Intercensal Prediction of Nigeria Population Based on Growth Models with Hyperbolic Restrictions

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***Abstract—Intercensal* estimate is an estimate of population between official census dates with both of the census counts being known. This was observed for three cases using three growth models so as to determine the effectiveness of models in predicting correctly the census figure. Case 1 was the use of the 1963 population census result as the base year and 1991 population census result as the launch year. Case 2 was the use of the 1991 population census result as the base year and 2006 population census result as the launch year and case 3 was the use of the 1963 population census result as the base year and 2006 population census result as the launch year. The Nigeria population census figure for the year 1963, 1991 and 2006 were used for intercensal prediction while nonlinear estimation was applied on the data sourced online from 1955-2016 for model validation. A modified Hyperbolic Exponential Growth Model (HEGM) was used along with Exponential Growth Model (EGM) in predicting population figures and Mean Square Error (MSE), Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) were used to assess the suitability of the model on population prediction. Different values of shape parameter in the hyperbolic model were assumed to be small, moderate and high with ±0.1, ±0.5 and ±0.9 for Case 1, 2, and 3. HEGM gave the best intercensal estimate for the three cases and was preferred based on the AIC, BIC and MSE results with theta stabilized at ±0.1.**

Keywords--*Intercensal, Growth models, Hyperbolic Growth Model, Nigeria Population.*

I. Introduction

In demographics, an intercensal estimate is an estimate of population between official census dates with both of the census counts being known. Some nations produce regular intercensal estimates while others do not. Intercensal estimates can be less or more informative than official census figures, depending on methodology, completeness, accuracy (as they can have significant undercounts or overestimates) and date of data, and can be released by nations, subnational entities, or other organizations including those not affiliated with governments. Being able to forecast population and even being able to answer some interesting questions about population in the past, depends on developing accurate mathematical models of population growth.

Analysis and projection of population are based on population figure as it informs an objective distribution of social amenities by government. Hence, knowledge about past, current and future population are fundamental in every aspect of decision making. In view of this, current projection of this population figures is of great necessity.

Ofori et.al (2013) applied exponential and logistic growth models to model the population growth of Ghana using data from 1960 to 2011. Dean Hathout (2012) modelled population growth using exponential and hyperbolic models. Oyamakin et.al (2013) compare exponential and hyperbolic growth models in height and diameter increment of Pine (Pinuscaribaea).

The aim of this study is to investigate the property of Exponential Growth Model (EGM) and Hyperbolic Exponential Growth Model (HEGM) in modeling Nigeria population data and to determine the intercensal estimate, and compare the predictive performance of the two models.

II. Materials and Methods

A. Data Description

The Nigeria population census figure for the year 1963, 1991 and 2006 were used for intercensal prediction and nonlinear estimation was applied on the data sourced online (www.worldometers.info) from 1955-2016 for model validation. Intercensal estimation was observed for three cases using three growth models. Cases 1 was the use of the 1963 population census result as the base year and 1991 population census result as the launch year, case 2 was the use of the 1991 population census result as the base year and 2006 population census result as the launch year and case 3 was the use of the 1963 population census result as the base year and 2006 population census result as the launch year.

B. *Methodology*

1. Exponential Growth Model (EGM)

Suppose that P_t is the total number of individuals in the population at time (t) such that;

$$\frac{\partial P_t}{\partial t} = rP_t \qquad (1)$$

Separating variables we have that;

$$\frac{1}{P_t}\partial P_t = r\partial t \qquad (2)$$

Integrating both sides we have;

$$\ln P_t = rt + c_1 \qquad (3)$$

Taking the exponential of both sides we have;

$$P_t = e^{rt+c_1} \qquad (4)$$

$$P_t = e^{rt}e^{c_1} \qquad (5)$$

$$P_t = P_0 e^{rt} \qquad (6)$$

The growth rate will be

$$r = \frac{1}{t}(\ln P_t - \ln P_0) \qquad (7)$$

Equation (6) is the Exponential Growth Model

2. Hyperbolic Exponential Growth Model(HEGM)

Studies have shown that majority of the growth models emanated from the Malthusian Growth Equation (MGE), which is limited to growing without bounds. This study was designed to develop alternative growth models flexible to enhance internal prediction of population figures based on hyperbolic sine function with bound.

Suppose that P_t is the total number of individuals in the population at time (t) such that;

$$\frac{\partial P_t}{\partial t} = P_t\left[r + \frac{\theta}{\sqrt{1+t^2}}\right] \qquad (8)$$

Separating variables we have that;

$$\frac{\partial P_t}{P_t} = \left[r + \frac{\theta}{\sqrt{1+t^2}}\right]\partial t \qquad (9)$$

Integrating both sides we have;

$$\ln P_t = rt + \theta \arcsin h(t) + c_2 \qquad (10)$$

Taking the exponential of both sides we have;

$$P_t = P_0 e^{rt+\theta \arcsin h(t)} \qquad (11)$$

The growth rate will be

$$r = \frac{1}{t}(\ln P_t - \ln P_0 - \theta \arcsin h(t)) \qquad (12)$$

Equation (11) is the Hyperbolic Exponential Growth Model where P_0 is the population value for the base year, P_t is the population value for the target year, θ is the shape parameter which act as a stabilizing factor and can take both negative and positive value depending on the intrinsic nature of the growth rate.

III. Analysis And Results

Table 1: Growth rates for different cases based on the Models

	Geometric	EGM	HEGM					
			θ(0.1)	θ (0.5)	θ (0.9)	θ (0.1)	θ (-0.5)	θ (-0.9)
Case 1	1.69%	1.68%	0.24%	-5.51%*	-11.30%*	3.12%	8.87%	14.60%
Case 2	3.20%	3.14%	0.78%	-8.30%*	-17.40%*	5.31%	14.40%	23.50%
Case 3	1.09%	1.09%	1.12%	-12.20%*	-7.17%*	3.19%	17.50%	11.50%

Source: Computed by the authors * indicates values of the shape parameter that produces negative growth rate

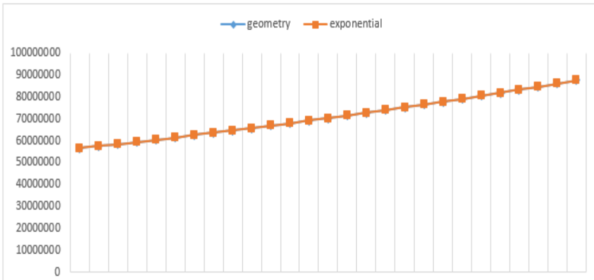

Fig 1: Intercensal estimate for case 1 using Geometry and EGM

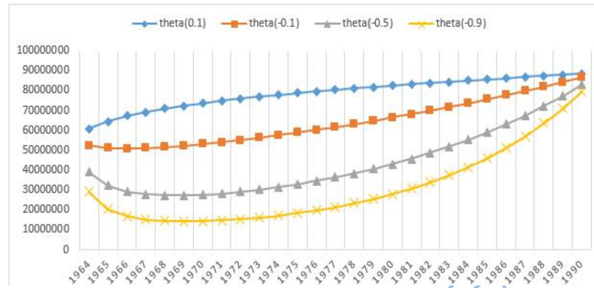

Fig 2: Intercensal estimate for case 1 using HEGM with $\theta(0.1), \theta(-0.1), \theta(-0.5)$ and $\theta(-0.9)$

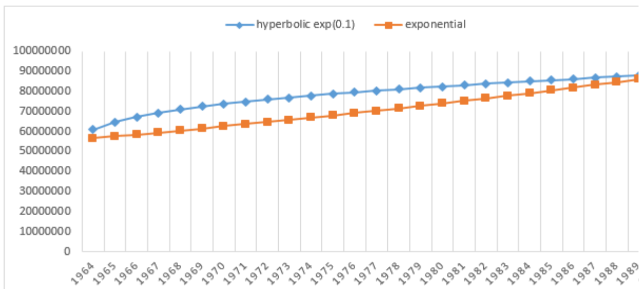

Fig 3: Intercensal estimate for case 1 using EGM and HEGM ($\theta(0.1)$)

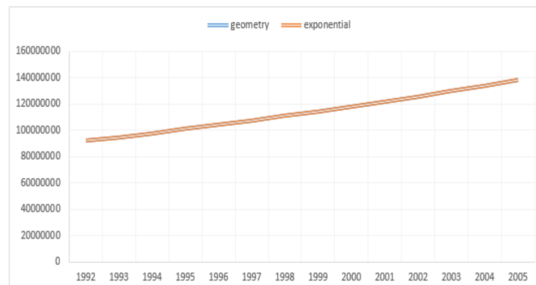

Fig 4: Intercensal estimate for case 2 using Geometry and EGM

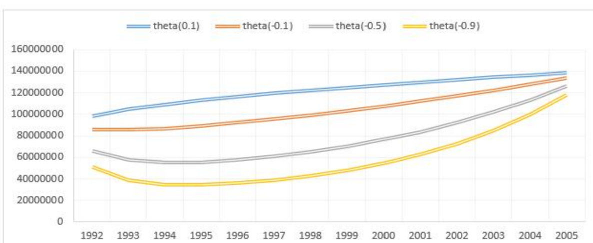

Fig 5: Intercensal estimate for case 2 using HEGM with $\theta(0.1), \theta(-0.1), \theta(-0.5)$ and $\theta(-0.9)$

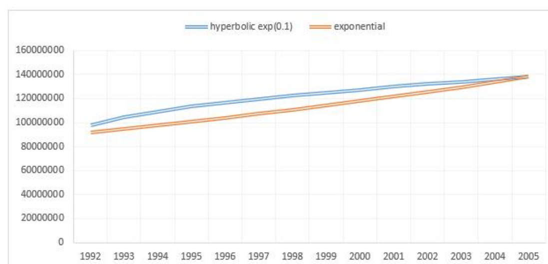

Fig 6: Intercensal estimate for case 2 using EGM and HEGM ($\theta(0.1)$)

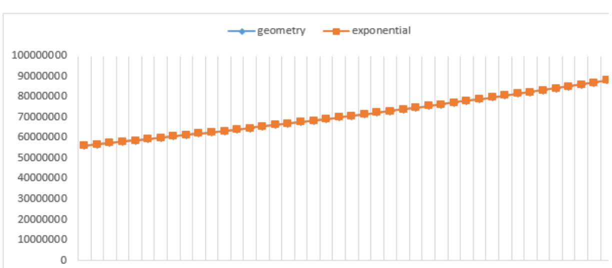

Fig 7: Intercensal estimate for case 3 using Geometry and EGM

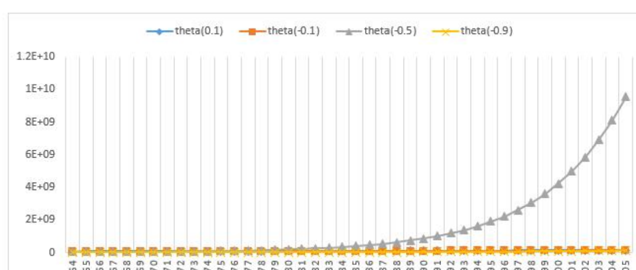

Fig 8: Intercensal estimate for case 3 using HEGM with $\theta(0.1)$, $\theta(-0.1)$, $\theta(-0.5)$ and $\theta(-0.9)$

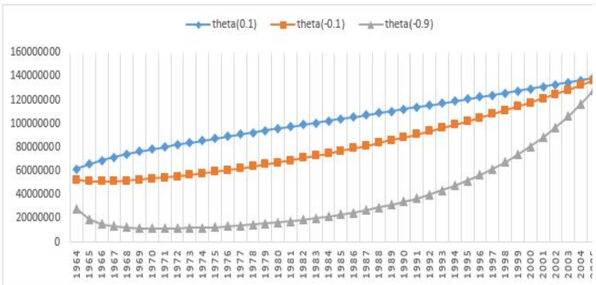

Fig 9: Intercensal estimate for case 3 using HEGM with θ(0.1), θ(-0.1) and θ(-0.9)

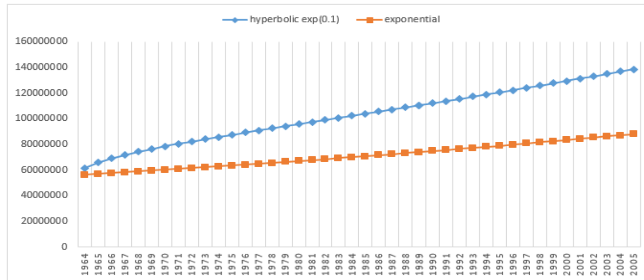

Fig 10: Intercensal estimate for case 3 using EGM and HEGM (θ(0.1))

Table 2: Parameter Estimation using HEGM

Parameter	Estimate	Std. Error	t-value	Pr(>|t|)
c	41.14676	0.52821	77.899	< 2.97e-16
r	0.0265	0.000203	130.747	< 2e-16
theta	-0.02156	0.004497	-4.792	0.000733

Source: Computed by the authors from data sourced from (www.worldometers.info)

Table 3: Parameter Estimation using EGM

Parameter	Estimate	Std. Error	t-value	Pr(>|t|)
c	38.8493	0.32932	118	< 2e-16
r	0.02568	0.000178	144.3	< 2e-16

Source: Computed by the authors from data sourced from (www.worldometers.info)

Table 4: Selection criteria

	MSE	R2	AIC	BIC
HEGM	0.2988	0.9998	-11.8753	-10.1804
EGM	0.9571	0.9996	3.2584	2.3883

Source: Computed by the authors

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IV. Discussion

Hyperbolic exponential growth rate was used for intercensal estimation using different values of theta which ranges from small to moderate to high with ±0.1, ±0.5 and ±0.9. Theta can take positive value if the growth rate of a population is low and it can take negative value if the growth rate of a population is high, therefore theta is said to be a stabilizing factor. In table 1, the growth rate estimate for different cases using Geometry Growth model (GGM), EGM and HEGM. HEGM with theta (0.5) and (0.9) produced negative growth rate for the three cases and the implication of this is that human population is going into extinction.

Figure 1, 4 and 7 suggested that EGM and GGM predicted the same intercensal estimate for the three cases, which prompted us to compare the HEGM with the EGM.

Figure 2, 5, and 9 present the intercensal estimate using HEGM with theta (0.1), (-0.1), (-0.5) and (-0.9) for the three cases. For case 1 and 2, HEGM with theta (0.1) performed best, followed by HEGM with theta (-0.1), theta (-0.5) and theta (-0.9) while for case 3, HEGM with theta (0.1) performed best, followed by HEGM with theta (-0.1) and theta (-0.9). Figure 8 suggested that HEGM with theta (-0.5) does not converge. Figure 3, 6 and 10 suggested that HEGM with theta (0.1) performed better than EGM in predicting the intercensal estimate for the three cases.

Table 2 and 3 present the result of the nonlinear estimation and the fitted exponential and hyperbolic exponential growth model were $P_t = 38.849262e^{0.025675t}$ and $P_t = 41.14675e^{0.025675t - 0.0215562\text{arcsinh}(t)}$ respectively. In Table 4, the hyperbolic exponential growth model was preferred based on the AIC, BIC and MSE results given as -11.8753, -10.18044 and 0.9571 respectively.

V. Conclusion

Different values of theta in the hyperbolic model were assumed to be small, moderate and high with ±0.1, ±0.5 and ±0.9 respectively for Case 1, 2, and 3. HEGM gave the best intercensal estimate for the three cases and was preferred based on the AIC, BIC and MSE results with theta stabilized at ±0.1.

Acknowledgement

We gratefully acknowledge the suggestions given by the anonymous referees, which have helped to improve the presentation of the paper.

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