The Generalized Beta Modified Weighted Exponential Distribution: Theory and Application

N. I. Badmus[1]; A. A. Olufolabo[2]; T. A. Bamiduro[3]

[1]Department of Statistics,
Yaba College of Technology,
Lagos, Nigeria.
e-mail: idowu.badmus@yabatech.edu.ng[1]

[2]Federal School of Statistics (FSS),
Ibadan, Oyo State, Nigeria.
Email: iteneyeolufolabo@yahoo.co.uk[2]

[3]Department of Statistics,
University of Ibadan,
Ibadan, Nigeria
Email: adebayobamiduro@yahoo.com[3]

***Abstract*— Flexible models are designed for modeling non-normal data in survival analysis and this has become the concern of statisticians in data analysis. A new flexible and continuous distribution called Beta modified Weighted Exponential distribution that extends modified Weighted Exponential distribution and some of its special cases are proposed. The essence is to compare the effectiveness of Beta modified Weighted Exponential distribution against modified Weighted Exponential distribution in terms of parameter estimation. The statistical properties of the proposed model are provided and the method of Maximum Likelihood Estimation was used in estimating its parameters. Real data is used to establish the validity of the developed model.**

Keywords: *Beta modified Weighted Exponential, hazard rate, kurtosis, skewness, survival rate*

I. Introduction

Flexible parametric models are useful for modeling survival data and this has become an important aspect in the field of statistics which is now a concern to statisticians in data analysis. This paper therefore presents a univariate model called Beta modified Weighted Exponential (BMWE) distribution constructed from modified Weighted Exponential (MWE) distribution as the base line distribution (see Aleem*et al.*, [2]). This new distribution is obtained by adding two shape parameters to the existing MWE distribution using logit of beta function by Jones [15]. This approach has been used by several researchers in literature.

Some of the works related to beta link function (Jones, [15]) include; beta Frechet distribution (Nadarajah and Gupta, [17]), Exponentiated exponential family: an alternative to gamma and weibull distributions (Gupta and Kundu, [12]), beta generalised exponential (Barreto-Souza *et al.*, [5]), a new family of generalized distribution (Cordeiro*et al.*, [7]), the beta exponentiatedweibull distribution (Cordeiro*et al.*, [8]), , beta weighted weibull (Badmus and Bamiduro, [3]), beta weighted exponential distribution (Badmus*et al.*, [4]), and beta exponential Frechet distribution (Mead *et al.*, [16]) among others.

The rest of the paper is divided into six (6) sections as follows: section 2, we present the probability (pdf), cumulative (cdf), reliability function, hazard rate, reverse hazard functions and various sub-models. Section 3 contains the derivation of the moments and moment generating function, skewness, kurtosis and entropy. Estimation of model parameters using method of maximum likelihood estimation is presented in section 4. Application of the proposed model is considered in section 5, followed by conclusion in section 6.

II. Research Methodology

The pdf of the BMWE Distribution

The baseline as stated in Aleem*et al.* [2] is given as:

$$f_{MWE}\ (t) = \alpha(\beta\gamma + 1)e^{(-\alpha(\beta\gamma+1)t} \qquad (1)$$

where, α is a scale parameter while, β and γ are shape parameters; and the corresponding distribution function is:

$$F_{MWE}(t) = 1 - e^{(-\alpha(\beta\gamma+1)t} \quad (2)$$

Here we explore a new beta modified weighted exponential distribution using generalized beta by (Jones, [15]) in which many authors have used in literature. Some more recent extensions on generalizations are the beta modified Weibull distribution (Silva *et al*., [18]), the Kumaraswamy modified Weibull distribution (Cordeiro*et al*., [9]) are the generalized modified Weibull (GMW) (Carassco *et al*., [6]).

The beta link function is given as

$$g(t) = \frac{f(t)[F(t)]^{u-1}[1-F(t)]^{v-1}}{B(u,\ v)},\ \mathrm{u,v} > 0 \quad (3)$$

where, $u, v\ and\ t > 0$, u and v are two shape parameters, in the addition of those in the baseline distribution, $f(t)$ and $F(t)$ are the pdf and cdf of base line distribution respectively.

III. Analysis

Application to Real-life Data

We fit BMWE, MWE and BWE distributions to a real data set to establish the supremacy of the BMWE distribution over both MWE and BWE distributions. The data contain life of fatigue fracture of Kevlar 373/epoxy subjected to constant pressure at 90 % stress level until all had failed. The data was extracted from (Abdul-Moniem and Seham, [1]). MLEs method is used to obtain the estimates of parameters, standard errors and p-values. Models are compared employing Akaike Information Criterion, Bayesian Information Criterion and Consistent Akaike Information Criterion. However, the Exploratory Data Analysis (EDA) is presented in Table 2.

Furthermore, since the coefficient of skewness of a normal distribution is 0, while the kurtosis value quantifies the weight of tails in comparison to the normal distribution for which the kurtosis equals 3 (see Ezequiel, [11]and Delignette-Muller and Dutang, [10]) Then, Table 2 revealed the nature of the data i.e the skewness is non-zero and kurtosis is greater than 3. Automatically, this implies that the data is non-normal data (skewed data); and it requires flexible distribution to model such data.

IV. Results

Table 3 contains the maximum likelihood estimates, standard error (in parenthesis), and model selection criteria (AIC, BIC and CAIC) of the BMWE, MWE and BWE. The log-likelihood of the BMWE ($\hat{l}_{BMWE} = 203.694$), MWE ($\hat{l}_{MWE} = 191.031$) and BWE ($\hat{l}_{BWE} = 198.114$). Thelikelihood ratio (LR) statistic for testing the Hypothesis $H_0: a = b = 1\ versus\ H_1: H_0\ is\ not\ true$i.e., we estimated LR of the models as follows: the BMWE and MWE models are $w_1 = 2(-\hat{l}_{BMWE} - (-\hat{l}_{MWE})) = -25.326$, BMWE and BWE models, $w_2 = 2(-\hat{l}_{BMWE} - (-\hat{l}_{BWE})) = -11.160$; and BWE and MWE, is $w_3 = 2(-\hat{l}_{BWE} - (-\hat{l}_{MWE})) = -14.166$. Then, (p-value < *2e-16****) all through while, the p-value of the baseline distribution are (< *2e-16****) and < *4.9e-10****); and whereby p-value for BWE are (< *2e-16****), *8.0e-05* and < *2.1e-12****). Therefore, it is evident that the BMWE distribution is a better model than MWE distribution with respect to values of the AIC, BIC and CAIC.

A. Equations

Now, putting equations (1) and (2) into (3); we obtain the density function of BMWE as:

$$f_{BMWE|}(t) = \frac{f_{MWE}(t)[F_{MWE}(t)]^{u-1}[\bar{F}_{MWE}(t)]^{v-1}}{B(u\ ,v)},$$

$$\alpha, \beta, \gamma, u, v\ \approx BMWE\ (\alpha, \beta, \gamma, u, v, t > 0) \quad (4)$$

where, $\bar{F}_{MWE}(t) = 1 - [F_{MWE}(t)]^{v-1} = \left[e^{(-\alpha(\beta\gamma+1)t}\right]^{v-1}$

$$f_{BMWE|\{\alpha,\beta,\gamma,u,v\}}(t) = \frac{1}{B(u\ ,v)}\left[1 - e^{(-\alpha(\beta\gamma+1)t}\right]^{u-1}$$
$$\left[e^{(-\alpha(\beta\gamma+1)t}\right]^{v-1}\alpha(\beta\gamma+1)e^{(-\alpha(\beta\gamma+1)t} \quad (5)$$

where, $f_{MWE}(t)$ and $F_{MWE}(t)$ is the pdf and cdf of base distribution; and expression (4) becomes the pdf of the new BMWE distribution.

Letting

$$W(t) = F_{MWE}(t) = 1 - e^{(-\alpha(\beta\gamma+1)t)}$$

then,

$$\frac{dW}{dt} = [f_{MWE}(t)][F_{MWE}(t)] \quad (6)$$

Consequently, expression (4) becomes

$$f_{BMWE}(t) = \frac{1}{B(u,v)}[W(t)]^{u-1}[\bar{W}(t)]^{v-1}dW(t) \quad (7)$$

Cumulative Distribution Function (CDF)

Here, we used equation (7) to obtained cumulative distribution function of BMWE with variable T and is given by

$$F_{BMWE}(t) = \int_0^t f_{BMWE}(t) = \frac{\int_0^t [W(t)]^{u-1}[\bar{W}(t)]^{v-1}dW(t)}{B(u,\ v)} \quad (8)$$

$$F_{BMWE}(t) = \frac{B(g;u,\ v)}{B(u,\ v)} \quad (9)$$

where

$I_t(u,v) = \frac{1}{B(u,\ v)}\int_0^t y^{u-1}(1-y)^{v-1}dy$ denotes the incomplete beta function ratio.

The Reliability Function

The survival function $RL_{BMWE\{\alpha,\beta,\gamma,u,v\}}$ of a random BMWE $(\alpha,\beta,\ \gamma,u,v)$ variable T with distribution function $F(t)$ is defined as

$$\{\alpha,\beta,\ \gamma,u,v\}(t) = 1 - F_{BMWE}(t) = \bar{F}_{BMWE} \qquad (10)$$

$$S_{BMWE}\{\alpha,\beta,\ \gamma,u,v\}(t) = \frac{B(u,\ v\,b) - B(t;u,\ v)}{B(u,\ v)} \qquad (11)$$

The Hazard Rate Function

The hazard rate function $HZ_{BMWE\{\alpha,\beta,\gamma,u,v\}}$ of a random BMWE $(\alpha,\beta,\ \gamma,u,v)$ variable T with cdf $F(t)$ is given by

$$hz_{BMWE\{\alpha,\beta,\gamma,u,v\}}(t) = \frac{f_{BMWE\{\alpha,\beta,\gamma,u,v\}}(t)}{\bar{F}_{BMWE}(t)E\{\alpha,\beta,\gamma,u,v\}(t)} \qquad (12)$$

Substituting equations (7) and (11) yield

$$hz_{BMWE\{\alpha,\beta,\gamma,u,v\}}(t) = \frac{[W(t)]^{u-1}[\bar{W}(t)]^{v-1}dW(t)}{B(u,\ v) - B(t;u,\ v)} \qquad (13)$$

Some Special Cases of the Generalized Modified Weighted Exponential Distribution

Some new distributions emanate from the BMWE distribution depending on the values of the parameter; these include:

- If $u = 1$ in (5) we have Lehmann Type II Modified Weighted Exponential (LMWE) distribution

$$f_{LMWE|\{\alpha,\beta,\gamma,1,v\}}(t) = v\left[e^{(-\alpha(\beta\gamma+1)t}\right]^{v-1}\alpha(\beta\gamma + 1)e^{(-\alpha(\beta\gamma+1)t} \qquad (14)$$

- For v= 1 in (3) we obtain Exponentiated Modified Weighted Exponential (EMWE) distribution

$$f_{EMWE|\{\alpha,\beta,\gamma,u,1\}}(t) = u\left[e^{(-\alpha(\beta\gamma+1)t}\right]^{u-1}\alpha(\beta\gamma + 1)e^{(-\alpha(\beta\gamma+1)t} \qquad (15)$$

Moments and Generating Function

Moment Generating Function

The moment generating function (MGF) of the generalized BMWE distribution is obtained using the idea of Hosking [14] used in Beta generated distributions. The MGF $M(s) = E(e^{st})$ is given as

$$M(t) = \frac{1}{B(u,v)}\sum_{i=0}^{\infty}(-1)^i\binom{v-1}{i}$$

$$\int_{-\infty}^{\infty} e^{st} f_{MWE}(t)[F_{MWE}(t)]^{u(i+1)-1} \qquad (16)$$

Substituting the $pdf\ f_{MWE}(t)$ and $cdf\ F_{MWE}(t)$ as defined in (1) and (2) above into $MGFM(s)$ in equation (16) gives

$$M_{BMWE}(t) = \frac{1}{B(u,v)}\sum_{i=0}^{\infty}(-1)^i\binom{v-1}{i}\int_{-\infty}^{\infty} e^{st}\left[e^{(-\alpha(\beta\gamma+1)t}\right]^{u(i+1)-1}\left[\alpha(\beta\gamma+1)e^{(-\alpha(\beta\gamma+1)t}\right]dt \qquad (17)$$

Moments

Using Aleem*et al.* [2], the r^th non-central moment of the class of MWE distribution $MWE(\alpha,\beta,\gamma)$ is given by:

$$\mu'_{MWEr} = E(X^r) = (\beta\gamma+1)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\beta\gamma+1)^{-\frac{r}{2}-1} \qquad (18)$$

The r^th non-central moment of the BMWE distribution would be given as

$$\mu'_{BMWE(r)} = \int_0^{\infty} t^r f_{BMWE}(t)dt$$

That is

$$\mu'_{BMWE(r)} = \int_0^{\infty} t^r\left\{\frac{1}{B(u,v)}[W(t)]^{u-1}[1 - W(t)]^{v-1}dW(t)\right\}$$

where

$W(t) = e^{(-\alpha(\beta\gamma+1)t}$, $m(t) = e^{-\alpha t}$, $\theta = (\beta\gamma+1)$

Then,

$$\mu'_{BMWE(r)} = \left[\frac{(\delta)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\delta)^{-\frac{r}{2}-1}}{B(u,\ v)}\right]\sum_{i=0}^{\infty}(-1)^i\binom{v-1}{i}.$$

$$\left\{\int_0^{\infty}[\mathrm{m(t)}(\delta)]^{u(i+1)-1}dt\right\}$$

$$= P\left[(\delta)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\delta)^{-\frac{r}{2}+1}\right] \qquad (19)$$

Badmus*et al.* [4]

where $P = \frac{\sum_{i=0}^{\infty}(-1)^i\binom{v-1}{i}\int_0^{\infty}[\mathrm{m(t)}(\delta)]^{u(i+1)-1}dt}{B(u,v)}$,

We also derived the first four non-central moments μ'_r, by letting $r = 1, 2, 3\ and\ 4$ respectively in equation 19; i.e.μ'_1 is given as

$$\mu'_1 = E_{BMWE}(t) = \left[\frac{(\delta)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\delta)^{-\frac{3}{2}}}{B(u,v)}\right]\left[\sum_{i=0}^{\infty}(-1)^i\binom{v-1}{i}\right]$$

Hence, central moments $\mu_r, r = 1, 2, 3, 4, \ldots$ are related to noncentral moments μ_r' as

$$\mu_r = \sum_{w=0}^{r} \binom{r}{k} \mu_{r-k}' \, \mu_k'^{k}, \text{ where } \mu_1' = \mu \text{ and } \mu_0' = 1 \qquad (20)$$

Conversely, the mean and variance, 3rd and 4th moments of the BMWR distribution are given by

$$\mu = \mu_1'$$
$$\mu_2 = \mu_2' - \mu^2$$
$$\mu_3 = \mu_3' - 3\mu\mu_2' + 2\mu^3 \text{and}$$
$$\mu_4 = \mu_4' - 4\mu\mu_3' + 6\mu^2\mu_2' - 3\mu^4$$

where,

$$\mu_1' = P\left[(\delta)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\delta)^{-\frac{3}{2}}\right] \qquad (21)$$

$$\mu_2' = P\left[(\delta)\alpha^{-\frac{2}{2}}\Gamma\left(\frac{2+2}{2}\right)(\delta)^{-\frac{(2+2)}{2}}\right]$$

$$= 2P[(\delta)\alpha^{-1}(\delta)^{-2}] (22)$$

$$\mu_3' = P\left[(\delta)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{3+2}{2}\right)(\delta)^{-\frac{(3+2)}{2}}\right] = 6P\left[(\delta)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{5}{2}\right)(\delta)^{-\frac{5}{2}}\right] (23)$$

$$\mu_4' = P\left[(\delta)\alpha^{-\frac{4}{2}}\Gamma\left(\frac{4+2}{2}\right)(\delta)^{-\frac{(4+2)}{2}}\right] = 24P[(\delta)\alpha^{-2}2(\delta)^{-3}] (24)$$

Moments measures of Skewness, ω_1 and of excess kurtosis, ω_2, are respectively given as

$$\omega_1 = \frac{\mu_3}{\sqrt[2]{\mu_2^3}} \qquad (25)$$

$$\omega_2 = \frac{\mu_4}{\mu_2^2} \qquad (26)$$

Renyi Entropy

Entropy is a useful tool of measuring uncertainty variation and it has been used in different situations in science, engineering and medicine. The Renyi entropy is defined as

$$I_R(\theta) = (1-\theta)^{-1}\log\left(\int_{-\infty}^{\infty} f(k)^{\theta} dk\right)$$

where $\theta > 0$ and $\theta \neq 1$, (see Handique and Chakraborty, [13]). Then, using binomial expansion in (5) we write $f_{BMWE|}(t;\alpha,\beta,\gamma,u,v) =$

$$\left[\frac{1}{B(u,\ v)} f_{MWE}(t)[F_{MWE}(t)]^{u-1}\right]^{\theta}[1-[1-F_{MWE}(t)]]^{(v-1)\theta}$$

$$\frac{1}{B(u,\ v)^{\theta}}[f_{MWE}(t)[F_{MWE}(t)]^{u-1}]^{\theta} \sum_{i=0}^{(v-1)\theta} \binom{(v-1)\theta}{i}(-1)^i[1-F_{MWE}(t)]^i$$

Now the entropy of T can be obtained as

$$I_R(\theta) = (1-\theta)^{-1}\log\left(\sum_{i=0}^{(v-1)\theta} R_i \int_{-\infty}^{\infty}[f_{MWE}(t)[F_{MWE}(t)]^{u-1}]^{\theta}[1-F_{MWE}(t)]^i dt\right)$$

$$= (1-\theta)^{-1}\log\left(\sum_{i=0}^{(v-1)\theta} R_i \int_{-\infty}^{\infty}[f_{MWE}(t;\alpha,\beta,\gamma,u)]^{\theta}[\bar{F}_{MWE}(t;\alpha,\beta,\gamma)]^i dt\right)$$

where $R_i = \frac{1}{B(u,v)^{\theta}}\binom{(v-1)\theta}{i}(-1)^i$.

Parameter Estimation

We derived the MLEs of the parameter of $BMWE(\alpha, \beta, \gamma, u, v)$ distribution following Cordeiro*et al.* [7] and Badmus*et al.* [4];

Putting $\varphi = (u, v, \omega, \tau)$, where $\tau = (\beta,\ \gamma, \theta)$ and is a vector of parameters.

We had the likelihood

$$L_{BMWE}(\varphi) = m\log\omega - n\log[B(u,v)] + \sum_{i=1}^{n}\log[f(t;\varphi)] + (u-1)\sum_{i=1}^{n}\log[F(t;\varphi)]$$
$$(v-1)\sum_{i=1}^{n}\log[1-F(t;\varphi)] \qquad (27)$$

$$L_{BMWR}(\varphi) = Const - v\log[B(u,v)] + \sum_{i=1}^{n}\log[f(t;\varphi)]$$
$$+(u-1)\sum_{i=1}^{n}\log[F(t;\varphi)](v-1)\sum_{i=1}^{n}\log[1-F(t;\varphi)] \qquad (28)$$

Taking partial derivative of (28) with respect to $(u, v, \alpha, \beta, \gamma)$, we get

$$\frac{\partial L_{BMWE}(\varphi)}{\partial u} = -n\log(u,v) + (u-1)\sum_{t=1}^{n}\log[F(t;\varphi] (29)$$

$$\frac{\partial L_{BMWE}(\varphi)}{\partial v} = -n\log(u,v) + (v-1)\sum_{t=1}^{n}\log[1-F(t;\varphi] \qquad (30)$$

$$\frac{\partial L_{BMWE}(\varphi)}{\partial \alpha} = \sum_{t=1}^{n} \log\left[\frac{\frac{\partial}{\partial\alpha}[f(t;\varphi)]}{f[(t,\ \varphi)]}\right] + (\mathrm{u}-1)\sum_{t=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}[F(t;\varphi)]}{F(t;\varphi)}\right] + (\mathrm{v}-1)\sum_{t=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}[1-F(t;\varphi)]}{1-F(t;\varphi)}\right] (31)$$

$$\frac{\partial L_{BMWE}(\varphi)}{\partial \beta} = \sum_{t=1}^{n} \log\left[\frac{\frac{\partial}{\partial\beta}[f(t;\varphi)]}{f[(t;\varphi)]}\right] + (\mathrm{u}-1)\sum_{t=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}[F(t;\varphi)]}{F(t;\varphi)}\right] + (\mathrm{n}-1)\sum_{t=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}[1-F(t;\varphi)]}{1-F(t;\varphi)}\right]$$

(32)

$$\frac{\partial L_{BMWE}(\varphi)}{\partial \gamma} = \sum_{g=1}^{v} log\left[\frac{\frac{\partial}{\partial \gamma}[f(t;\varphi)]}{f[(t;\varphi)]}\right] + (u-1)\sum_{t=1}^{n} \log\left[\frac{\frac{\partial}{\partial \gamma}[F(t;\varphi)]}{F(t;\varphi)}\right] + (v-1)\sum_{t=1}^{n} \log\left[\frac{\frac{\partial}{\partial \gamma}[1-F(t;\varphi)]}{1-F(t;\varphi)}\right] \quad (33)$$

Expressions (29) to (33) can be solved using iteration method (Newton Raphson) to obtain $\hat{u}, \hat{v}, \hat{\alpha}, \hat{\beta}, \hat{\gamma}$ the MLE of $(m, n, \alpha, \beta, \gamma)$ respectively.

B. Figures and Tables

The plots (i – iv) shown below are the pdf plots of the propose distribution, its special cases and the baseline distributions (BMWE, LMWE, EMWE and MWE distributions) at several values of $u = (10, 1, 50, 2.5, 1.5)$ and $v = (3.5, 1, 2.5, 5, 1.5)$ when $c = \alpha, d = \beta$ and $e = \gamma$ are fixed at (0.3, 5, 0.3).
From the plot, as values of m and n increases, the skewness of the BMWE (in red ink) decreases and as values reduces the graphs of the BMWE (in red ink) skewed to the right and so on.

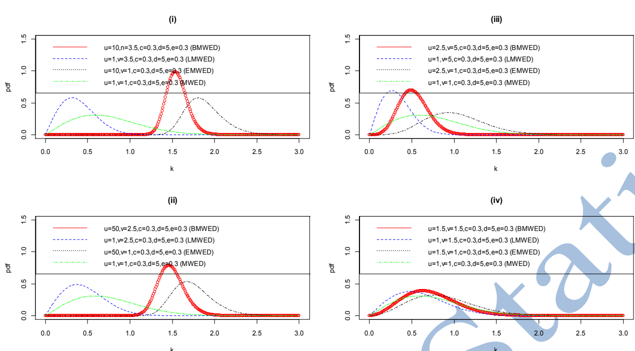

Figure 1. The pdf plots of the BMWE, LMWE, EMWE and MWE distributions

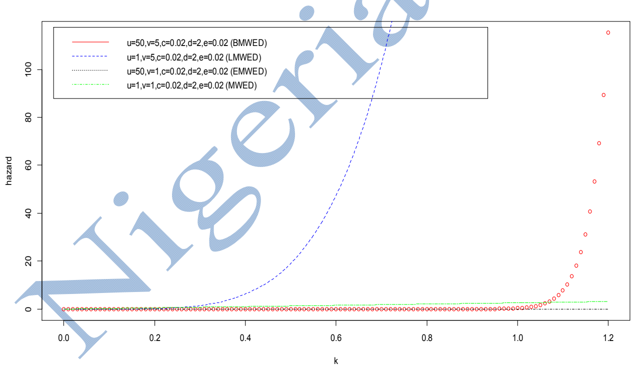

Figure 2. Plot of hazard rates of the BMWE $(u, v, \alpha, \beta, \gamma)$, L MWE= $(1, v, \alpha, \beta, \gamma)$, EMWE $(u, 1, \alpha, \beta, \gamma)$ and MWE = $(u, v, \alpha, \beta, \gamma)$, ((0.3,0.3,0.15,0.01,0.28), (1,0.3,0.15,0.01,0.28), (0.3,1,0.15,0.01,0.28), (1,1,0.15,0.01,0.28)), respectively

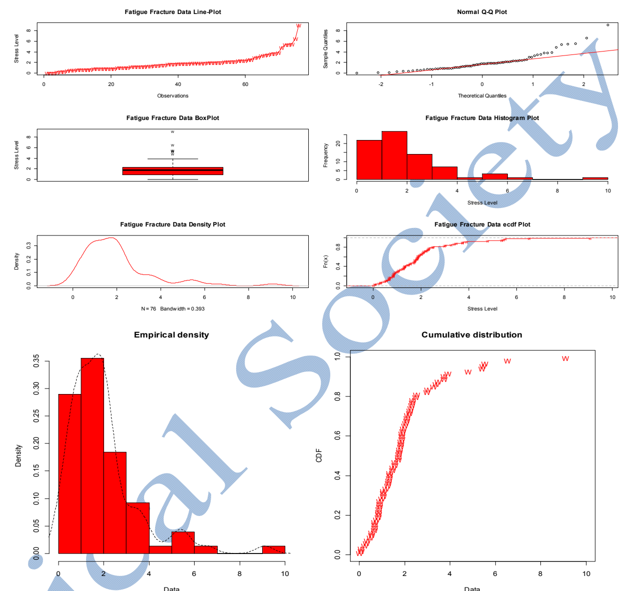

Figure 3: Consists line plot, normal QQ plot, box-plot, Empirical density and cumulative distribution.

Table 1: Summary of Some New Distributions Emanate from BMWW Distribution

Distribution/Parameter	u	v	α	β	γ
LMWE	1	--	--	--	--
EMWE	--	1	--	--	--

TABLE 2: Summary

Descriptive Statistics		
Min	***1st Qt.***	***Median***
0.0251	0.9048	1.7360
3rdQt.	***Mean***	***Max***
2.2960	1.9590	9.0960
Skewness	***Kurtosis***	
1.9796	8.1608	

TABLE 3: MLES OF THE PARAMETERS FROM THE DEVELOPED AND BASELINE DISTRIBUTION FITTED TO THE FATIGUE DATA SET,THE CORRESPONDING SES (GIVEN IN PARENTHESES) AND P-VALUE IN [.].

	Distributions and Parameters		
	BMWE	***MWE***	***BME***
u	8.336 (0.103) [<2e-16]	1	2.046 (0.112) [<2e-16]
v	0.248 (0.006) [<2e-16]	1	4.220 (1.070) [8.0e-05]
α	0.979 (0.006) [<2e-16]	3.570 (0.314) [<2e-16]	2.537 (0.361) [2.1e-12]
β	0.112 (0.001) [<2e-16]	-0.038 (0.006) [4.9e-10]	1 1 1
γ	4.852 (0.060) [<2e-16]	2.842 (0.316) [<2e-16]	0.018 (0.000) [<2e-16]
AIC	- 397.387	- 386.006	- 385.004
BIC	- 377.031	- 370.201	- 369.201
CAIC	- 388.229	- 379.122	- 378.122

IV. DISCUSSIONS

From all indications the BMWE distribution performed better than other competing distributions, due to the results from AIC, BIC, CAIC in Table 3. However, it implies that the BMWE distribution has capability of accommodating non-normal or skewed data and a good representation of the data used for the analysis.

V. CONCLUSION

A new BMWE distribution which includes two of its sub models namely the LMWE and EMWE distribution and some of its important properties were studied. The parameters of the distributions were estimated by the method of MLEs. In conclusion, the derived distribution revealed its superiority and has better representation of the data than other distributions when compared. However, the two sub models are introduced for future study.

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