

An Extended Goal Programming Model with the Dash Diet Plan for Hypertension Patients

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Abstract— The DASH diet plan is a heart-healthy daily eating plan that lowers high blood pressure. It requires foods that have low sodium, saturated fat, total fat, and cholesterol nutrient content while rich in potassium, magnesium, calcium, and fiber. A Weighted Goal Programming (WGP) model for the DASH diet plan has been previously been formulated. The satisficing diet plan obtained from the model gave a diet plan that minimizes the target tolerable nutrient limit but has imbalanced food servings. The Extended Goal Programming (EGP) is a modeling tool that combines the multiple philosophies of satisficing, optimizing, and balancing in multi-objective problems. In this paper, an EGP model for the DASH diet plan is formulated. This is to be able to obtain a satisficing diet plan with balanced food servings. The formulated model was illustrated with sample foods from the DASH food chart for a 1500mg daily sodium level and calories levels of 1600, 2000, and 3000 calories per day. The daily diet plans were also obtained for varying meta-weights for three meta-goals which included minimizing the normalized weighted sum of unwanted deviations from the set goals, minimizing the normalized maximum unwanted deviation from the set goals, and minimizing the number of unmet goals from the set goals. The solution obtained from the use of the LINGO software presented a satisficing and balanced diet.

Keywords - Goal programming, Extended Goal programming, DASH diet, multi-objective modelling, balanced diet plan.

I. INTRODUCTION

Goal programming (GP) developed by [1] presents a technique for solving multi-objective models in which the several objectives may be conflicting and do not have the same units of measurement. The basic idea of the goal programming technique is to convert the original multiple objectives into a single goal which is solved to obtain what is referred to as a satisficing solution. The resulting solution is termed as satisficing or efficient because it may not be optimal with respect to all the conflicting objectives of the problem. In [2] the goal programming technique was proven to be an appropriate method for achieving nutritional balance in selected diets.

It has also been shown as a popular theoretical method for dealing with multiple objective decision-making problems in [3]. In the GP model, the achievement function is a key element which represents a mathematical expression of the unwanted deviational variables. The most widely used GP achievement function variants include the weighted GP variants, the lexicographic variant and the MinMax GP variants. While the weighted, lexicographic and MinMax forms of the achievement function are the most widely GP variants used, extensions may be made in the achievement function under certain conditions to suit the decision maker's preferences and to achieve more accuracy.

The Extended Goal Programming (EGP) model introduced by [4] is an extension to the goal programming model. It allows a parametric analysis of trade-off between efficiency and balance on the levels of achievement of the goal target values. Lexicographic and non-lexicographic forms of the achievement function of the model have been presented in [4]. In [5] an Extended GP methodology to achieve a balance and optimal solution across a hierarchical decision network was proposed. Meanwhile in [6] an application of Extended GP in the field of offshore wind farm site selection using the farm sites in United Kingdom as an example was presented.

On the other hand, two new meta-objectives were introduced into the EGP framework in [7]. In [8] a weighted GP diet model was formulated for the Dietary Approach to Stop Hypertension (DASH) problem aimed at minimizing the daily cost of the DASH eating plan as well as the deviations from the DASH diet's nutrients tolerable intake levels. From the results obtained the weighted GP model minimized deviations from the DASH diet nutrient's tolerable intake levels as well as the target daily cost. One of the shortcomings of the weighted GP for the DASH diet problem though is its inability to give a solution in which there is balance between efficiency and equity in the minimization of deviations from nutrients tolerable limit targets as well as the recommended number of daily servings of the foods. This shortcoming of the weighted GP model is been taken care of in the Extended GP model proposed in [4]. Hence in this paper we achieve the efficiency-equity

balance in minimizing deviations from target goals of diet cost and nutrient's tolerable intake limits together with the daily food servings in the DASH diet plan for hypertension patients.

II. MATERIALS AND METHODS

The DASH diet plan

The DASH diet plan follows a healthy guideline for hypertension patients: see [9]. It focuses on the intake of foods rich in nutrients that are expected to lower blood pressure mainly minerals (like potassium, calcium and magnesium and fiber). These foods also have low quantity of nutrients that increase blood pressure like sodium, fats, cholesterol, carbohydrate and protein. The DASH eating plan calls for a certain number of daily servings from various food groups which includes grains, vegetables, fruits, fat-free or low-fat milk and milk products, nuts, lean meats, poultry and fish. The number of serving being dependent on the required calorie level of the patients which depends on the patient's daily life style. We have DASH eating plans for different daily calorie levels like 1600, 2000, 2600 and 3100 calories eating plans which is recommended for persons of different lifestyles as shown in table 1.

Table 1: DASH daily calorie need chart for different level of activities.

| | | Calories needed for each activity level | | |
|--------|---------|---|-------------------|-------------|
| Gender | Age | Sedentary | Moderately active | Active |
| Female | 19 - 30 | 2000 | 2000 - 2200 | 2400 |
| | 31 - 50 | 1800 | 2000 | 2200 |
| | 51+ | 1600 | 1800 | 2000 - 2200 |
| Male | 19 - 30 | 2400 | 2600 - 2800 | 3000 |
| | 31 - 50 | 2200 | 2400 - 2600 | 2800 - 3000 |

A. Non-lexicographic Extended GP model

The non-lexicographic EGP model as introduced by [4] is a combination of the weighted GP and MinMax GP models which balances efficiency and equity in minimizing deviations from given target of goals. It is presented as follows;

Achievement function

$$\text{Min } (1 - \lambda)D + \lambda \sum_{i=1}^q (\alpha_i n_i + \beta_i p_i) \quad (1)$$

Goals and constraints

$$(\alpha_i n_i + \beta_i p_i) - D \leq 0 \quad (2)$$

$$f_i(x) + n_i - p_i = t_i, \quad i \in \{1, \dots, q\} \quad (3)$$

$$x \in F, n \geq 0, p \geq 0, \lambda \in (0,1) \quad (4)$$

λ weights the importance attached to the minimization of the weighted sum of unwanted deviation variables.

n_i and p_i are the negative and positive deviations from the goal's target.

D is maximum deviation from goal's target.

The EGP model as presented in [10] in a production data with normalized weighted deviational variables is as follows

$$\text{Min } a = \alpha\lambda + (1 - \alpha) \sum_{i=1}^m \left(\frac{u_i n_i}{k_i} - \frac{v_i p_i}{k_i} \right) \quad (5)$$

Subject to

$$\frac{u_i n_i}{k_i} + \frac{v_i p_i}{k_i} \leq \lambda \quad (6)$$

$$\sum a_{ij} x_j + n_i - p_i = b_i \dots \dots \text{Goal constraints} \quad (7)$$

$$, \quad i = 1, 2, \dots, Q, \quad j = 1, 2, \dots, n.$$

$$\sum_{j=1}^m c_{jk} x_j (\leq, =, \geq) g_j \dots \dots \text{System constraints} \quad (8),$$

$$k = 1, 2, \dots, m$$

$$x_j, n_i, p_i \geq 0 \quad (9)$$

With k as the normalization constant, λ as the maximum deviation and α as parameter weights the importance attached to the minimization of the weighted sum of unwanted deviation variables.

B. Extended Goal Programming DASH diet model Distribution

In this section we present the EGP model for achieving a balanced and optimal daily DASH diet plan that minimizes deviations from DASH nutrient's tolerable lower and upper target levels as well as the recommended daily servings of foods. A three meta-objective EGP model is used to allow for a combination of balancing, optimizing and goal-achieving philosophies. Assuming a linear form of the achievement function, percentage normalization and positive continuous decision variables, the Extended Goal Programming DASH diet model is presented as follows;

$$\text{Min } D = \alpha\lambda + \beta \sum_{q=1}^Q \left(\frac{u_q n_q}{k_q} - \frac{v_q p_q}{k_q} \right),$$

$$+ \gamma \left(\sum_{q=1}^Q t_q y_q + \sum_{q=1}^Q w_q r_q \right), \quad q = 1, \dots, Q \quad (10)$$

Subject to;

$$\sum_{j=1}^n a_{ij} x_j + n_q - p_q = b_q, \quad j = 1, \dots, n \quad (11)$$

$$\sum_{i=1}^m a_{ij} x_j (\leq, =, \geq) g_j, \quad i = 1, 2, \dots, m \quad (12)$$

$$x_j \leq s_j \quad (13)$$

$$\frac{u_q n_q}{k_q} \leq \lambda, \quad q \in Q_1 \quad (14)$$

$$\frac{v_q p_q}{k_q} \leq \lambda, \quad q \in Q_2 \quad (15)$$

$$n_q - M y_q \leq 0, \quad q \in Q_1 \quad (16)$$

$$p_q - M r_q \leq 0, \quad q \in Q_2 \quad (17)$$

$$n_q \geq 0, p_q \geq 0, x_j \geq 0, n_q p_q = 0, y, r = 0 \text{ or } 1, \quad Q_1, Q_2 \in Q. \quad (18)$$

$$y_q = \begin{cases} 1, & \text{if achieved value is less than target value} \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

$$r_q = \begin{cases} 1, & \text{if achieved value is greater than target value} \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

With

k_q = Normalization constant associated with the q th goal.

a_{ij} = quantity of i th nutrient in one serving of food x_j , $j = 1, 2, \dots, m$.

u_q = relative level of importance associated with per unit minimization of the negative deviational variable from the target value of the q th goal.

v_q = relative level of importance associated with per unit minimization of the positive deviational variable from the target value of the q th goal.

y_q = binary variable that takes value 1 if the achieved value of the q th goal is less than the target value and 0 if otherwise.

r_q = binary variable that takes value 1 if the achieved value of the q th is greater than the target value and 0 otherwise.

t_q = relative weights representing the penalty applied for not meeting the q th goal in the negative direction.

w_q = relative weights representing the penalty applied for not meeting the q th goal in the positive direction.

n_q = negative deviational variable of the q th goal

p_q = positive deviational variable of the q th goal

Q_1 = ordered set of the indices of unwanted negative deviational variables.

Q_2 = ordered set of the indices of unwanted positive deviational variables.

b_q = estimated target level for q th goal.

g_j = total calorie content of nutrient j in diet plan.

s_j = recommended daily servings of food j

λ = maximum weighted unwanted deviation

The parameters α, β and γ represents the relative importance of the meta-objectives “minimization of the normalized maximum unwanted deviations from the set of goals, “minimization of the normalized weighted sum of unwanted deviations from the set of goals and “minimization of the number of unmet goals from the set of goals respectively. The parameters are constrained as $\alpha + \beta + \gamma = 1$ as used in [12].

III. RESULTS AND DISCUSSION

A. Data Analysis

The Extended GP DASH diet model will be illustrated using some selected food items from the DASH sample food charts. The sample foods together with the tolerable intake

level for each of the ‘DASH’ nutrients were gotten from [9]. The sample foods includes carrot, cooked groundnut, whole wheat bread, boiled sweet potato, low fat yoghurt, orange juice, watermelon and grilled fish. The nutrient content of the foods were gotten from the food composition table in [11]. The servings of the foods were measured in the Nutrition and dietetics lab of Michael Okpara university of Agriculture Umudike while the costs per serving of the sample foods were obtained in Ubani central market Umuahia, Abia State. The data is shown in Table 2. It also contains the weight per serving of the foods. Meanwhile in Table 3 we have the tolerable intake level of the nutrients which are the target values of each goal. The objective is to minimize deviations from the desired target levels of the considered goals and extended goals as stated below.

The goals considered as the original goals of the decision maker in this work includes;

- Minimizing the overachievement of the desired daily diet cost.
- Minimizing the overachievement of the tolerable intake level of each of the undesired nutrients (saturated fat, total fat, sodium, protein, carbohydrate and cholesterol).
- Minimizing the underachievement of the tolerable intake level of each of the desired nutrients (potassium, magnesium, calcium, fiber).

The system constraints of the diet plan are as follows;

- Total fat should be 27% of total calorie level
- Saturated fat should be 6% of total calorie level
- Protein should be 18% of total calorie level
- Carbohydrate should be 55% of total calorie level.

The meta-objectives for the achievement of the above given goals are as follows;

- Minimizing the normalized weighted sum of unwanted deviations from the set goals.
 - Minimization of normalized maximum unwanted deviation from the set goals.
 - Minimization of number of unmet goals from the set goals.
- These will be referred in this work as meta objectives 1, 2 and 3 and their parameters are α, β , and γ respectively. So we are considering the achievement of 12 goals and 3 meta-goals in this illustration

B. Results

The summary of the result of the Extended Goal programming DASH diet model is given in table 4. The model was analyzed with the given data using the LINGO

optimization software. The analysis for the minimization of the deviations from the targets of the different goals was carried out for 1500mg daily sodium level on 1600, 2000 and 3000 daily calorie levels. The solution was obtained using the weight sensitivity analysis algorithm in [12] for the meta-weight space (α, β, γ) . This is used to investigate the effect varying the mix of underlying philosophies will have on the values of the decision variables and deviational variables. The input parameters of the algorithm are set as TMax = 2 (vary at most two parameters at a time). The normalizing $\alpha + \beta + \gamma = 1$ constraint is placed on the

values of (α, β, γ) to get a wide range of solutions. From the result an average of 9 goals is achieved from the 12 set goals across the different daily calorie levels of 1600, 2000 and 3000 and for the varying level of importance of the meta-weights. The solution values are seen not to vary much among the varying levels of the meta-weights. The meta-weights were given varying levels of importance to see how the decision variables (x_j 's) and deviational variables (n_q 's and p_q 's) vary with the different levels of importance.

Table 2. Nutrient content per serving of foods, cost and weight per serving

| NUTRIENTS | FOODS | | | | | | | |
|---------------------------------------|---------------|---------------------|---------------------------|------------------------------------|-------------------|----------------|-------------|-----------------------------|
| | Carrot (raw) | Ground nut (cooked) | Wheat Bread (whole wheat) | Sweet potato (boiled without salt) | Yoghurt (low fat) | Orange (juice) | Water melon | Fish (grilled without salt) |
| Total Fat (g) | 0.24 | 11.48 | 0.58 | 0.30 | 0.10 | 0.48 | 0.16 | 4.10 |
| Sodium (mg) | 33.60 | 1.50 | 124.80 | 15.00 | 8.10 | 3.20 | 2.40 | 73.00 |
| Cholesterol (mg) | 0 | 0 | 0 | 0 | 3.00 | 0 | 0 | 0.29 |
| Saturated Fat (g) | 0 | 1.55 | 0.20 | 0 | 0.60 | 0 | 0 | 34.00 |
| Protein (mg) | 0.25 | 5.1 | 2.23 | 0.38 | 0.95 | 0.18 | 0.13 | 7.45 |
| Carbohydrate (mg) | 1.43 | 4.13 | 10.38 | 6.38 | 1.7 | 2.25 | 1.55 | 0 |
| Calcium (mg) | 28.00 | 4.25 | 12.25 | 24.00 | 25.00 | 49.60 | 5.60 | 40.00 |
| Magnesium (mg) | 9.60 | 47.75 | 13.25 | 14.00 | 2.40 | 17.60 | 8.00 | 43.00 |
| Fiber (g) | 2.48 | 2.33 | 1.55 | 3.00 | 0 | 2.72 | 0.29 | 0.00 |
| Potassium (mg) | 212.8 | 181.75 | 56.50 | 264.0 | 31.0 | 265.6 | 87.2 | 549.0 |
| Calorie | 28 | 144.50 | 58.5 | 90.00 | 7.00 | 72.0 | 23.2 | 151.00 |
| Weight per serving of food (in grams) | 80 g | 25 g | 25 g | 100 g | 20 g | 160 g | 80 g | 100 g |
| Recommended number of daily servings | 1600 calories | 4 | 0.5 | 6 | 4 | 3 | 4 | 5 |
| | 2000 calories | 5 | 0.7 | 8 | 5 | 3 | 5 | 6 |
| | 3000 cal. | 6 | 1 | 13 | 6 | 4 | 6 | 9 |
| Cost of per serving of food | ₦15 | ₦20 | ₦15 | ₦15 | ₦30 | ₦15 | ₦15 | ₦50 |

Table 3. Target levels of goals and calorie level constraints.

| | |
|--------|---|
| Goals | Tolerable intake level of nutrients (either Upper or Lower) |
| Sodium | 1500mg (UTIL) |

| | | |
|---------------|--------|--------|
| Cholesterol | 129mg | (UTIL) |
| Total fat | 68g | (UTIL) |
| Saturated fat | 16g | (UTIL) |
| Protein | 95mg | (UTIL) |
| Carbohydrate | 285mg | (UTIL) |
| Calcium | 1334mg | (LTIL) |
| Magnesium | 542 | (LTIL) |
| Potassium | 5471mg | (LTIL) |
| Fiber | 34g | (LTIL) |

| | |
|--|------------|
| Calorie | About 2000 |
| 27% of calories should be from saturated fat | |
| 6% of calories should be from total fat | |
| 11% of calories should be from carbohydrate | |
| 55% of calories should be from Protein | |
| Estimated daily diet cost = ₦950 | |

Table 4. Summary of solution of Extended GP DASH diet model

| | 1500mg daily sodium level | | | | | | | | | | | |
|-----------|---------------------------|--------|--------|--------|---------------------------|--------|--------|--------|---------------------------|--------|--------|--------|
| | 1600 calories daily level | | | | 2000 calories daily level | | | | 3000 calories daily level | | | |
| | (a) | (b) | (c) | (d) | (a) | (b) | (c) | (d) | (a) | (b) | (c) | (d) |
| α | 0.333 | 0.900 | 0.050 | 0.050 | 0.333 | 0.900 | 0.050 | 0.050 | 0.333 | 0.900 | 0.050 | 0.050 |
| β | 0.333 | 0.050 | 0.900 | 0.050 | 0.333 | 0.050 | 0.900 | 0.050 | 0.333 | 0.050 | 0.900 | 0.050 |
| γ | 0.333 | 0.050 | 0.050 | 0.900 | 0.333 | 0.050 | 0.050 | 0.900 | 0.333 | 0.050 | 0.050 | 0.900 |
| λ | 0.478 | 0.4789 | 0.4789 | 0.4789 | 0.3085 | 0.2952 | 0.3085 | 0.3085 | 0.2706 | 0.2264 | 0.2706 | 0.2706 |
| n_1 | 375.4 | 375.4 | 375.4 | 375.4 | 227.7 | 209.9 | 227.7 | 227.7 | 184.9 | 105.2 | 184.9 | 184.9 |
| n_2 | 298.3 | 298.3 | 298.3 | 298.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n_3 | 119.2 | 119.2 | 119.2 | 119.2 | 118.9 | 118.9 | 118.9 | 118.9 | 115.9 | 115.8 | 115.9 | 115.9 |
| n_4 | 41.9 | 41.9 | 41.9 | 41.9 | 38.5 | 40.6 | 38.5 | 38.5 | 41.7 | 32.2 | 41.7 | 41.7 |
| n_5 | 2.4 | 2.4 | 2.4 | 2.4 | 0.1 | 0 | 0.1 | 0.1 | 0 | 0 | 0 | 0 |
| n_6 | 110.3 | 110.3 | 110.3 | 110.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n_7 | 16.3 | 16.3 | 16.3 | 16.3 | 10.5 | 10.03 | 10.5 | 10.5 | 9.2 | 7.7 | 9.2 | 9.2 |
| n_8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n_9 | 638.9 | 638.9 | 638.9 | 638.9 | 411.5 | 393.8 | 411.5 | 411.5 | 360.9 | 302.1 | 360.9 | 360.9 |
| n_{10} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 886.7 | 577 | 886.7 | 886.7 |
| n_{11} | 50.9 | 50.9 | 50.9 | 50.9 | 43.1 | 41.1 | 43.1 | 43.1 | 43.1 | 30.3 | 43.1 | 43.1 |
| n_{12} | 169.1 | 169.1 | 169.1 | 169.1 | 146.1 | 134.2 | 146.1 | 146.1 | 135.3 | 109.6 | 135.3 | 135.3 |
| p_1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p_2 | 0 | 0 | 0 | 0 | 0 | 162.7 | 0 | 0 | 0 | 339.7 | 0 | 0 |
| p_3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p_4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p_5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3.6 | 0 | 0 |
| p_6 | 0 | 0 | 0 | 0 | 0 | 7.4 | 0 | 0 | 10.5 | 93.8 | 10.5 | 10.5 |
| p_7 | 26.5 | 26.5 | 26.5 | 26.5 | 37.4 | 38.2 | 37.4 | 37.4 | 36.8 | 39.9 | 36.8 | 36.8 |
| p_8 | 268.3 | 268.3 | 268.3 | 268.3 | 2025.3 | 2081.7 | 2025.3 | 2025.3 | 2202.5 | 2701.9 | 2202.5 | 2202.5 |
| p_9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p_{10} | 101.7 | 101.7 | 101.7 | 101.7 | 92.7 | 140.1 | 92.7 | 92.7 | 0 | 0 | 0 | 0 |
| p_{11} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p_{12} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Goals met | 9 | 9 | 9 | 9 | 10 | 9 | 10 | 10 | 9 | 8 | 9 | 9 |

In columns (a) of table 4, we had $\alpha = \beta = \gamma = 0.333$. This indicates that all meta-objectives are given equal importance. In columns (b) α was given more importance than β and γ whereas in columns (c) β was given more importance than α and γ . In columns (d) though γ is given more importance than α and β . The value of λ for the

different calorie levels and across the varying meta-weights in the solution ranges from 0.2284 to 0.4789. This implies that the maximum deviation from targets of the goals ranges between 22.8% and 47.9%. From the results the deviation from the set goals for the 1600 calorie level was not met for magnesium, Fiber and calcium goals as their tolerable

nutrient target was underachieved when the goal was to prevent their under achievements. Their deviations from the

target were 20.4%, 47.9% and 47.9 respectively across the varying levels of meta-weights space.

Table 4. Summary of solution of Extended GP DASH diet model (Contd.)

| | 1500mg daily sodium level | | | | | | | | | | | |
|----------|---------------------------|-------|-------|-------|---------------------------|-------|-------|-------|---------------------------|-------|-------|-------|
| | 1600 calories daily level | | | | 2000 calories daily level | | | | 3000 calories daily level | | | |
| α | 0.333 | 0.900 | 0.050 | 0.050 | 0.333 | 0.900 | 0.050 | 0.050 | 0.333 | 0.900 | 0.050 | 0.050 |
| β | 0.333 | 0.050 | 0.900 | 0.050 | 0.333 | 0.050 | 0.900 | 0.050 | 0.333 | 0.050 | 0.900 | 0.050 |
| γ | 0.333 | 0.050 | 0.050 | 0.900 | 0.333 | 0.050 | 0.050 | 0.900 | 0.333 | 0.050 | 0.050 | 0.900 |
| x_1 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 6 | 6 | 6 | 6 |
| x_2 | 0.5 | 0.5 | 0.5 | 0.5 | 0.4 | 0.7 | 0.4 | 0.4 | 0 | 0.5 | 0 | 0 |
| x_3 | 6 | 6 | 6 | 6 | 6.7 | 8 | 6.7 | 6.7 | 7.1 | 9.4 | 7.1 | 7.1 |
| x_4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 |
| x_5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| x_6 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 |
| x_7 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 |
| x_8 | 2.8 | 2.8 | 2.8 | 2.8 | 3.6 | 3.7 | 3.6 | 3.6 | 3.6 | 4.3 | 3.6 | 3.6 |

The recommended servings were not also met for fish. For the 2000 calorie level, the target tolerable nutrient levels were not met for fiber and calcium and their deviations were an average of 30% from the target across the varying levels of meta-weights space. For the 3000 calorie level the target tolerable nutrient levels were not met also for fiber and calcium and the deviations were an average of 27% from the target across the varying level of meta-weights space. The recommended servings were not also met for bread and fish for both 2000 and 3000 calorie levels.

IV. CONCLUSION

In this paper we were concerned primarily with achieving efficiency-balance trade-offs in minimizing deviations from target goals of diet cost, nutrient’s tolerable intake limits and recommended daily food servings in the DASH diet plan for hypertension patients. Hence an EGP model for the DASH diet problem was formulated. The model was illustrated using some selected food items from the DASH sample food charts with their nutrient content and tolerable intake level for each of the ‘DASH’ nutrients as well as the cost per serving of food items. From the solution of the analysis done using LINGO optimization software an average of 9 goals were met for the 1600, 2000 and 3000 daily calorie levels considered and across the varying meta-weights. The target goals were not met for fiber and calcium for all the calorie levels across the varying meta-weights. Also the recommended servings of two foods were not bread and fish were not met for the 2000 and 3000 calorie levels. But in all the Extended GP DASH diet model achieve the efficiency-equity balance in minimizing deviations from target goals of

diet cost and nutrient’s tolerable intake limits together with the daily food servings in the DASH diet plan for hypertension patients.

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