Application of Run Sum Control Chart with Auxiliary Information to Spring Manufacturing Industry

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***Abstract* — In recent studies, application of auxiliary information technique in control charts have shown superior run length performances among other control charts developed without using it in the detection of out-of-control signals in engineering and manufacturing industries. In this paper, an application of run sum control chart based on auxiliary information (called RS-AI chart, hereafter) characteristics is proposed to efficiently monitor the production process shifts in a springs manufacturing industry. The statistical properties and optimal charting parameters presented in the RS-AI chart using the markov chain approach in minimizing the steady state out-of-control average run length (ARL_1) was adopted in this study. The results show that production processes have shifts to an out-of-control at some random interval and do not conforms to the springs manufacturing industry specification using the optimal RS-AI control chart. Therefore, there is need for the manufacturer to investigate the causes of this faults and make proper adjustment to attain sustainable production processes.**

Keywords - *Control chart; run sum (RS); auxiliary information (AI); average runs length (ARL); spring manufacturing industry.*

I. Introduction

A control chart is one of the most useful techniques in Statistical Process Control, for monitoring production process stability over period. As the global market is becoming more competitive, therefore, customer satisfaction and retention are key factors in manufacturing industries.

Statistical Quality Control plays a vital role in ensuring and maintaining the quality of manufactured products and performed services. For maintaining, improving and building production quality, control charts were first proposed by Walter A. Shewhart $\bar{X}$ in the 1920's to monitor and quickly detect assignable causes in the process mean and/or process variability of a production process Montgomery [11]. However, the standard Shewhart $\bar{X}$ chart is insensitive to small and moderate process shifts. To address this setback, numerous methods have been employed to improve its sensitivity as can be read in the available literature.

The more advanced control chart, such as the run sum chart proposed by Roberts [15] is also known as a zone chart by Jaehn [9] and Davis [5], where the region between the upper and lower control limits of the chart is divided into several scoring zones, and then compute the run sum of the scores. An out-of-control signal is issued by the run sum chart if the run sum of the scores exceed a critical value named the triggering score. Prompting an unforeseen shift immediately is important to practitioners to plan for corrective actions, if necessary. Unlike certain charts which were also designed for a quick detection of small and moderate shifts, like the EWMA and CUSUM charts, their charts' statistics are dependent on all previous data and thus, the charts' statistics drift slowly to the opposite direction where a sudden unforeseen shift occurs.

Since the pioneering work of Roberts [15], the run sum chart framework is continuously improving to more advanced run sum-type charts. Davis *et al* [5] showed that the modified zone control chart consistently outperforms the basic Shewhart $\bar{X}$ chart with runs rules.

The performance of a run sum chart is claimed to be as competitive as the EWMA and CUSUM charts when the number of regions is increased by Champ and Rigdon [4]. Some research work on run sum chart are conducted by Aguirre-Torres and Reyes-Lopez [2], Sitt et al. [16], Khoo et al [10] and Teoh et al. [17].

To further enhance the precision and efficiency in production process monitoring, the concept of auxiliary information was integrated with control charting techniques, where the linear correlation between the study variable and auxiliary variable is considered. In short, the idea of such a control chart is to use a regression estimator that is based on the information of more than one variable to estimate the performance of the study variable more efficiently. For example, the use of auxiliary information not only enhances the accuracy of the estimator but it also helps to increase the out-of-control detection ability of a control chart Haq and Khoo [8].

In a later research, Riaz [13, 14] proposed the auxiliary information based Shewhart charts to monitor the process mean and variability, whereby the chart is only sensitive to detect large shifts in the process parameters. To increase the efficiency in detecting small and moderate shifts, an EWMA-type based single auxiliary variable chart was proposed by Abbas, Riaz, and Does [1]. Their proposed chart is better than the classical CUSUM and classical EWMA charts in signaling an out-of-control. More recent works on auxiliary information based control charts were made by Haq and Khoo [7], Ahmad et al [3] and Umar *et al.* [18].

In line with this phenomenon which combine the advantages of the run sum chart and the auxiliary information technique, Ng *et al* [12] proposed an optimal run sum auxiliary information based chart (denoted as RS-AI). The RS-AI chart is optimally designed to minimize the steady state out-of-control ARL(ARL_1) using the markov chain approach. Prior to this research, an extensive application of the optimal RS-AI control chart to monitor the production process of springs manufacturing industry are carried out in this study.

II. METHODOLOGY

The industrial dataset used herein, and the method adopted for monitoring the production process mean shifts in spring manufacturing industry are discussed in this section.

A. *Spring Data*

The springs data in this study are obtained from Ghute and Shirke [6]. The data is from a springs manufacturing process that measured the spring elasticity (Y), which is the study variable, and the spring inner diameter (X) taken as the auxiliary variable for 15 samples each of size 5. Both the study (Y) and auxiliary (X) variables are corelated.

B. *Preliminary Concepts of Auxiliary Information Scheme*

Suppose that $(y_{r,1}, x_{r,1}), (y_{r,2}, x_{r,2}), \ldots, (y_{r,n}, x_{r,n})$ consist a random sample (of size n) of bivariate observations at sample number, $r = 1, 2, \ldots$, taken for a study variable Y. Let the auxiliary variable X be correlated with the study variable Y and the correlation coefficient between variables Y and X is denoted as ρ. Each pair of $(Y_{r,j}, X_{r,j})$, for $j = 1, 2, \ldots, n$, is assumed to follow a bivariate normal distribution $N_2(\mu_{Y0} + \delta\sigma_Y, \mu_X, \sigma_Y^2, \sigma_Y^2, \rho)$, where μ_{Y0} and σ_Y^2 are in-control population mean and population variance of the study variable Y, respectively, and both are assumed to be known; μ_X and σ_X^2 are population mean and variance of the auxiliary variable X, respectively, and both are assumed to be known; and $\delta = \frac{|\mu_{Y1} - \mu_{Y0}|}{\sigma_Y}$ is the size of a standardized mean shift in the study variable Y, where μ_{Y1} is the out-of-control mean of the process (or study variable, Y).

The regression estimator of μ_Y is given as Riaz et al [13, 14]

$$U_{Y_r}^* = \bar{Y}_r + \rho\left(\frac{\sigma_Y}{\sigma_X}\right)(\mu_X - \bar{X}_r), \quad (1)$$

where σ_X is the standard deviation of the auxiliary variable X, $\bar{Y}_r = \sum_{j=1}^{n} Y_{r,j} \big/ n$ and $\bar{X}_r = \sum_{j=1}^{n} X_{r,j} \big/ n$ are the sample mean of the study variable Y and auxiliary variable X, respectively, at time r. The mean and variance of estimator $U_{Y_r}^*$ are given by Ng *et al* [20]

$$E\left(U_{Y_r}^*\right) = \mu_Y, \quad \text{Var}\left(U_{Y_r}^*\right) = \frac{1}{n}\sigma_Y^2\left(1-\rho^2\right) \quad (2)$$

As the pair (Y, X) follows the bivariate normal distribution, it follows that $U_{Y_r}^*$ is a univariate normal random variable with mean $E\left(U_{Y_r}^*\right)$ and variance $\text{Var}\left(U_{Y_r}^*\right)$, i.e. $U_{Y_r}^* \sim N\left(\mu_Y, \frac{1}{n}\sigma_Y^2\left(1-\rho^2\right)\right)$.

Note that when the underlying process of the study variable Y is in-control, $\mu_Y = \mu_{Y0}$, and when it is out-of-control $\mu_Y = \mu_{Y1}$.

C. *Run Sum Control Chart with Auxiliary Information*

The RS-AI control chart aimed to effectively monitor the mean shifts of a single regression estimator. In similar concept to the basic run sum $\bar{X}$ chart, the RS-AI chart suggested by Ng *et al* [12] consists of q regions, each above and below the central line, CL, as presented in Figure 1.

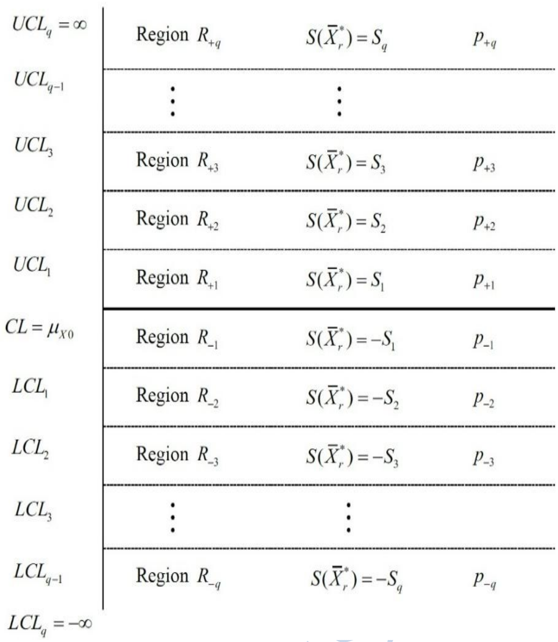

Fig: 1: 1–q regions (each above and below the CL) two sided RS-AI chart with corresponding control limits, scores and probabilities.

The regions above and below the CL are used to identify an increase and a decrease in the production process mean. A total of q upper control limits, i.e. $(\mu_{Y0} = CL < UCL_1 < UCL_2 < \ldots < UCL_{q-1} < UCL_q = \infty)$ above the CL and q lower control limits $(-\infty = LCL_q < LCL_{q-1} < \ldots < LCL_2 < LCL_1 < CL = \mu_{Y0})$ below the CL are used in the RS-AI chart (please see Figure 1). Based on the regression estimator in Equation. (1), the plotting statistic of the RS-AI chart is given by Ng *et al* [12]

$$\bar{Y}_r^* = U_{Y_r}^* = \bar{Y}_r + \rho\left(\frac{\sigma_Y}{\sigma_X}\right)\left(\mu_X - \bar{X}_r\right). \quad (3)$$

Suppose that the process parameters are known from the historical information. Based on Equation (2), the k^{th} lower and upper control limits LCL_k and UCL_k of the RS-AI chart are computed as Ng *et al*, [12]

$$UCL_k = \mu_{Y0} + A\left(\frac{3k}{q-1}\right)\frac{\sigma_Y}{\sqrt{n/(1-\rho^2)}}, \quad (4a)$$

for $k = 1, 2, \ldots, q-1$

and

$$LCL_k = \mu_{Y0} - A\left(\frac{3k}{q-1}\right)\frac{\sigma_Y}{\sqrt{n/(1-\rho^2)}}, \quad (4b)$$

for $k = 1, 2, \ldots, q-1$, where the parameter A in Equations. (4a) and (4b) is a constant selected to obtain the desired steady state in-control ARL (ARL_0) performance and σ_Y, μ_{Y0} and ρ have been defined in the previous section. Note that $LCL_q = -\infty$ and $UCL_q = \infty$. Also, Figure 1 illustrates the regions above the CL $\left(R_{+1}, R_{+2}, \ldots, R_{+q}\right)$ with the associated integer scores $(S_1, S_2, \ldots, S_q)$ and the regions below the CL $\left(R_{-1}, R_{-2}, \ldots, R_{-q}\right)$ with the associated integer scores $\left(-S_1, -S_2, \ldots, -S_q\right)$.

The RS-AI chart depends on the following cumulative scores U_r and L_r to operate:

$$U_r = \begin{cases} 0 & \text{if } \bar{Y}_r^* < CL \\ U_{r-1} + S(\bar{Y}_r^*) & \text{if } \bar{Y}_r^* \geq CL \end{cases} \quad (5a)$$

and

$$L_r = \begin{cases} 0 & \text{if } \bar{Y}_r^* > CL \\ L_{r-1} + S(\bar{Y}_r^*) & \text{if } \bar{Y}_r^* \leq CL \end{cases}, \quad (5b)$$

where $r = 1, 2, \ldots$, while the score function $S(\bar{Y}_r^*)$ is denoted as

$$S(\bar{Y}_r^*) = S_k \text{ if } \bar{Y}_r^* \in R_{+k}, \quad (6a)$$

for $k = 1, 2, \ldots, q$ and $r = 1, 2, \ldots$

and

$$S(\bar{Y}_r^*) = -S_k \text{ if } \bar{Y}_r^* \in R_{-k}, \quad (6b)$$

for $k = 1,2,...,q$ and $r = 1,2,...$

In this study, we consider the "no head start" feature, thus, U_0 and L_0 are set as zero. The RS-AI chart signals an out-of-control when the cumulative score $U_r \geq S_q$ or $L_r \leq -S_q$. Similarly, an out-of-control will be issued by the RS-AI chart when $\bar{Y}_r^*$ falls in the upper most region $\left(\bar{Y}_r^* \in R_{+q}\right)$ or in the lower most region, $\left(\bar{Y}_r^* \in R_{-q}\right)$. Note that S_q and $-S_q$ are the triggering scores for the upper-sided and lower-sided RS-AI charts, respectively.

The step-by-step operation of the RS-AI chart is discussed as follows:

Step 1: Specify the number of regions (q), sample size (n), correlation coefficient between variables Y and X, (ρ), and set the initial values U_0 and L_0 as zero.

Step 2: Determine the optimal parameter A and the cumulative scores $\pm S_k$, for k = 1, 2, …, q.

Step 3: Compute the charting statistic of the RS-AI chart, i.e. $\bar{Y}_r^*$, using Equation (3).

Step 4: Compute LCL_k and UCL_k, for k = 1, 2, …, q – 1, by using the Equations (4a) and (4b), respectively.

Step 5: If $\bar{Y}_r^*$ falls in the interval $[UCL_{k-1}, UCL_k)$, for k = 1, 2, …, q, the cumulative sum score U_r is calculated as $U_r = U_{r-1} + S_k$ (see Equation (5a)), while simultaneously resetting the cumulative score $L_r = 0$ (see Equation (5b). If $\bar{Y}_r^*$ falls in the interval $(LCL_k, LCL_{k-1}]$, for $k = 1,2,...,q$ (see Equation (5b)), compute the cumulative score $L_r = L_{r-1} + (-S_k)$ and reset $U_r = 0$ (see Equation (5a)).

Step 6: If $U_r \geq S_q$ or $L_r \leq -S_q$, the chart signals an out of-control at the r^{th} sample.

III. Results And Discussion

To demonstrate the application of the RS-AI chart on the spring manufacturing process adopted from Ghute and Shirke [6], the step-by-step operation of this control chart is illustrated herein. According to historical information, the in-control means, variances and correlation coefficient of Y and X are $\mu_{Y0} = 45.85$, $\mu_X = 28.29$, $\sigma_Y = 0.1503$, σ_X = 0.0592 and $\rho = 0.5$.

These given parameters values are used in the computation of the RS-AI chart's statistic. Ghute and Shirke [6] noted that the in-control process has a bivariate normal distribution and the first ten samples come from an in-control process. The process is out-of-control from sample eleven onwards.

The optimal parameters of the RS-AI chart computed by Ng *et al* [12] are adopted for this study. Suppose that the four regions RS-AI chart is considered, then from step 1, q = 4, n = 5, ρ = 0.5, δ = 0.8 and $U_0 = L_0 = 0$. Moreover, from step 2, the optimal parameters of the RS-AI chart are obtained from Table 1 are $\left(A, \{S_1, S_2, S_3, S_4\}\right)$ = (1.202, {0, 1, 2, 4}) provided by Ng *et al*, [12].

In step 3, the charting statistics of the RS-AI chart, i.e., $\bar{Y}_r^*$ are computed using Equation 3, from the bivariate (Y, X) observations for the springs manufacturing process. Table 2 gives the pairs (Y, X) observations, charting statistics $\bar{Y}_r^*$, scores $S\left(\bar{Y}_r^*\right)$, and cumulative score $\left(U_r, L_r\right)$ of the RS-AI chart, for the 15 samples. Step 4 provides the computed values of UCL_k and LCL_k of the RS-AI chart, for k = 1, 2 and 3 using Equations 4a and 4b as follows:

$UCL_1 = 45.85 + 1.202(1)(0.058) = 45.92$,

$UCL_2 = 45.85 + 1.202(2)(0.058) = 45.99$,

$UCL_3 = 45.85 + 1.202(3)(0.058) = 46.06$ and

$LCL_1 = 45.85 - 1.202(1)(0.058) = 45.78$,

$LCL_2 = 45.85 - 1.202(2)(0.058) = 45.71$,

$LCL_3 = 45.85 - 1.202(3)(0.058) = 45.64$. Note that $UCL_4 = \infty$, $CL = 0$ and $LCL_4 = -\infty$.

From Table 2, the RS-AI chart's statistic computed at sample 1 is $\bar{Y}_1^* = 45.90$, which falls in the interval $\left[CL, UCL_1\right)$ and thus, a score $S\left(\bar{Y}_1^*\right) = +0$ is assigned to U_1 giving $\left(U_1, L_1\right) = (+0, 0)$. As $\bar{Y}_2^* = 45.94 \in [UCL_1, UCL_2)$ for sample 2, a score

$S(\bar{Y}_2^*) = +1$ is allocated and (U_2, L_2) remains as $(+1, 0)$. Next, as $\bar{Y}_3^* = 45.71 \in [LCL_1, CL)$ gives a score $S(\bar{Y}_3^*) = -2$, the cumulative scores become (U_3, L_3) = $(0, -2)$. The process of allocating scores and updating (U_r, L_r) continues in the same manner until sample 14, where the chart's statistic $\bar{Y}_{14}^* = 45.96 \in [UCL_1, UCL_2)$ and therefore, a score $S(\bar{Y}_{14}^*) = +1$ is added to U_{13} (= +1) so that $(U_{14}, L_{14}) = (+2, 0)$. The next sample, $\bar{Y}_{15}^*$ falls in the interval $[UCL_2, UCL_3)$, thus, a score $S(\bar{Y}_{15}^*) = +3$ is added to U_{14} (= +2) and $(U_{15}, L_{15}) = (+4, 0)$ is obtained. The process is declared as out-of-control at sample 15 as $U_{15} \geq S_4$ (= +4) (see the boldfaced entries in Table 2).

The search for assignable cause(s) needs to be conducted by springs manufacturing industry quality management so that appropriate corrective actions can be initiated. The RS-AI chart is also depicted in Figure 2, where all the 15 sample means, $\bar{Y}_r^*$, in Table 2 are plotted on it (represented by the bold circle dots). As can be seen in Figure 2, the first out-of-control signal occurs at sample 15 because $U_{15} = +4 \geq S_4$ (= +4).

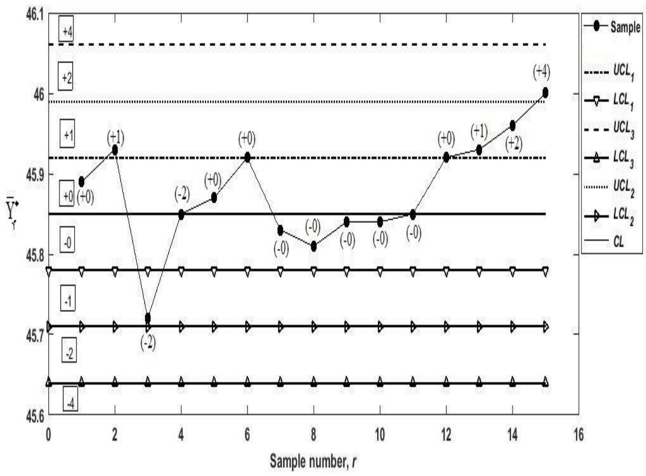

Fig 2: RS-AI chart for springs data with $(A, \{S_1, S_2, S_3, S_4\})$ = (1.202, {0, 1, 2, 4}).

Table 1: Optimal parameters $(A, \{S_1, S_2, ..., S_q\})$ in minimizing the steady state ARL_1 of the 4 and 7 regions RS-AI charts when n ∈ {**5**, 7}, ρ ∈ {0, 0.25, 0.5, 0.75, 0.9, 0.95}, δ ∈ {0.2, 0.4, 0.6, 0.8, 1, 1.25, 1.5, 1.75, 2} and ARL_0 = 370 (see Ng *et al*, 2018).

	***n* = 5**													***n* = 7**												
	***q* = 4**					*q* = 7								*q* = 4					*q* = 7							
ρ	A	S_1	S_2	S_3	S_4	A	S_1	S_2	S_3	S_4	S_5	S_6	S_7	A	S_1	S_2	S_3	S_4	A	S_1	S_2	S_3	S_4	S_5	S_6	S_7
$\delta = 0.6$																										
0	1.202	0	3	5	10	2.022	0	1	2	3	4	5	5	1.202	0	1	2	4	1.262	0	0	1	2	3	3	5
0.25	1.202	0	1	2	4	2.022	0	1	2	3	4	5	5	1.202	0	2	5	8	1.262	0	0	1	2	3	3	5
0.5	1.202	0	1	2	4	1.161	0	0	1	2	3	4	6	1.202	0	1	2	4	1.274	0	0	1	2	3	4	5
0.75	1.202	0	3	5	10	1.274	0	0	1	2	3	4	5	1.162	0	2	5	9	1.274	0	0	1	2	3	4	5
0.9	1.331	0	1	3	4	1.237	0	0	1	2	4	5	6	1.082	0	1	5	8	1.283	0	0	1	2	4	6	6
0.95	1.082	0	1	6	9	1.283	0	0	1	2	4	6	6	1.510	0	1	4	4	1.283	0	0	1	2	4	6	6
$\delta = \mathbf{0.8}$																										
0	1.202	0	2	5	8	1.274	0	0	1	2	3	4	5	1.202	0	3	5	10	1.274	0	0	1	2	3	4	5
0.25	1.202	0	1	2	4	1.274	0	0	1	2	3	4	5	1.202	0	3	5	10	1.274	0	0	1	2	3	4	5
0.5	**1.202**	**0**	**1**	**2**	**4**	1.274	0	0	1	2	3	4	5	1.162	0	2	5	9	1.274	0	0	1	2	3	4	5
0.75	1.331	0	1	3	4	1.237	0	0	1	2	4	5	6	1.331	0	2	5	7	1.283	0	0	1	2	4	6	6
0.9	1.082	0	1	5	8	1.283	0	0	1	2	4	6	6	1.059	0	1	5	9	1.283	0	0	1	2	4	6	6
0.95	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6
$\delta = 1$																										
0	1.202	0	2	5	8	1.274	0	0	1	2	3	4	5	1.331	0	2	5	7	1.237	0	0	1	2	4	5	6
0.25	1.202	0	2	5	8	1.274	0	0	1	2	3	4	5	1.331	0	1	3	4	1.237	0	0	1	2	4	5	6
0.5	1.331	0	2	5	7	1.274	0	0	1	2	3	4	5	1.331	0	1	3	4	1.237	0	0	1	2	4	5	6
0.75	1.331	0	1	3	4	1.283	0	0	1	2	4	6	6	1.082	0	1	5	8	1.283	0	0	1	2	4	6	6
0.9	1.510	0	1	4	4	1.283	0	0	1	2	4	6	6	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6
0.95	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6	1.510	0	1	4	4	1.005	0	0	0	1	1	4	7
$\delta = 1.25$																										
0	1.331	0	2	5	7	1.237	0	0	1	2	4	5	6	1.331	0	1	3	4	1.283	0	0	1	2	4	6	6
0.25	1.331	0	1	3	4	1.237	0	0	1	2	4	5	6	1.331	0	1	3	4	1.283	0	0	1	2	4	6	6
0.5	1.331	0	1	3	4	1.283	0	0	1	2	4	6	6	1.082	0	1	5	8	1.283	0	0	1	2	4	6	6
0.75	1.082	0	1	6	9	1.283	0	0	1	2	4	6	6	1.510	0	1	4	4	1.283	0	0	1	2	4	6	6
0.9	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6	1.510	0	1	4	4	1.219	0	0	1	1	3	6	6
0.95	1.510	0	1	4	4	1.005	0	0	0	1	1	3	6	1.000	0	0	1	10	1.000	0	0	0	0	1	1	4

Table 2: Application of RS-AI chart using spring dataset. Boldfaced denotes the out-of-control signal

Sample number	Springs observations										Sample means		RS-AI charting statistic		
r	Y_1	Y_2	Y_3	Y_4	Y_5	X_1	X_2	X_3	X_4	X_5	$\bar{Y}_r$	$\bar{X}_r$	$\bar{Y}_r^*$	$S(\bar{Y}_r^*)$	(U_r, L_r)
1	46.32	45.79	45.88	45.88	45.8	28.14	28.31	28.27	28.2	28.26	45.93	28.26	45.89	+0	(+0, 0)
2	45.85	45.91	45.8	45.91	45.93	28.5	28.35	28.3	28.32	28.2	45.88	28.33	45.93	+1	(+1, 0)
3	45.83	45.75	45.75	45.52	45.58	28.29	28.3	28.29	28.38	28.29	45.69	28.31	45.72	-2	(0, -2)
4	45.81	45.99	45.78	46.02	45.85	28.22	28.26	28.27	28.27	28.28	45.89	28.26	45.85	-0	(0, -2)
5	45.77	45.94	46.04	45.77	45.67	28.3	28.36	28.27	28.32	28.3	45.84	28.31	45.87	+0	(+0, 0)
6	45.77	45.93	45.77	45.92	46.04	28.34	28.29	28.32	28.27	28.19	45.89	28.31	45.92	+0	(+0, 0)
7	45.9	45.83	45.69	45.78	45.72	28.24	28.32	28.31	28.36	28.41	45.78	28.33	45.83	-0	(0, -0)
8	45.75	45.89	45.66	45.84	45.74	28.23	28.36	28.34	28.31	28.33	45.78	28.31	45.81	-0	(0, -0)
9	45.59	46.1	45.87	45.57	45.87	28.25	28.39	28.31	28.35	28.32	45.8	28.32	45.84	-0	(0, -0)
10	45.7	45.75	45.78	45.89	45.9	28.31	28.28	28.31	28.36	28.32	45.8	28.32	45.84	-0	(0, -0)
11	45.52	45.83	46.15	45.73	45.76	28.34	28.31	28.25	28.3	28.45	45.8	28.33	45.85	-0	(0, -0)
12	46.04	45.96	45.9	45.6	45.88	28.27	28.23	28.35	28.37	28.36	45.88	28.32	45.92	+0	(+0, 0)
13	45.87	45.87	45.79	45.82	45.79	28.35	28.44	28.42	28.32	28.31	45.83	28.37	45.93	+1	(+1, 0)
14	45.92	45.88	46.07	45.84	45.82	28.32	28.3	28.32	28.33	28.4	45.91	28.33	45.96	+1	(+2, 0)
15	46.02	45.83	45.94	45.97	45.76	28.27	28.33	28.41	28.44	28.41	45.9	28.37	**46**	**+2**	**(+4, 0)**

IV. Conclusion

The RS-AI chart is an attractive chart to practitioners as it has an excellent performance in detecting most levels of shifts by merely adjusting the value of ρ. As can be view in Figure 2, the RS-AI chart detects the first out-of-control signal at sample number 15 because $U_{15} = +4 \geq S_4$ (= +4). Hence, the production process of spring manufacturing industry is declared to be out-of-control at sample 15. Therefore, following an out-of-control signal being issued by the optimal RS-AI control chart, corrective actions are advisable for the quality management to thorough investigate the underlying process and remove the assignable causes to maintain the quality of their product for effective customers satisfaction and overcome market competition.

This paper focuses on application of the RS-AI chart for monitoring the process mean of spring manufacturing industry. In a future study, an EWMA-AI chart for monitoring the production process mean and/or variance may be introduced to the manufacturing industries.

Acknowledgment

This research is supported by Petroleum Trust Development Fund (PTDF), Nigeria, Kaduna State Government, Nigeria and the Universiti Sains Malaysia (USM), Graduate Assistant Scheme. Also, the authors are grateful to anonymous reviewers for their valuable comments that lead to an improve version of the article.

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