average Outgoing Quality Limit (AOQL) is about 0.0208 at p=0.03 for inspection with no error is assumed but decreases to 0.0009 at p =0.02 as type I inspection error increases and the type II inspection error is kept at zero.

,

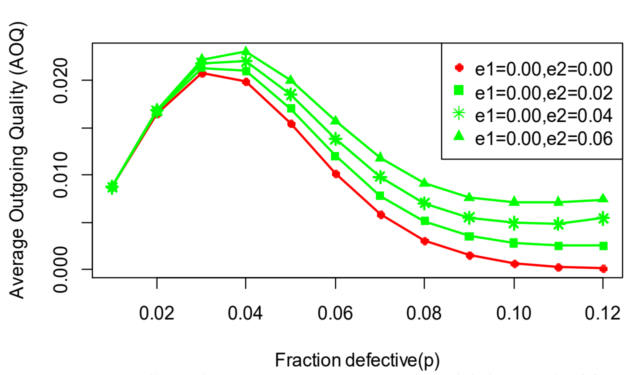


***Fig.3.2:Effect of type II inspection error on AOQ Curve of RSS Plan***

Fig 3.2 above, the AOQ for a situation when inspection with no error is assumed is uniformly lower than those with some form of type II inspection error. The values of AOQs however get higher as the type II inspection error increases and the type I inspection error is kept at zero. The Average Outgoing Quality Limit (AOQL) for inspection with no error assumption is 0.0208 at about p=0.03. However, as the type II inspection error increases and the type I inspection error is kept at zero, the AOQL increases to 0.0230 at p=0.04

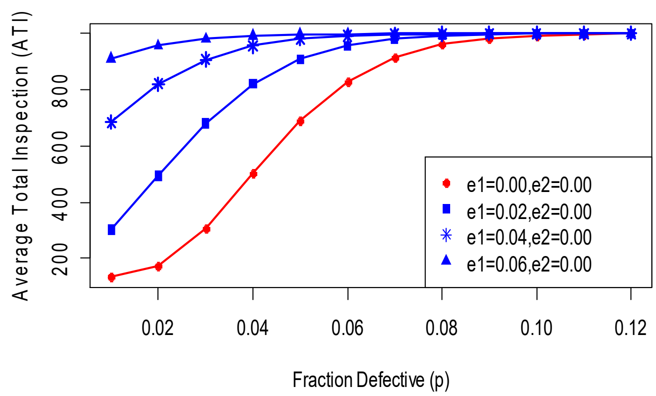


***Fig.3.3:Effect of type I inspection error on the ATI Curve of RSS Plan***

From fig. 3.3 above, the ATI when no error is assumed is uniformly lower than those of when type I inspection error is increased and type II inspection error is kept at zero. However, they all converge at about p=0.1.

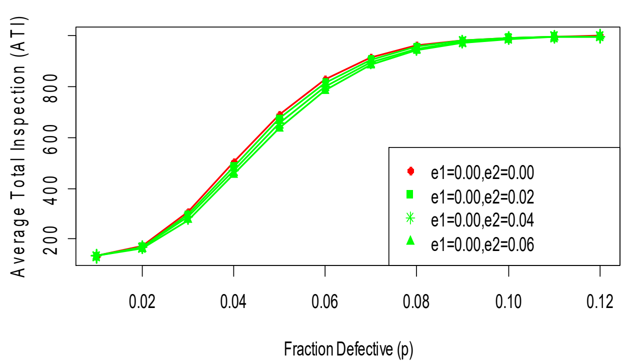


***Fig.3.4:Effect of type II inspection error on the ATI Curve of RSS Plan***

The ATI when no error is assumed is uniformly higher than those of when type II inspection error increased and type I inspection error is kept at zero. They are however very close and converge at about p=0.1.

**3.2 Determination of optimal Sampling Plans for RSS plan in the existing model and the modified model.**

.

**Table 1**: Rectifying Single Sampling (RSS) Plans with error-free inspection assumption satisfying the parameters AQL=0.02, LTPD=0.07, α=0.05, β=0.1, p=0.03 with$n \leq 250$

| $n$ | $c$ | $AOQ$ | $ATI$ | $D_n$ | $D_d$ | $1 - P_a(AQL)$ | $P_a(LTPD)$ | $P_a(p)$ | $TC$ |
|---|---|---|---|---|---|---|---|---|---|
| 196 | 7 | 0.0184 | 386.83 | 18.40 | 11.60 | 0.0448 | 0.0322 | 0.7627 | 593.99 |
| 196 | 8 | 0.0208 | 306.47 | 20.81 | 9.19 | 0.0180 | 0.0642 | 0.8626 | 532.92 |
| 197 | 7 | 0.0183 | 390.88 | 18.27 | 11.73 | 0.0459 | 0.0309 | 0.7586 | 597.07 |
| 197 | 8 | 0.0207 | 309.74 | 20.71 | 9.29 | 0.0185 | 0.0620 | 0.8596 | 535.41 |
| 198 | 7 | 0.0182 | 394.95 | 18.15 | 11.85 | 0.0470 | 0.0297 | 0.7544 | 600.16 |
| 198 | 8 | 0.0206 | 313.04 | 20.61 | 9.39 | 0.0190 | 0.0598 | 0.8566 | 537.91 |
| 199 | 7 | 0.0180 | 399.03 | 18.03 | 11.97 | 0.0482 | 0.0285 | 0.7503 | 603.26 |
| 199 | 8 | 0.0205 | 316.35 | 20.51 | 9.49 | 0.0196 | 0.0577 | 0.8535 | 540.42 |
| 200 | 7 | 0.0179 | 403.12 | 17.91 | 12,09 | 0.0493 | 0.0274 | 0.7461 | 606.37 |
| 200 | 8 | 0.0204 | 319.68 | 20.41 | 9.59 | 0.0202 | 0.0556 | 0.8504 | 542.95 |
| 201 | 8 | 0.0203 | 323.03 | 20.31 | 9.69 | 0.0208 | 0.0537 | 0.8473 | 545.50 |
| **201** | **9** | **0.0220** | **267.19** | **21.98** | **8.02** | **0.0077** | **0.0979** | **0.9172** | **503.07** |
| 202 | 8 | 0.0202 | 326.40 | 20.21 | 9.79 | 0.0214 | 0.0518 | 0.8441 | 548.06 |
| 202 | 9 | 0.0219 | 269.78 | 21.91 | 8.09 | 0.0080 | 0.0947 | 0.9151 | 505.03 |
| 203 | 8 | 0.0201 | 329.79 | 20.11 | 9.89 | 0.0220 | 0.0499 | 0.8409 | 550.64 |
| 203 | 9 | 0.0218 | 272.39 | 21.83 | 8.17 | 0.0083 | 0.0917 | 0.9129 | 507.02 |
| 204 | 8 | 0.0200 | 333.19 | 20.00 | 10.00 | 0.0226 | 0.0481 | 0.8377 | 553.23 |
| 204 | 9 | 0.0217 | 275.03 | 21.75 | 8.25 | 0.0085 | 0.0888 | 0.9108 | 509.02 |
| 205 | 8 | 0.0199 | 336.62 | 19.90 | 10.10 | 0.0232 | 0.0464 | 0.8344 | 555.83 |
| 205 | 9 | 0.0217 | 277.68 | 21.67 | 8.33 | 0.0088 | 0.0859 | 0.9086 | 511.04 |

**Table 2**: Rectifying Single Sampling (RSS) Plans with inspection error satisfying the parameters AQL=0.02, LTPD=0.07, α=0.05, β=0.1, p=0.03, with $n \leq 250$

| $n$ | $c$ | $AOQ_e$ | $ATI_e$ | $D_{ne}$ | $D_{de}$ | $1 - P_{ae}(AQL_e)$ | $P_{ae}(LTPD_e)$ | $P_{ae}(p)$ | $TC$ |
|---|---|---|---|---|---|---|---|---|---|
| 13 | 5 | 0.0237 | 212.67 | 23.68 | 6.32 | 0.0471 | 0.0967 | 0.7977 | 462.14 |
| 16 | 7 | 0.0231 | 231.49 | 23.12 | 6.88 | 0.0388 | 0.0412 | 0.7810 | 476.49 |
| 17 | 8 | 0.0241 | 199.03 | 24.09 | 5.91 | 0.0265 | 0.0413 | 0.8148 | 451.74 |
| 18 | 9 | 0.0248 | 177.35 | 24.82 | 5.18 | 0.0185 | 0.0402 | 0.8404 | 432.92 |
| 19 | 9 | 0.0217 | 279.32 | 21.70 | 8.30 | 0.0395 | 0.0125 | 0.7346 | 512.96 |
| 19 | 10 | 0.0254 | 155.50 | 25.38 | 4.62 | 0.0133 | 0.0382 | 0.8609 | 418.55 |
| 19 | 11 | 0.0276 | 79.26 | 27.65 | 2.35 | 0.0037 | 0.0974 | 0.9386 | 360.43 |
| 20 | 10 | 0.0225 | 253.65 | 22.47 | 7.53 | 0.0295 | 0.0117 | 0.7616 | 493.38 |
| 20 | 11 | 0.0258 | 141.07 | 25.81 | 4.19 | 0.0097 | 0.0356 | 0.8765 | 407.55 |
| 20 | 12 | 0.0278 | 73.04 | 27.83 | 2.17 | 0.0027 | 0.0910 | 0.9459 | 355.69 |
| 21 | 11 | 0.0231 | 233.27 | 23.07 | 6.93 | 0.0225 | 0.0107 | 0.7832 | 477.84 |
| 21 | 12 | 0.0261 | 130.03 | 26.14 | 3.86 | 0.0073 | 0.0326 | 0.8886 | 399.13 |
| 21 | 13 | 0.0280 | 68.52 | 27.96 | 2.04 | 0.0020 | 0.0838 | 0.9515 | 352.24 |
| 22 | 11 | 0.0195 | 352.56 | 19.53 | 10.47 | 0.0462 | 0.0027 | 0.6620 | 568.79 |
| 22 | 12 | 0.0236 | 217.12 | 23.55 | 6.45 | 0.0174 | 0.0096 | 0.8005 | 465.53 |
| 22 | 13 | 0.0264 | 121.59 | 26.39 | 3.61 | 0.0056 | 0.0293 | 0.8982 | 392.70 |
| 22 | 14 | 0.0281 | 65.28 | 28.06 | 1.94 | 0.0015 | 0.0763 | 0.9557 | 349.77 |

Table 2 continues

| $n$ | $c$ | $AOQ_e$ | $ATI_e$ | $D_{ne}$ | $D_{de}$ | $1-P_{ae}(AQL_e)$ | $P_{ae}(LTPD_e)$ | $P_{ae}(p)$ | $TC$ |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 12 | 0.0201 | 332.46 | 20.13 | 9.87 | 0.0371 | 0.0023 | 0.6833 | 553.47 |
| 23 | 13 | 0.0239 | 204.40 | 23.83 | 6.07 | 0.0138 | 0.0084 | 0.8143 | 455.84 |
| 23 | 14 | 0.0266 | 115.21 | 26.58 | 3.42 | 0.0044 | 0.0261 | 0.9056 | 387.83 |
| **23** | **15** | **0.0281** | **63.02** | **28.13** | **1.87** | **0.0012** | **0.0687** | **0.9590** | **348.05** |
| 24 | 13 | 0.0206 | 316.24 | 20.61 | 9.39 | 0.0302 | 0.0020 | 0.7006 | 541.10 |
| 24 | 14 | 0.0242 | 194.47 | 24.22 | 5.78 | 0.0111 | 0.0073 | 0.8253 | 448.26 |
| 24 | 15 | 0.0267 | 110.45 | 26.72 | 3.28 | 0.0035 | 0.0228 | 0.9114 | 384.21 |
| 25 | 14 | 0.0210 | 303.28 | 20.99 | 9.01 | 0.0249 | 0.0017 | 0.7146 | 531.22 |
| 25 | 15 | 0.0245 | 186.82 | 24.45 | 5.55 | 0.0091 | 0.0062 | 0.8341 | 442.43 |
| 26 | 15 | 0.0213 | 293.11 | 21.29 | 8.71 | 0.0209 | 0.0014 | 0.7258 | 523.46 |
| 27 | 15 | 0.0175 | 422.00 | 17.47 | 12.53 | 0.0431 | 0.0003 | 0.5940 | 621.73 |

Tables 2 and 3 represent the sampling plans generated that satisfy the conditions in the two models stated above. The existing model shows an optimal sampling plan of $n = 201, C = 9$ with a minimum total cost of 503.07. In the modified model the optimal sampling plan is $n = 23, C = 15$ with a minimum total cost of 348.05. Table 3 shows minimum values for sample size$(n)$, the ATI, producer's risk $(1 - P_{ae}(AQLe))$, consumer's risk $\left(P_{ae}(LTPDe)\right)$ and the Total Cost (TC) in the optimal sampling plan of the modified model than the optimal values in the existing model. This suggests that the modified model is more economical than the existing model.

**Table 3:** Comparison of the probability of acceptance of optimal sampling plans for RSS plans under the existing model and the modified model

| $\alpha = 0.05$ | | $\beta = 0.1$ | |
|---|---|---|---|
| Existing model optimal sampling plan: $n = 201, c = 9$ | | modified model optimal sampling plan: $n = 23, c = 15$ | |
| $p$ | $P_a$(p) | $p$ | $Pa_e$(p) |
| 0.01 | 1.0000 | 0.01 | 1.0000 |
| 0.02(AQL) | 0.9923=$P_a(AQL)$ | 0.02 | 0.9988=$P_{ae}(AQL_e)$ |
| 0.03 | 0.9172 | 0.03 | 0.9590 |
| 0.04 | 0.7141 | 0.04 | 0.7659 |
| 0.05 | 0.4483 | 0.05 | 0.4534 |
| 0.06 | 0.2290 | 0.06 | 0.1994 |
| 0.07(LTPD) | 0.0978=$P_a(LTPD)$ | 0.07 | 0.0687=$P_{ae}(LTPD_e)$ |
| 0.08 | 0.0358 | 0.08 | 0.0197 |
| 0.09 | 0.0115 | 0.09 | 0.0049 |
| 0.1 | 0.0033 | 0.1 | 0.0011 |

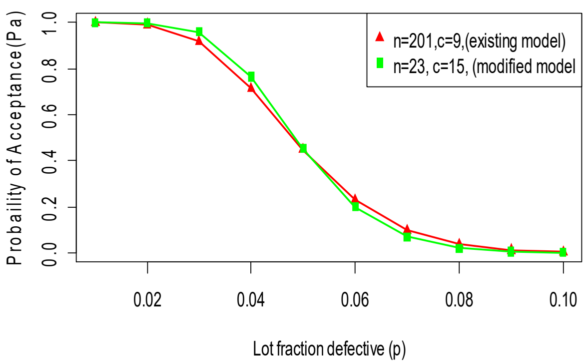


***Fig.3.5:OC curves for the RSS plans under the existing and modified models***

Fig.3.5 above is the operating characteristics curve for the RSS plan under the existing model and the modified model. Generally, the probability of acceptance for all the sampling plans decreased as the fraction defective units of the lot increased. All the models show a high probability of acceptance at $AQL \leq 0.02$ and a low probability of acceptance at $LTPD \geq 0.07$. However the probability of acceptance of the modified model is higher than that of the existing model $AQL \leq 0.02$ and lower than the existing model at $LTPD \geq 0.07$. The producer's risk (α) and the consumer's risk(β) in the optimal sampling plan of the modified model are 0.0012 or 0.12% and 0.0687 or 6.87% respectively which is lower than the producer's risk (α) of 0.0077 or 0.77% and consumer's risk (β) of 0.0978 or 9.78% respectively in the optimal sampling plan of the existing model. This showed that the modified model performed better and is more discriminatory than the existing model.

**Table 4**: The Average Total Inspection (ATI) for optimal RSS plans under the existing model and the modified model

| Optimal RSS plan (existing model): $n = 201, c = 9$ | | Optimal RSS plan (modified model): $n = 23, c = 15$ | |
|---|---|---|---|
| $p$ | ATI | $p$ | ATI |
| 0.01 | 201.03 | 0.01 | 23.00 |
| 0.02 | 207.18 | 0.02 | 24.14 |
| 0.03 | 267.19 | 0.03 | 63.02 |
| 0.04 | 429.41 | 0.04 | 251.71 |
| 0.05 | 641.79 | 0.05 | 557.07 |
| 0.06 | 816.99 | 0.06 | 805.21 |
| 0.07 | 921.86 | 0.07 | 932.84 |
| 0.08 | 971.36 | 0.08 | 980.76 |
| 0.09 | 990.79 | 0.09 | 995.19 |
| 0.1 | 997.35 | 0.1 | 998.90 |

From table 5 and fig.3.6, the Average Total Inspection (ATI) for the two models generally increase as the lot fraction defective ($p$) units increased. The Average Total Inspection (ATI) in the modified model is lower than that of the existing model at $AQL \leq 0.02$ and higher than that of the existing model at $LTPD \geq 0.07$. The lower ATI of the modified model at $AQL \leq 0.02$ is due to a higher probability of acceptance which resulted to less inspection of the lot. The ATI in the modified model also increased more than that of the existing model at $LTPD \geq 0.07$ quality level. The increase in ATI is a result of 100% inspection of the rejected lot which is caused by the increased probability of lot rejection at $LTPD \geq 0.07$

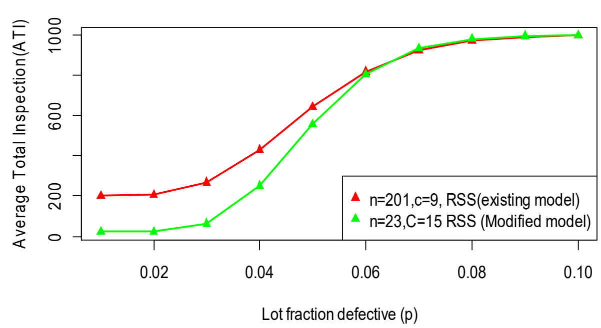


***Fig.3.6:Average Total Inspection(ATI) for Optimal Sampling Plans for RSS***

**Table 6**: Total Cost for optimal RSS plans of the existing model and the modified model

| RSS plan(existing model) | | | | RSS plan(modified model) | | | |
|---|---|---|---|---|---|---|---|
| $p$ | $n$ | $c$ | $TC$ | $n$ | $c$ | $TC$ | Difference in $TC$ |
| 0.01 | 201 | 9 | 284.95 | 23 | 15 | 121.18 | 57.47% |
| 0.02 | 201 | 9 | 374.03 | 23 | 15 | 220.31 | 41.09% |
| 0.03 | 201 | 9 | 503.07 | 23 | 15 | 348.05 | 30.81% |
| 0.04 | 201 | 9 | 692.00 | 23 | 15 | 571.92 | 17.35% |
| 0.05 | 201 | 9 | 885.07 | 23 | 15 | 836.47 | 5.49% |
| 0.06 | 201 | 9 | 1024.84 | 23 | 15 | 1022.57 | 0.22% |
| 0.07 | 201 | 9 | 1105.62 | 23 | 15 | 1115.68 | -0.91% |
| 0.08 | 201 | 9 | 1149.69 | 23 | 15 | 1159.35 | -0.84% |
| 0.09 | 201 | 9 | 1177.42 | 23 | 15 | 1185.82 | -0.71% |
| 0.1 | 201 | 9 | 1199.47 | 23 | 15 | 1207.77 | -0.69% |

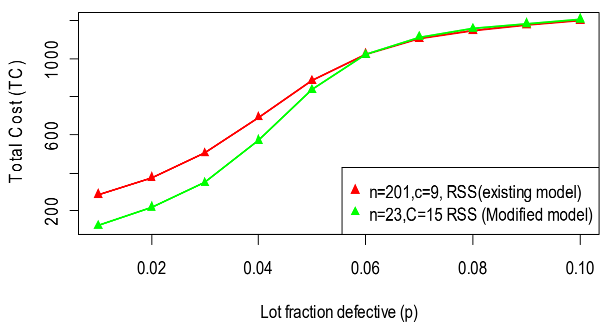

***Fig.3.7:Total Cost(TC) for Optimal Sampling Plans for RSS***

From table 6 and fig.3.7 above, it is also noted that the total cost generally increased with an increase in the fraction defective unit (p) of the lot in all the models. However, the total cost in the modified model is lower at $AQL \leq 0.02$ but higher than the existing model at $LTPD \geq 0.07$. This is because the probability of acceptance of the modified model is higher at $AQL \leq 0.02$ resulting to less inspection hence the decrease in total cost. On the other, the probability of rejection in the modified model is higher at $LTPD \geq 0.07$ than in the existing model. Therefore 100% inspection is carried out on the rejected lot hence the increase in the total cost.

## IV. Conclusion

In this paper, the effect of sampling inspection error on the performance measures of Rectifying Single Sampling (RSS) is investigated. A modified Kumar (2018) economic cost model is used to determine the optimal sampling plan for RSS that minimized the total cost. A comparison between the existing model and the modified model is made. The results showed that incorporating inspection error into Kumar's model yielded a better and more economical result.

## References

Amitava, M. (2016). *Fundamentals of Quality Control and Improvement*. 4th Edition, John Willey & Sons, Inc, New Jersey, Canada, Pp.502-510

Anuja, S., Ratika, S., Tulka, S., and Singh, V. (2013). A Novel Method for Dynamic Sampling Plan and Inspection Policies for Quality Assurance. *Asian Research Journal of Business Management*, 2(1):34-43

Banovac, E., Darko, P., and Nikola, V. (2012). Mathematical Aspect of Acceptance Sampling Procedure. *International Journal of Mathematical Models and Methods in Applied Sciences,* 5(6):1-9

Castillo –Villar, K. K.., Smith, N. R.., and Simonton, J. L. (2012). The Impact of the Cost of Quality on Serial-chain Network Design. *International Journal of Production Research,* 50(19): 5544-5566.

Chen, C., Espanchin-Niell, R. S., and Haight, G. R. (2016). Optimal Inspection of Imports to prevent Invasive Pest introduction.pp.1-40

Ching-Ho, Y., Heng, M., Chi-Huang, Y., and Chia-Hao, C. (2015).The Economic Design of Variable Acceptance Sampling Plan with Rectifying Inspection. *Kybernetes,* 44(3):440-450.

Ezzatallah, B., and Bahram, S. (2011). Chain Sampling Plan using Fuzzy Probability Theory. *Journal of Applied Sciences.* 11(24): 3830-3838.

Fallahnezhad, M. S., Sajjadieh M., Abdollahi, P. (2014). An Iterative Decision Rule to Minimize Cost of Acceptance Sampling Plan in Machine Replacement Problem. *International Journal of Engineering,* 27(7): 1099-1106.

Fallahnezhad, M. S., Ahmadi, Y. (2016). A New Optimization Model for Designing Acceptance Sampling plan Based on Run Length of conforming units. *Journal of Industrial and System Engineering,* 9(2): 67-87.

Fallahnezhad,M,Qazvini,E and Abessi, M. (2017).Designing and Economical Acceptance Sampling plan in the presence of inspection errors based on Maxima Nomination Sampling Method. *Scientia Iranica,* 25(3): 1701-1711.

Frishman, F. (1960). An extended Chain Sampling Inspection Plan. *Journal of Industrial Quality Control,* 17: 10-12.

Gao, F. (2003). Studies on Chain Sampling Schemes in Quality and Reliability Engineering. A Thesis Submitted for the Degree of Doctor of Philosophy Department of System and Industrial Engineering. National University of Singapore.

Hagenimana et al. (2016). Technical Application for Inspection Sampling for Repairable Systems in an Economic System. SpringPlus.Open Acess.pp.1-20.

Hoang, P. (2006). *Springer Handbook of Engineering Statistics*. Springer-verlag London Limited, Pp.264.

Hsu, L., and Hsu, J. (2012). Economic Design of Acceptance Sampling plans in a Two-Stage Supply Chain. *Advances in decision Science,* 2012: 1-14.

Jalbout,A Hadi,S Alkahby,Y , Fouad N. Jalbout , Abdulla Darwish (2002). The Effects of Inspection Errors on the Average Outgoing Quality in an Industrial Process. *Electronic Journal of Mathematical and Physical Sciences.vol.1, pp.15-21*

kumar S. (2018).An optimal Single Sampling Plan for Minimizing the Producer's total cost and providing protection for both producer and consumer. *International Journal of current Microbiology and applied sciences.7(2):Pp.* 3759-3768

Mistra, R. C., and Sandilya, A. (2009). *Reliability and Quality management*. New Age International Publishers, Ansari Road, Daryaganj, New Delhi, Pp.132-133.

Montgomery. D., and Jenings, C. (2011). *Managing, Controlling and Improving Quality*. New Jersey: John Willey and Sons, Pp.632-636

Montgomery, D. C. (2009). *Introduction to Statistical Quality Control*. 6th Edition, John Willey & Sons, Inc.Arizona USA, Pp. 632-650.

Muhammad, B., and Chang, W. (2016). Minimization of Inspection Cost by Determining the Optimal Number of Quality Inspectors in the Garment Industry. *Indian Journal of fibre and textile research, 41*: 346-350.

Muhammad, A. S., Randolph, K. Henriqueta, N., and Antonio, A. (2017). Cost of Quality: Evaluating Cost –Quality Trade-offs for Inspections Strategies of Manufacturing Processes. *International Journal of Production Economics,* 188: 156-166.

Namin, S., Pakzad, A., and Nezhad, M. (2017). A DEA – Bases Approach for Multi-Objective Design of Attribute Acceptance Sampling Plans. *International Journal of Data Envelopment Analysis,* 5(2): 12-24.

Nezhad, M. S., and Nasab, H. H. (2011). Designing a Single Stage Acceptance Sampling Plan Based on the Control Threshold Policy. *International Journal of Industrial and Production Research,* 22(3): 143-150.

Nirma, M., Sabramania, R., and Uma, G. (2016). Selection of Optimum Single Sampling Plans under Prior Binomial Distribution by Minimizing the Average Acceptance Cost. *International Journal of Scientific and Research Publications,* 6(9):325-338

Subramaniam, S., and Yuvaraj, V. (2011). Bayesian Single Sampling Plan for Attributes based on a Two point Prior Distribution. *International Journal of Civil Engineering Research,* 2 (2): 283-290.

Valtteri, T. (2012). Cost Modelling of Inspection Strategies in Automotive Quality Control. *Journal of Engineering Management Research,* vol.1(2): 33-38

Yinfeng , G, , and Loon, C. T.,(2006).Chain Sampling Scheme under Constant Inspection Errors.*Quality and Reliability Engineering Journal.Vol.22, pp.889-903.*

.