Improved Ratio-Cum-Product Estimator for Estimation of Finite Population Mean

J. O. Muili[1*]; A. Audu[2]; A. Adebiyi[2]

[1]Department of Mathematics,
Kebbi State University of Science and Technology Aliero, Nigeria.

[2]Department of Statistics,
Usmanu Danfodiyo University, Sokoto, Nigeria.
e-mail: jamiunice@yahoo.com[1]

***Abstract* - A ratio-cum-product estimator for estimating the finite population mean of the study variable in simple random sampling without replacement (SRSWOR) has been suggested. The aim is to develop an efficient estimator to improve the precision of the estimate using the information of auxiliary variable for both positive and negative correlations. The expressions of the bias and mean square error (MSE) of the suggested estimator are derived by Taylor's series method up to the first degree of approximation. The efficiency conditions under which the proposed ratio-cum-product estimator is better than the sample mean, product estimator, and other estimators considered in this study have been established. The numerical results show that the suggested estimator is better and more efficient than the sample mean, product estimator and other existing estimators.**

Keywords - *Product estimator, Finite Population, Auxiliary variable, and Population mean.*

I. Introduction

Survey sampling is a branch of statistics that deals with the estimation of population parameters (mean, variance, or total population) under study with the aid of auxiliary variable(s) to increase the precision of the estimate of the study variable. In a situation where auxiliary information is available, it is possible to devise suitable ways of using it in obtaining the sample strategies which are better than those in which no such information is used. When the information on an auxiliary variable X is known, a ratio, product, or linear regression estimator could be employed for the estimation of finite population mean. The ratio and product methods of estimation are used for the estimation of finite population mean when the association (correlation) or relationship between the study variable and the auxiliary variable is positive and negative respectively.

Cochran (1940) initiated an important contribution to the modern sampling theory by suggesting methods of using auxiliary information for the estimation of population mean to increase the precision of the estimates. Many researchers have suggested ratio and product type estimators for the estimation of finite population mean of study variable using different population parameters (auxiliary variables) such as Upadhyaya and Singh (1999), Abu-Dayyeh (2003), Singh *et al*. (2004), Kadilar and Cingi (2004), Yan and Tian (2010), Tailor *et al*. (2011), Jeelani *et al*. (2013), Gupta and Yadav (2017), Muili and Audu (2019), Muili *et al*. (2019), Muili *et al*. (2020), etc.

Let $U=\{U_1,U_2,U_3,...,U_N\}$ be a finite population having N units and each $U_i=(X_i,Y_i)$, $i=1,2,3,...,N$ has a pair of values. Y is the study variable and X is the auxiliary variable which is correlated with Y. $y=\{y_1,y_2,...,y_n\}$ and $x=\{x_1,x_2,...,x_n\}$ be n sample values. $\bar{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables respectively. S_y^2 and S_x^2 are the population mean squares of Y and X respectively. s_y^2 and s_x^2 be respective sample mean squares based on the random sample of size n drawn without replacement. N : Population size, n: Sample size, $\bar{Y},\bar{X}$: Population means of study and auxiliary variables ρ_{yx} : Coefficient of correlation, C_y, C_x : Coefficient of variations of study and auxiliary variables,

$\beta_{2(x)}$: Coefficient of Kurtosis of auxiliary variable, M_d :Median of the auxiliary variable, TM : Tri-Mean

$$\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,\ \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i, \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\ \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,$$

$$s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2,\ \ s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2,$$

$$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2,\ S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2,$$

$$\gamma=\frac{1-f}{n},\ f=n/N,\ C_y^2=\frac{S_y^2}{\bar{Y}^2},\ and\ C_x^2=\frac{S_x^2}{\bar{X}^2}$$

II. Some Existing Estimators of Population Mean

The usual sample mean $(\bar{y})$ in simple random sampling without replacement is given as:

$$\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i \tag{1.0}$$

$$V(\bar{y})=\gamma\bar{Y}^2C_y^2 \tag{1.1}$$

Cochran (1940) proposed a ratio estimator for the estimation of the population mean $(\bar{Y})$ of the study variable (Y) which can only be used when the coefficient of correlation between the study variable and the auxiliary variable is positive. The ratio estimator, bias and mean square error are given respectively as:

$$t_R=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.2}$$

$$Bias\left(t_R\right)=\gamma\bar{Y}\left(C_x^2-\rho_{yx}C_yC_x\right) \tag{1.3}$$

$$MSE\left(t_R\right)=\gamma\bar{Y}^2\left(C_y^2+C_x^2-2\rho_{yx}C_yC_x\right) \tag{1.4}$$

Robson (1957) proposed a product estimator for estimating the population mean $(\bar{Y})$ of the study variable (Y) given as:

$$t_P=\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \tag{1.5}$$

$$Bias\left(t_P\right)=\gamma\bar{Y}\left(C_x^2+\rho C_yC_x\right) \tag{1.6}$$

$$MSE\left(t_P\right)=\gamma\bar{Y}^2\left(C_y^2+C_x^2+2\rho C_yC_x\right) \tag{1.7}$$

Upadhyaya and Singh (1999) developed ratio and product estimators for the estimation of population mean using known values of coefficient of variation(C_x) and coefficient of kurtosis $(\beta_2(x))$ of variable variables with their biases and mean squares errors (MSEs) given as:

$$t_1=\bar{y}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right) \tag{1.8}$$

$$t_2=\bar{y}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}\right) \tag{1.9}$$

$$t_3=\bar{y}\left(\frac{\bar{x}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right) \tag{1.10}$$

$$t_4=\bar{y}\left(\frac{\bar{x}\beta_2(x)+C_x}{\bar{X}\beta_2(x)+C_x}\right) \tag{1.11}$$

$$Bias\left(t_1\right)=\gamma\bar{Y}\left(\lambda_1^2C_x^2-\lambda_1\rho C_yC_x\right) \tag{1.12}$$

$$Bias\left(t_2\right)=\gamma\bar{Y}\left(\lambda_2^2C_x^2-\lambda_2\rho C_yC_x\right) \tag{1.13}$$

$$Bias\left(t_3\right)=\gamma\bar{Y}\left(\lambda_1^2C_x^2+\lambda_1\rho C_yC_x\right) \tag{1.14}$$

$$Bias\left(t_4\right)=\gamma\bar{Y}\left(\lambda_2^2C_x^2+\lambda_2\rho C_yC_x\right) \tag{1.15}$$

$$MSE\left(t_1\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_1^2C_x^2-2\lambda_1\rho C_yC_x\right) \tag{1.16}$$

$$MSE\left(t_2\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_2^2C_x^2-2\lambda_2\rho C_yC_x\right) \tag{1.17}$$

$$MSE\left(t_3\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_1^2C_x^2+2\lambda_1\rho C_yC_x\right) \tag{1.18}$$

$$MSE\left(t_4\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_2^2C_x^2+2\lambda_2\rho C_yC_x\right) \tag{1.19}$$

where $\lambda_1=\dfrac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$ *and* $\lambda_2=\dfrac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$

Singh *et al.* (2004) developed ratio and product types estimators for estimating the population mean $(\bar{Y})$ of the study variable (Y). The biases and mean square error are given as:

$$t_5=\bar{y}\left(\frac{\bar{X}+\beta_{2(x)}}{\bar{x}+\beta_{2(x)}}\right) \tag{1.20}$$

$$t_6=\bar{y}\left(\frac{\bar{x}+\beta_{2(x)}}{\bar{X}+\beta_{2(x)}}\right) \tag{1.21}$$

$$Bias\left(t_5\right)=\gamma\bar{Y}\left(\lambda_3^2C_x^2-\lambda_3\rho C_yC_x\right) \tag{1.22}$$

$$MSE\left(t_5\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_3^2C_x^2-2\lambda_3\rho C_yC_x\right) \quad (1.23)$$

$$Bias\left(t_6\right)=\gamma\bar{Y}\lambda_3\rho C_yC_x \quad (1.24)$$

$$MSE\left(t_6\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_3^2C_x^2+2\lambda_3\rho C_yC_x\right)$$
(1.25)

where $\lambda_3=\dfrac{\bar{X}}{\bar{X}+\beta_{2(x)}}$

Yousuf and Rather (2021) modified a product-type estimator for the estimation of population mean using a linear combination of Coefficient of Correlation and Median of auxiliary variables as:

$$t_7=\bar{y}\left(\frac{\rho\bar{x}+M_d}{\rho\bar{X}+M_d}\right) \quad (1.26)$$

$$Bias\left(t_7\right)=\gamma\bar{Y}\lambda_4\rho C_yC_x \quad (1.27)$$

$$MSE\left(t_7\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_4^2C_x^2+2\lambda_4\rho C_yC_x\right) \quad (1.28)$$

where $\lambda_4=\dfrac{\rho\bar{X}}{\rho\bar{X}+M_d}$,

III. The Proposed Estimator

We proposed a ratio-cum-product estimator for estimating the population mean using a linear combination of coefficient of correlation and median as:

$$\hat{\bar{Y}}_p=\bar{y}\left[\beta\left(\frac{\rho\bar{X}+M_d}{\rho\bar{x}+M_d}\right)+\left(1-\beta\right)\left(\frac{\rho\bar{x}+M_d}{\rho\bar{X}+M_d}\right)\right] \quad (1.29)$$

To derive the bias and MSE, we define $e_0=\dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$, and $e_1=\dfrac{\bar{x}-\bar{X}}{\bar{X}}$ such that $\bar{y}=\bar{Y}\left(1+e_0\right)$, $\bar{x}=\bar{X}\left(1+e\right)$,

from the definitions of e_0, and e_1, we obtain

$$\left.\begin{array}{l}E\left(e_0\right)=E\left(e_1\right)=0,\ E\left(e_0^2\right)=\gamma C_y^2\\ E\left(e_1^2\right)=\gamma C_x^2,\ E\left(e_0e_1\right)=\gamma\rho_{yx}C_yC_x\end{array}\right\} \quad (1.30)$$

Expressing (1.29) in terms of e_0 and e_1 we have

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left[\begin{array}{l}\beta\left(\dfrac{\rho\bar{X}+M_d}{\rho\left(1+e_1\right)\bar{X}+M_d}\right)\\+\left(1-\beta\right)\left(\dfrac{\rho\left(1+e_1\right)\bar{X}+M_d}{\bar{X}+M_d}\right)\end{array}\right] \quad (1.31)$$

Reducing (1.31), we have

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left[\beta\left(1+\lambda_pe_1\right)^{-1}+\left(1-\beta\right)\left(1+\lambda_pe_1\right)\right] \quad (1.32)$$

where $\lambda_p=\dfrac{\rho\bar{X}}{\rho\bar{X}+M_d}$,

Simplifying (1.32) up to first-order approximation, it reduces to (1.33) as:

$$\hat{\bar{Y}}_p=\bar{Y}\left[\begin{array}{l}1+e_0+\left(1-2\beta\right)\lambda_pe_1\\+\left(1-2\beta\right)\lambda_pe_0e_1+\rho\lambda_p^2e_1^2\end{array}\right] \quad (1.33)$$

Applying the results of (1.30) to (1.34) gives the bias as:

$$Bias\left(\hat{\bar{Y}}_p\right)=\gamma\bar{Y}\left[\beta\lambda_p^2C_x^2+\left(1-2\beta\right)\lambda_p\rho C_yC_x\right] \quad (1.34)$$

where $\beta=\dfrac{1}{2}\left(1+\dfrac{\rho C_y}{\lambda_pC_x}\right)$

Squaring and taking the expectation of (1.33), gives

$$MSE\left(\hat{\bar{Y}}_p\right)=\bar{Y}^2E\left[e_0+\left(1-2\beta\right)\lambda_pe_1\right]^2 \quad (1.35)$$

Expanding and applying the results of (1.30) to (1.36), gives

$$MSE\left(\hat{\bar{Y}}_p\right)=\gamma\bar{Y}^2\left[\begin{array}{l}C_y^2+\left(1-2\beta\right)^2\lambda_p^2C_x^2\\+2\left(1-2\beta\right)\lambda_p\rho C_yC_x\end{array}\right]$$
(1.36)

Obtaining the expression for the value of β, differentiate $MSE\left(\hat{\bar{Y}}_p\right)$ partially with respect to β and equate to zero then simplifying for β, obtaining an optimum value of β and Substitute in (1.36) gives:

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min}=\gamma\bar{Y}^2C_y^2\left[1-\rho^2\right] \quad (1.37)$$

3.1 Comparison of Efficiency

The condition under which the proposed estimator will have minimum mean square error compared to the sample mean, ratio estimator, product estimator and other existing estimators have been derived as follows:

The proposed estimator of the population mean is more efficient than the sample mean if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < V\left(\bar{y}\right)$$

$$\gamma \bar{Y}^2 C_y^2 \left[1-\rho^2\right] < \gamma \bar{Y}^2 C_y^2 \qquad (1.38)$$

The proposed estimator of the population mean is more efficient than the ratio estimator if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < MSE\left(\hat{\bar{Y}}_R\right)$$

$$\gamma \bar{Y}^2 C_y^2 \left[1-\rho^2\right] < \gamma \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho C_y C_x\right)$$

(1.39)

The proposed estimator of the population mean is more efficient than ratio-type estimators if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < MSE\left(\hat{\bar{Y}}_j\right) \qquad j = 1,2,3$$

$$\gamma \bar{Y}^2 C_y^2 \left[1-\rho^2\right] < \gamma \bar{Y}^2 \left(C_y^2 + \theta \lambda_j^2 C_x^2 + 2\lambda_j \rho C_y C_x\right)$$

(1.40)

When conditions (1.38), (1.39) and (1.40) are satisfied, we conclude that the proposed estimator is more efficient than the sample mean, ratio estimator, product estimator and other existing estimators considered in the study.

3.2 Empirical Study

To assess the performance of the proposed estimator, we considered the two populations as:
Auxiliary variable (X) = Fixed Capital. Study variable (Y) = Output of 80 factories

Table 1: Parameters of the Populations

Parameter	Population I	Population II
N	30	80
n	10	20
$\bar{Y}$	17.5	51.8264
$\bar{X}$	4.4637	11.2646
ρ	-0.1994	0.9413
C_y	0.4758	0.3542
C_x	0.8727	0.750
$\beta_{2(x)}$	0.2296	2.866
$\beta_{1(x)}$	1.36	1.05
M_d	2.27	7.575

Source: [Population I: Yadav *et al*. (2016). Population II: Murthy (1967)]
Table 1 shows the descriptive statistics of the two populations.

Table 2: The Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the Proposed and Other Estimators

Estimator	Population I		Population II	
	MSE	PRE	MSE	PRE
Ratio Estimator	341.5092	100	498.2401	100
Product Estimator	243.4638	140.271	3151.239	15.81093
Upadhyaya and Singh (1999) t_1	314.3803	108.130	174.1301	286.1309
Upadhyaya and Singh (1999) t_2	159.2658	214.4272	461.379	107.9893
Upadhyaya and Singh (1999) t_3	221.7920	153.9772	2155.113	23.11898
Upadhyaya and Singh (1999) t_4	106.3119	321.2333	3054.145	16.31357
Singh *et al*. (2004) t_5	317.5905	107.5313	223.4473	222.9788
Singh *et al*. (2004) t_6	224.3415	152.2274	2338.359	21.30725
Yousuf and Rather (2021) t_7	192.4395	177.4632	1614.127	30.86747
Proposed Estimator	**64.3549**	**530.6654**	**37.9199**	**1313.928**

The Values of MSE and PRE of the Existing and Proposed Estimators

IV. Result and Discussion

A ratio-cum-product estimator for the estimation of the population mean of the study variable is proposed. The bias and mean square error (MSE) of the proposed estimator are derived up to the first order of appreciation. Theoretical comparison of the proposed ratio-cum-product estimator of the population mean with sample mean, ratio estimator and other existing estimators considered in the study were established. The values of mean square errors (MSE) of the proposed estimator are smaller than the sample mean, ratio estimator, product estimator and other estimators considered in the study. The performance of the proposed estimator over the sample mean, ratio estimator, product estimator and other selected existing estimators using two real populations were obtained. The results show that the proposed estimator is more efficient that the sample mean, ratio estimator, product estimator, Upadhyaya and Singh (1999), Singh *et al*. (2004) and Yousuf and Rather (2021) estimators.

V. Conclusion

The results in Table 1 clearly showed that the proposed ratio-cum-product estimator performed better than the sample mean, ratio estimator, product estimator, Upadhyaya and Singh (1999), Singh *et al*. (2004) and Yousuf and Rather (2021) estimators considered in the study having least Mean Square Error (MSE) and higher Percentage Relative Efficiency (PRE). Base on the results, the proposed estimator increases the efficiency of the estimate in estimating finite population mean.

References

Abu-Dayyeh, W.A., Ahmed, M.S., Ahmed, R.A. and Muttlak, H.A. (2003). Some Estimators of a Finite Population Mean using Auxiliary Information, *Applied Mathematics and Computation*. 139, 287– 298.

Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce, *The Journal of Agricultural Science*, 30, 262–275.

Gupta, R. K. and Yadav, S. K. (2017). New Efficient Estimators of Population Mean Using Non-Traditional Measures of Dispersion. *Open Journal of Statistics*, 7, 394-404.

Jeelani M. I., Maqbool S., and Mir S. A. (2013). Modified Ratio Estimators of Population Mean using Linear Combination of Estimators of Population Mean using Linear Combination of Coefficient of Skewness and Quartile Deviation. *International Journal of Modern Mathematical Sciences* 6: 174 -183.

Kadilar C. and Cingi H. (2004). Ratio Estimators in Simple Random Sampling. *Applied Mathematics and Computation*. 151: 893 – 902.

Muili, J. O., Adebiyi, A. and Agwamba, E. N. (2020). Improved Ratio Estimators for Estimating Population Mean Using Auxiliary Information. *International Journal of Scientific and Research Publications*. 10(5): 937-945.http://dx.doi.org/10.29322/IJSRP.10.05.2020.p101111

Muili, J. O., and Audu, A. (2019). Modification of Ratio Estimator for Population Mean. *Annals. Computer Science Series*. 17, 2, 74-78.

Muili, J. O., Audu, A., Singh, R.V.K., and Yunusa, I. A. (2019). Improved Estimators of Finite Population Variance Using Unknown Weight of Auxiliary Variable. *Annals. Computer Science Series*. 17, 1, 148-153.

Murthy, M. N. (1967). *Sampling Theory and Methods*. Statistical Publishing Society, Calcutta, India.

Robson, D. S. (1957). Application of multivariate polykays to the theory of unbiased ratio-type estimation, *Journal of the American Statistical Association*, 52, 511–522.

Singh, H.P., Tailor, R. and Kakran, M. S. (2004). An Improved Estimation of Population Mean using Power Transformation. *Journal of the Indian Society of Agricultural Statistics*, 8(2): 223–230.

Tailor, R., Parmar, R., Kim, J. M. and Tailor, R. (2011). Ratio-Cum-Product Estimators of Population Mean using known Population Parameters of Auxiliary Variates, *Communications of the Korean Statistical Society*, Vol. 18(2), pp. 155–164. DOI: https://doi.org/10.5351/CKSS.2011.18.2.155

Upadhyaya, L.N. and Singh, H. P. (1999). Use of Transformed Auxiliary Variable in Estimating the Finite Population Mean. *Biometrical Journal,* 41:627 – 636.

Yadav, S. K., Subramani, J., Mishra, S. S. and Shukla, A. K. (2016). Improved Ratio-Cum-Product Estimators of Population Mean Using Known Population Parameters of Auxiliary Variables. *American Journal of Operational Research*, 6(2): 48-54. https://doi.org/10.5923/j.ajor.20160602.03

Yan, Z. and Tian, B. (2010). Ratio Method to the Mean Estimation using Coefficient of Skewness of Auxiliary Variable. *In: Information Computing and Applications. ICICA*.

Yousuf, R. and Rather, K. U. I. (2021). Improved Product Type Estimator for Estimation of Population Mean. *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 8(2): 5- 7. https://doi.org/10.26438/ijsrmss/v8i2.57

Appendix

Population I: R-software code for Population I

```
N=30;n=10;ybar=17.5;xbar=4.4637;rho=-0.1994;cy=0.4758;cx=0.8727;b2=0.2296;b1=1.36;md=2.27;
f=n/N;g=1-f/n;
teta1=xbar*cx/(xbar*cx+b2);
teta2=xbar*b2/(xbar*b2+cx);
teta3=xbar/(xbar+b2);
teta4=xbar*rho/(xbar*rho+md);
mseratio=g*ybar^2*(cy^2+cx^2-2*rho*cy*cx);
mseproduct=g*ybar^2*(cy^2+cx^2+2*rho*cy*cx);
mseup1=g*ybar^2*(cy^2+teta1^2*cx^2-2*teta1*rho*cy*cx);
mseup2=g*ybar^2*(cy^2+teta2^2*cx^2-2*teta2*rho*cy*cx);
mseup3=g*ybar^2*(cy^2+teta1^2*cx^2+2*teta1*rho*cy*cx);
mseup4=g*ybar^2*(cy^2+teta2^2*cx^2+2*teta2*rho*cy*cx);
msesingh1=g*ybar^2*(cy^2+teta3^2*cx^2-2*teta3*rho*cy*cx);
msesingh2=g*ybar^2*(cy^2+teta3^2*cx^2+2*teta3*rho*cy*cx);
mseyr=g*ybar^2*(cy^2+teta4^2*cx^2+2*teta4*rho*cy*cx);
msep=g*ybar^2*cy^2*(1-rho^2);
mseratio;mseproduct;mseup1;mseup2;mseup3;mseup4;msesingh1;msesingh2;mseyr;msep;
```

Population II: R-software code for Population II

```
N=80;n=20;ybar=51.8264;xbar=11.2646;rho=0.9413;cy=0.3542;cx=0.75;b2=2.866;b1=1.05;md=7.575;
f=n/N;g=1-f/n;
teta1=xbar*cx/(xbar*cx+b2);
teta2=xbar*b2/(xbar*b2+cx);
teta3=xbar/(xbar+b2);
teta4=xbar*rho/(xbar*rho+md);
mseratio=g*ybar^2*(cy^2+cx^2-2*rho*cy*cx);
mseproduct=g*ybar^2*(cy^2+cx^2+2*rho*cy*cx);
mseup1=g*ybar^2*(cy^2+teta1^2*cx^2-2*teta1*rho*cy*cx);
mseup2=g*ybar^2*(cy^2+teta2^2*cx^2-2*teta2*rho*cy*cx);
mseup3=g*ybar^2*(cy^2+teta1^2*cx^2+2*teta1*rho*cy*cx);
mseup4=g*ybar^2*(cy^2+teta2^2*cx^2+2*teta2*rho*cy*cx);
msesingh1=g*ybar^2*(cy^2+teta3^2*cx^2-2*teta3*rho*cy*cx);
msesingh2=g*ybar^2*(cy^2+teta3^2*cx^2+2*teta3*rho*cy*cx);
mseyr=g*ybar^2*(cy^2+teta4^2*cx^2+2*teta4*rho*cy*cx);
msep=g*ybar^2*cy^2*(1-rho^2);
mseratio;mseproduct;mseup1;mseup2;mseup3;mseup4;msesingh1;msesingh2;mseyr;msep;
```