A Mixed-Integer Lexicographic Goal Programming Model for achieving Estimated Targets in Multi-Product Systems

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***Abstract* — Multi-product systems, mostly business entities, are often faced with the challenge of achieving several target goals within a given period of time which are often conflicting and not measurable in same units. Even as it is most times impossible for such conflicting goals to be entirely optimally achieved, effort is made to minimize deviations from the estimated target of such goals. Priorities are sometimes also given to these goals such that the compromised solution obtained minimizes the deviation from these estimated targets according to a given prioritized order of importance. The Lexicographic Goal Programming technique is an appropriate method for solving such problems. In this paper, we present a Mixed Integer Lexicographic Goal Programming model for minimizing deviations from estimated target of goals set by multi-product systems. To demonstrate the model, we focused on a multi-product production company (Nigerian Breweries PLC) to develop a Mixed-integer lexicographic model for its production process based on the monthly targets established by one of the production factories of the company for the year 2016. The goals considered include the estimated monthly profit target, monthly production target of each of the drinks Star, Gulder, Maltina, Goldsberg, 33 Export and Fayrouz, estimated machine production time and estimated target distribution cost. These goals were categorized into three priority levels with the second priority normalized. The LINGO Optimization software was used to obtain the optimal solution based on the collected data. The result obtained showed that all the target goals were met. This shows that the Mixed-integer Lexicographic goal programming model is an appropriate technique for solving multi-objective problems in multi-product systems.**

Keywords- *Goal programming model, Lexicographic Goal Programming, Multi-product Goal Programming model, Mixed-integer Goal Programming model, Goal programming variants.*

I. Introduction

Goal programming (GP) is a mathematical technique for solving multiple objective decision making problems in which the objectives may be conflicting. The Goal Programming technique has been applied in a wide range of planning, resource allocation, policy analysis and functional management problems [1]. Hence the design and application of GP models in solving multi objective models, especially in industry, is a popular problem area. Lexicographic Goal programming (LGP) is a special case of the Goal Programming technique.

The distinguishing feature of the LGP variant from other Goal Programming variants is the existence of a number of priority levels with each priority level containing a number of unwanted deviations to be minimized. In the lexicographic GP approach in a multi-product system, management sets some estimated for its goals for a certain period of time and assigned priorities to them. In applying this technique management only has to say which goal is more important than the other, it need not say how much more. With this information the LGP model tries to minimize the deviations from the estimated targets that were set by beginning with the most important goal and continue in such a way that the less important goal is considered only after the most important ones are satisfied or have reached the point beyond which no further improvement is desired [2]. In the final solution even if all the goals are not fulfilled to the fullest extent (of estimated targets) to give an optimal solution , the deviations will be the minimum possible giving what is called a satisficing solution.

The LGP approach as well as other GP variants has been studied by many researchers and successfully applied to many diverse real life problems especially in industry. [3] proposed a Lexicographic goal Programming approach with different scenarios to solve the aggregate production planning model with conflicting multi-objective functions in order to maximize the total net profit with limited investments (budget, limited storage space, production capacity and resources of the company. [4] also presented a Lexicographic GP model for multi-assembly multi-manned assembly line balancing problem with the primary objective of minimizing the total number of multi-manned stations (line length), minimizing the total number of workers as the secondary objective and smoothing the number of workers at the stations as the tertiary objective. Meanwhile a Lexicographic GP approach was presented for the optimal deployment of traffic police with different roads segments and shifts considered by [5]. The model was demonstrated with focus on the metropolitan city of Delhi (central) as a case study.

[6] on the other hand presented an efficient solution for solving Lexicographic linear GP problems. The procedure considered goal constraints as both objective function and constraints and then the problem is solved in an iterative tabular procedure in which the objective function becomes the prioritized deviational variables and is solved sequentially from the highest priority level to the lowest.

[7] developed an aggregate production planning model that best serves those companies whose aim is to have the best utilization of their resource in an uncertain environment while trying to keep an acceptable degree of quality and customer service level simultaneously taking into account the performance and availability of production lines. The proposed model which was a fuzzy model was first converted to an equivalent crisp multi-objective model and then Goal Programming was applied to the converted model.

[8] suggested a procedure based on Analytic hierarchy process combines with a mean variance and Goal Programming model that makes integrated asset management possible. The objective was to suggest a flexible approach that is relatively simple to use and that makes it possible to incorporate all factors , both objective and subjective, that are likely to influence the asset allocation decision.

[9] constructed a multi-objective integrated production planning model which was constructed doubly by resource and materials. The model took delivery-on-time, reduced inventory, reduced overtime work, maintained safety inventory and its optimization objectives and achieved the integrated optimization of production planning, material requirement planning, resource requirement planning, inventory planning and overtime work planning.

[10] proposed a multi-period and multi-stage model with multi-choice goals under inventory management constraints formulated by 0-1 mixed integer linear programming. The design task of the problem involved the choice of the pop up stores to be opened and the distribution network design to satisfy the demand with three multi-choice goals. The Revised multi-choice goal programming approach was applied to solve the mixed integer linear programming model and optimal solution was obtained that satisfied the demands with the three multi-choice goals.

In this paper, we present a mixed-integer Lexicographic Goal programming (MILGP) model for attainment of management objectives in multi-product systems. The achievement function of the MILGP model presented is such that some priority levels might be combination of goals while some others contains just single goals and also variables of the model are both non-negatively and integer constrained while others are just non-negatively constrained.

II. Research Methodology

A. *The Lexicographic Linear Goal Programming model*

The general lexicographic goal programming model with m goals and priority levels as presented by [11] is given as:

Find $\bar{X} = x_1, x_2, \dots, x_n$ so as to

$Minimize = P_1(w_1^+ d_1^+ + w_1^- d_1^-) + P_2(w_2^+ d_2^+ + w_2^- d_2^-) + \cdots + P_m(w_m^+ d_m^+ + w_m^- d_m^-)$ (1)

Subject to $f_j(\bar{x}) + d_i^- - d_i^+ = b_j$ (2)

$x_1, x_2, \dots, x_n$ and $d_j^-, d_j^+, w_j^-, w_j^+ \geq 0$ (3)

$j = 1, \dots, m$, $k = 1, \dots, n$

Where $P_j(w_j^+ d_j^+ + w_j^- d_j^-)$ is the subset of unwanted deviational variables for priority j

d_j^- = negative deviational variable of the *j*th goal

d_j^+ = positive deviational variable of the *j*th goal

w_j^+ represents the numerical weight associated with the positive deviational variable

w_j^- represents the numerical weight associated with the negative deviational variable

$f_j(\bar{x})$ = function of the decision variable

b_j represents the target level of the *j*th goal

$\bar{X} = x_1, x_2, \dots, x_n$ represents the vector of n decision variables.

B. *Mixed-integer lexicographic Goal programming model (MILGPM) for multi-product systems.*

The mixed-integer lexicographic goal Programming model is hereby presented as follows.

For a multi-product system with products given as $\bar{x}$ $(x_1, x_2, \dots, x_n)$ with q goals and l priority levels with $i = q = 1,2,3, \dots, Q$ or $\sum q's$ for a given priority level as the case may be.

Let $u_i^l =$ Preferential weights associated with the minimization of n_i in the lth prioriy level

v_i^l = Preferential weights associated with the minimization of p_i in the lth prioriy level

k_i = Normalization constant associated with the *i*th goal

a_{kr} = amount of contribution of qth goal on product x_k , $k = 1,2, \dots, n$

c_{jk} = amount of resource j necessary to manufacture one unit of product x_k
c_j = total availability of the jth resource for a given period
n_i = negative deviational variable of the ith goal(s) in the achievement function
n_q = negative deviational variable of the qth goal in the goal constraints
p_i = positive deviational variable of the ith goal(s) in the achievement function
p_q = positive deviational variable of the qth goal in the goal constraints
$g(x)$ = function of the deviational variable for the system constraints
b_q = estimated target level for qth goal
Then the MILGP model will have the achievement function given as

$$Lex\ Min\ d = \left[\sum_i \left(\frac{u_i^1 n_i}{k_i} + \frac{v_i^1 n_i}{k_i} \right), \sum_i \left(\frac{u_i^2 n_i}{k_i} \frac{v_i^2 n_i}{k_i} \right), \ldots, \sum_i \left(\frac{u_i^L n_i}{k_i} + \frac{v_i^L n_i}{k_i} \right) \right], \quad l = 1,2,\ldots,L \qquad (4)$$

Subject to

$$\sum a_{kq} x_k + n_q - p_q = b_q \qquad (5)$$

$$\sum_{j=1}^{m} c_{jk}\, x_k \ (\le, =, \ge) g_j \qquad (6)$$

$$j = 1,2,\ldots,m \ , \ k = 1,2,\ldots,n$$

$x_1, x_2, \ldots, x_n \ge 0$ and integer, $n_q, p_q \ge 0$ and integer for some q. $u_i^l, v_i^l, k_i \ge 0$ (7)
With either u_i^l or $v_i^l = 0$ when not included in a priority level
i.e. its minimization is considered not important
So $u_i^l \times v_i^l = 0$

C. *Assumptions of the model.*

The following assumptions are made for the MILGP model presented above

- ✓ Additivity: the level of penalization for an unwanted deviation from a target level is independent of the levels of unwanted deviations from other goals.
- ✓ Proportionality : The penalization for an unwanted deviation from a target level is directly proportional to the distance away from the target level.
- ✓ Mixed-integer value constraints : Some decision variables are integer constrained while some others just non-negatively constrained.
- ✓ Certainty : The data coefficients of the model are known with certainty while the goal targets are regarded as initial estimates.

Priority Levels: Some priority levels have just one goal while some are the combination of goals which are taken to be of equal importance.

III. Data illustration of MILGPM

The mixed-integer Lexicographic GP model formulated above will be illustrated using data collected from a multi-product manufacturing company. Data was collected from a production factory of Nigerian Breweries Plc on production of the drinks- Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export.

The data which are estimated values is shown in appendix A and includes the average monthly production quantity (in crate), average number of brews produced in a month, average number of cartons produced per brew for a month, quantity of raw material used per brew, average machine bottling time per truckload in a month, estimated monthly profit per truckload, and estimated distribution cost(fueling and drivers allowance) per truckload for each drink for the year 2016. The objective is the minimization of the following- underachievement of the estimated profit target, underachievement of the estimated production target levels for each of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export, overachievement of the available machine bottling time and overachievement of the estimated distribution cost. This gives us 9 goals which are grouped into three priority levels as follows;
Priority 1 : Achieve the Profit goal.
Priority 2 : Achieve the production targets goals for each drink and production time goal.
Priority 3 : Achieve the distribution cost goal.
With
x_1 = number of truckload of Star produced
x_2 = number of truckload of Gulder produced
x_3 = number of truckload of Maltina produced
x_4 = number of truckload of Fayrouz produced
x_5 = number of truckload of Goldsberg produced
x_6 = number of truckload of 33 export produced
$For\ q = 1,2,\ldots,9, j = 1,\ldots,5, l = 1,2,3.$
The achievement function of the model is given as a vector to be lexicographically minimized:

$$Lex\ Min\ d = \left[(n_1), \left(\frac{n_2}{506} + \frac{n_3}{250} + \frac{n_4}{72} + \frac{n_5}{220} + \frac{n_6}{379} + \frac{n_7}{264} + \frac{p_8}{720} \right), (p_9) \right] \qquad (8)$$

Subject to
<u>Goal constraints</u>

$$127470\, x_1 + 140700\, x_2 + 133700\, x_3 + 93310\, x_4 + 95900\, x_5 + 105000\, x_6 + n_1 - p_1 = 113{,}000{,}000 \qquad (9)$$

(profit goal)

$$x_1 + n_2 - p_2 = 506 \qquad (10)$$

$$x_2 + n_3 - p_3 = 250 \qquad (11)$$

$$x_3 + n_4 - p_4 = 72 \qquad (12)$$

$x_4 + n_5 - p_5 = 220$ (13)
$x_5 + n_6 - p_6 = 397$ (14)
$x_6 + n_7 - p_7 = 264$ (15)
$0.36\, x_1 + 0.36\, x_2 + 0.71\, x_3 + 0.71\, x_4 + 0.36\, x_5 + 0.36\, x_6 + n_8 - p_8 = 720$ (16)
(machine production time goal)
$11750\, x_1 + 11750\, x_2 + 11750\, x_3 + 11750\, x_4 + 11750\, x_5 + 11750\, x_6 + n_9 - p_9 = 20{,}080{,}750$ (Distribution cost goal) (17)

System constraints

$116\, x_1 + 59\, x_2 + 10\, x_3 + 0\, x_4 + 0\, x_5 + 0\, x_6 \leq 129780$ (18)
(*malted sorghum constraint*)

$178\, x_1 + 91\, x_2 + 6\, x_3 + 38\, x_4 + 114\, x_5 + 92\, x_6 \leq 362556$ (19)
(*malted barley constraint*)
$312\, x_1 + 159\, x_2 + 38\, x_3 + 0\, x_4 + 282\, x_5 + 221\, x_6 \leq 707540$ (20)
(*white sorghum constraint*)
$6\, x_1 + 0\, x_2 + 43\, x_3 + 138\, x_4 + 0\, x_5 + 0\, x_6 \leq 130950$ (21)
(*sugar constraint*)
$50.4\, x_1 + 50.3\, x_2 + 55.7\, x_3 + 48.8\, x_4 + 50.5\, x_5 + 50.9\, x_6 \leq 86317$ (22)
(*brewing constraint*)

$x_1, x_2, \ldots, x_6 \geq 0$ and integer
, $n_1, n_8, n_9, p_1, p_8, p_9 \geq 0$, (23)

$n_2, n_3, n_4, n_5, n_6, n_7, p_2, p_3, p_4, p_5, p_6, p_7 \geq 0$ and integer

IV RESULTS AND DISCUSSION

The mixed-integer lexicographic GP model on Nigerian Breweries Plc was solved using the LINGO optimization software. The formulated LINGO programme for the problem and the results are shown in the appendix B. The result is summarized in Table 1.

Table 1. Summary of LINGO solution of Mixed-integer Lexicographic GP model

Priority level/Goals analysis				
Priority levels	**Goals**	**Target Level**	**Achieved value**	**Goal achievement**
Priority 1	Profit goal	118,000,000.00 Naira	118,002,452.26 Naira	Achieved
Priority 2	Star Production target goal	506 trucks	506 trucks	Achieved
	Gulder Production target goal	250 trucks	250 trucks	Achieved
	Maltina Production target goal	72 trucks	72 trucks	Achieved
	Fayrouz Production target goal	220 trucks	220 trucks	Achieved
	Goldsberg Production target goal	397 trucks	397 trucks	Achieved
	33 export Production target goal	264 trucks	264 trucks	Achieved
	Machine production time goal	720 hours	628.2 hours	Achieved
Priority 3	Distribution cost goal	20,080,750 Naira	20,080,750 Naira	Achieved

Table 2: System constraint Analysis of LINGO solution

RAW MATERIALS	TOTAL AVAILABLE MONTHLY QUANTITY	SLACK VALUE	QUANTITY USED IN PRODUCTION
Malted sorghum	129780 kg	55644.0	74166kg
Malted barley	362556kg	135400.0	227156kg
White sorghum	707540kg	395228.0	312312kg
Sugar	130950kg	94458.0	36492kg
Brew	86317 hectoliters	7.10	86309.9 hectoliters

From the summary of the results shown in table 1 above we can see that each the goals in the different priority levels were satisfied. The monthly production targets of 506, 250, 72, 220, 397, 264 truckloads for each of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export respectively were met, the monthly distribution cost was satisfied as targeted while there was a slight overachievement by 2452.26 Naira in the profit target as well as the machine bottling time for the production of all the drinks been achieved in 628.2 hrs out of the targeted 720 in the month. Also from table 2 we see that the quantity of the raw materials- malted sorghum, malted barley, white sorghum and sugar used in the production was 74166kg, 227156kg, 312312kg and 36492kg respectively while the quantity of brews used was 86309.0 hectoliters.

Table 3. Values of decision variables on solution for the different priority levels

	Solution for first priority level	*Solution for second priority level*	*Solution for third priority level*
X_1	0	0	506
X_2	0	804	250
X_3	0	0	72
X_4	0	0	220
X_5	0	0	397
X_6	0	0	264

Checking solution for Lexicographic redundancy.

The solution obtained was checked for lexicographic redundancy by checking the solutions for each of the priority levels 1,2 and 3 obtained and it was seen that the satisfaction of the goals keeps improving as each priority level is considered from priority 1 to 3. From the solution in the solutions for priority levels 1,2 and 3 as shown in table 3 above, it can be seen that the final solution for the third priority level has a non-singular solution and hence lexicographic redundancy did not occur in the solution

V. Conclusion

This paper presents the a mixed-integer lexicographic goal programming (MILGP)model that can be applied by management of for multi-product systems in achieving set targets for a given period of time. Although in most goal programming problems it is difficult to guarantee an optimal solution nor a solution in which all the goal targets are achieved, but from the data illustration of the MILGP model in this work we have achieved a satisficing solution.

The MILGP model presented in this paper has an achievement function in which a priority level can be either a single goal or a combination of goals and the variables are mixed-integer constrained. The functional constraints of the model are made up of both the goals constraints and the system constraints. The MILGP model which can be applied in any system with multiple goals and given priority levels was illustrated with data from Nigerian Breweries PLC with the objective of minimizing the underachievement of the estimated profit target, underachievement of the estimated production target levels for each of the drinks Star, Gulder, Maltina, Fayrouz, Goldsberg, and 33 export, overachievement of the available machine bottling time and overachievement of the estimated distribution cost.

The satisficing solution obtained using the LINGO software showed that all the goals according to the priority levels were achieved and lexicographic redundancy was not found in the solution. Hence the model can be considered as efficient and can be applied in other multi-product multi-goals systems. The model may still be improved upon by considering the case of an increase or decrease in per unit penalty function instead of the per unit penalty of the weights for every unit of deviational variable from the target level as considered in this paper.

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References

[1] Sharma, J. (2013). *Operations research: Theory and applications (fifth edition)*. Haryana, India: Macmillan publishers.

[2] Gupta, P.K. and Hira, D.S. (2012). *Operations Research*. New Delhi, India: S.Chang & company.

[3] Gasin, M. (2013). Aggregate production planning using GoalnProgramming. *Al-Rafadain Engineering Journal, 21*(3), 78-86.

[4] Kellegoz, T. (2016). Balancing Lexicographic multi-objective Assembly lines with multi-manned stations. *mathematical problems in Engineering*, 1-20.

[5] Sharma, D.K. and Ghosh,D. (2007). Lexocigraphic Goal Programming model for police patrol cars deployment in metropolitan cities. *information and management sciences, 18*(2), 173-188.

[6] Orumie,U.C. and Ebong, D.W. (2013). An efficient method of solving Lexicographic Linear Goal Programming problem. *international journal of scientific and research publications, 3*(10), 1-8.
[7] Maladi, N. and Wong, K.Y. (2014). A multiobjective fuzzy aggregate production planning model considering real capacity and quality of products. *mathematical problems in Engineering*, 1-15.
[8] Sedzo, K., Marouane, A. and Assogbavi,T. (2011). Analytical hierarchy process and Goal programming approach for Asset allocation. *journal of mathematical finance, 2*, 96-104.
[9] Wang, c., Liu, X., Zhao, G.Z. and Chin,K.O. (2014). Multi-objective integrated production plannung model and similation constrained doubly by resource and materials. *international journal of simulation modeling, 13*(2), 243-251.
[10] Chang, H. (2013). A satisfactory method for fuzzy goal programming problems with different importance and priorities. *Quality and Quantity, 47*(1), 485-498.
[11] Ignizio, J. (1976). *Goal Programming and Extensions.* Lexington, USA: D.C. Heath