Logarithmic-Type Ratio Estimator for Estimation of Finite Population Mean

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***Abstract* — Logarithm is the inverse function to exponentiation. In this paper, a logarithmic-type ratio estimator is suggested for estimating the finite population mean of characteristics under study. Taylor series approximation was used up to first-order approximation in the derivation of bias and mean square error. Theoretically, the bias and mean square error equations of the suggested estimator are obtained and then compared with the finding which are supported by numerical illustration using a real dataset. The results revealed that the suggested estimator is more efficient than other existing estimators considered in the study.**

Keywords - *Logarithmic ratio estimator, Auxiliary variable, Efficiency, Mean square error.*

I. Introduction

Logarithms were introduced by John Napier in 1614 as a means of simplifying calculations. They were rapidly adopted by navigators, scientists, engineers, surveyors and others to perform high-accuracy computations more easily. The concept of the logarithm as the inverse of exponentiation extends to other statistical structures as well. A deeper study of logarithms requires the concept of a function. A function is a rule that, given one number, produces another number. An example is the function is an estimator. The search for efficient estimators leads us to consider logarithmic-type estimators, knowing that logarithm is the inverse operation to exponentiation. Hence, the relevance of the present study. A sampling survey is a field of statistics that deals with methods of sampling and estimation from a sample, it is essential to use sampling to reduce cost and time. At the design stage or the estimation stage or both stages, the auxiliary information is used to achieve enhanced precision and efficiency. Researchers have constructed estimators for the estimation of parameters by modifying existing estimators using a known function of an auxiliary variable, authors like Cochran (1940), Srivastava (1967), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Abu-Dayyeh (2003), Singh and Tailor (2003), Singh *et al.* (2004), Singh and Tailor (2005), Kadilar and Cingi (2006), Singh *et al.* (2008), Yan and Tian (2010), Tailor *et al.* (2011), Singh and Solanki (2012), Kazeem and Olanrewaju (2013), Yadav *et al.* (2016), Gupta and Yadav (2017), Muili and Audu (2019), Muili *et al.* (2019), Muili *et al.* (2020), etc.

Consider a finite population $U=\left(U_1,U_2,...,U_N\right)$. A sample size n is drawn from the population using a simple random sampling without replacement (SRSWOR) scheme. Let Y and X be the study and the auxiliary variables respectively. y_i and x_i be the observations on the ith unit. Let $\bar{y}$ and $\bar{x}$ be the sample means of the study and auxiliary variables. $\bar{Y}$ and $\bar{X}$ are the population means of the study and auxiliary variables respectively. Let s_y^2 and s_x^2 be the sample mean squares and S_y^2 and S_x^2, be the corresponding population mean squares. ρ is the correlation coefficient between Y and X. C_y and C_x respectively be the coefficients of variation for Y and X. N: Population size, n: Sample size, $\bar{Y},\bar{X}$: Population means of study and auxiliary variables. C_y, C_x: Coefficient of variations of study and auxiliary variables, $\beta_{2(x)}$: Coefficient of Kurtosis of auxiliary variable, M_d: Median of the auxiliary variable, TM: Tri-Mean $\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i$, $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$, $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$, $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$,

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2, \quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2, \quad \gamma = \frac{1-f}{n}, \ f = n/N,$$

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2}, TM = \frac{\left(Q_1 + 2Q_2 + Q_3\right)}{4},$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2, and \ C_x^2 = \frac{S_x^2}{\bar{X}^2}$$

A logarithmic ratio estimator is developed to improve the precision of the estimation of population mean using auxiliary information.

II. Some Existing Estimators of Population Mean

The sample mean $\left(\bar{y}\right)$ in simple random sampling without replacement is given as:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1.0}$$

$$V\left(\bar{y}\right) = \gamma \bar{Y}^2 C_y^2 \tag{1.1}$$

Cochran (1940) proposed a ratio estimator for the estimation of the population mean $\left(\bar{Y}\right)$ of the study variable $\left(Y\right)$ which can only be used when the coefficient of correlation between the study variable and the auxiliary variable is positive. The ratio estimator, bias and mean square error are given respectively as:

$$t_R = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1.2}$$

$$Bias\left(t_R\right) = \gamma \bar{Y}\left(C_x^2 - \rho C_y C_x\right) \tag{1.3}$$

$$MSE\left(t_R\right) = \gamma \bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_y C_x\right) \tag{1.4}$$

Yan and Tian (2010) developed two ratio-type estimators for estimating the population mean $\left(\bar{Y}\right)$ of the study variable $\left(Y\right)$ which can be applied when the correlation between the study variable and the auxiliary variable is positive. The ratio-type estimators, bias and mean square error are given respectively as:

$$\hat{\bar{Y}}_1 = \bar{y}\left(\frac{\bar{X}\beta_{2(x)} + \beta_{1(x)}}{\bar{x}\beta_{2(x)} + \beta_{1(x)}}\right) \tag{1.5}$$

$$Bias\left(\hat{\bar{Y}}_1\right) = \gamma \bar{Y}\left(\theta_1^2 C_x^2 - \theta_1 \rho C_y C_x\right) \tag{1.6}$$

$$MSE\left(\hat{\bar{Y}}_1\right) = \gamma \bar{Y}^2\left(C_y^2 + \theta_1^2 C_x^2 + 2\theta_1 \rho C_y C_x\right) \tag{1.7}$$

$$\hat{\bar{Y}}_2 = \bar{y}\left(\frac{\bar{X}\beta_{1(x)} + \beta_{2(x)}}{\bar{x}\beta_{1(x)} + \beta_{2(x)}}\right) \tag{1.8}$$

$$Bias\left(\hat{\bar{Y}}_2\right) = \gamma \bar{Y}\left(\theta_2^2 C_x^2 - \theta_2 \rho C_y C_x\right) \tag{1.9}$$

$$MSE\left(\hat{\bar{Y}}_2\right) = \gamma \bar{Y}^2\left(C_y^2 + \theta_2^2 C_x^2 + 2\theta_2 \rho C_y C_x\right) \tag{2.0}$$

where $\theta_1 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + \beta_{1(x)}}$ and $\theta_2 = \frac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{1(x)} + \beta_{2(x)}}$

Subramani and Kumarapandiyan (2012) proposed a ratio estimator of the population mean using a linear combination of the coefficient of kurtosis and median of auxiliary variables as:

$$\bar{Y}_3 = \bar{y}\left(\frac{\beta_{2(x)}\bar{X} + M_d}{\beta_{2(x)}\bar{x} + M_d}\right) \tag{2.1}$$

$$Bias\left(\hat{\bar{Y}}_3\right) = \gamma \bar{Y}\left(\theta_3^2 C_x^2 - \theta_3 \rho C_y C_x\right) \tag{2.2}$$

$$MSE\left(\hat{\bar{Y}}_3\right) = \gamma \bar{Y}^2\left(C_y^2 + \theta_3^2 C_x^2 + 2\theta_3 \rho C_y C_x\right) \tag{2.3}$$

where $\theta_3 = \frac{\beta_{2(x)}\bar{X}}{\beta_{2(x)}\bar{X} + M_d}$

Subramani and Kumarapandiyan (2013) proposed a ratio estimator of the population mean using the information of coefficient of variation of auxiliary variable as:

$$\bar{Y}_4 = \bar{y}\left(\frac{\bar{X} + \beta_{2(x)}}{\bar{x} + \beta_{2(x)}}\right) \tag{2.4}$$

$$Bias\left(\hat{\bar{Y}}_4\right) = \gamma \bar{Y}\left(\theta_4^2 C_x^2 - \theta_4 \rho C_y C_x\right) \tag{2.5}$$

$$MSE\left(\hat{\bar{Y}}_4\right) = \gamma \bar{Y}^2\left(C_y^2 + \theta_4^2 C_x^2 + 2\theta_4 \rho C_y C_x\right) \tag{2.6}$$

where $\theta_4 = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}}$

Jerajuddin and Kishun (2016) modified the ratio estimator of the population mean using the information of sample size (n) as:

$$\hat{\bar{Y}}_{JK} = \bar{y}\left(\frac{\bar{X} + n}{\bar{x} + n}\right) \tag{2.7}$$

$$Bias\left(\hat{\bar{Y}}_{JK}\right)=\gamma\bar{Y}\left(\theta_5^2C_x^2-\theta_5\rho C_yC_x\right) \qquad (2.8)$$

$$MSE\left(\hat{\bar{Y}}_{JK}\right)=\gamma\bar{Y}^2\left(C_y^2+\theta_5^2C_x^2+2\theta_5\rho C_yC_x\right) \qquad (2.9)$$

where $\theta_5=\dfrac{\bar{X}}{\bar{X}+n}$

Raja and Maqbool (2021) modified the ratio estimator using the linear combination of the Coefficient of Kurtosis and Tri-Mean (TM) of the auxiliary variable as:

$$\hat{\bar{Y}}_{RM}=\bar{y}\left(\frac{\bar{X}\beta_{2(x)}+TM}{\bar{x}\beta_{2(x)}+TM}\right) \qquad (3.0)$$

$$Bias\left(\hat{\bar{Y}}_{RM}\right)=\gamma\bar{Y}\left(\theta_6^2C_x^2-\theta_6\rho C_yC_x\right) \qquad (3.1)$$

$$MSE\left(\hat{\bar{Y}}_{RM}\right)=\gamma\bar{Y}^2\left(C_y^2+\theta_6^2C_x^2-2\theta_6\rho C_yC_x\right) \qquad (3.2)$$

$$\theta_6=\frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)}+TM}$$

III. The Proposed Estimator

$$\hat{\bar{Y}}_p=\bar{y}\left(1+\log\left(\frac{\bar{X}}{\bar{x}}\right)\right)^{\alpha} \qquad (3.3)$$

where $e_0=\dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1=\dfrac{\bar{x}-\bar{X}}{\bar{X}}$ such that $\bar{y}=\bar{Y}\left(1+e_0\right)$ *and* $\bar{x}=\bar{X}\left(1+e_1\right)$, from the definition of e_0 and e_1, we obtained

$$\left.\begin{aligned}&E\left(e_0\right)=E\left(e_1\right)=0,\ E\left(e_0^2\right)=\gamma C_y^2\\&E\left(e_1^2\right)=\gamma C_x^2,\ E\left(e_0e_1\right)=\gamma C_{yx}=\gamma\rho C_yC_x\end{aligned}\right\} \qquad (3.4)$$

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1+\log\left(1-e_1+e_1^2\right)\right)^{\alpha} \qquad (3.5)$$

Simplifying the log

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1+\left(-e_1+e_1^2\right)-\frac{\left(-e_1+e_1^2\right)^2}{2}\right)^{\alpha} \qquad (3.6)$$

Expanding the inner brackets

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1-e_1+e_1^2-\frac{\left(e_1^2+e_1^4-2e_1^3\right)}{2}\right)^{\alpha} \qquad (3.7)$$

Simplifying and reducing (3.7) to first-order approximation by neglecting power three and above, gives (3.8)

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1-e_1+e_1^2-\frac{e_1^2}{2}\right)^{\alpha} \qquad (3.8)$$

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1-e_1+\frac{e_1^2}{2}\right)^{\alpha} \qquad (3.9)$$

Simplifying (3.9) gives (4.0)

$$\hat{\bar{Y}}_p=\bar{Y}\left(1+e_0\right)\left(1-\alpha e_1+\alpha\frac{e_1^2}{2}+\frac{\left(\alpha^2-\alpha\right)}{2}e_1^2\right) \qquad (4.0)$$

Multiplying the brackets and Subtracting $\bar{Y}$ from both sides of (4.1), gives,

$$E\left(\hat{\bar{Y}}_p-\bar{Y}\right)=\bar{Y}E\left(\begin{aligned}&e_0-\alpha e_1-\alpha e_0e_1+\alpha\frac{e_1^2}{2}\\&+\frac{\left(\alpha^2-\alpha\right)}{2}e_1^2\end{aligned}\right) \qquad (4.1)$$

Applying the results of (3.4) to (4.1) gives (4.2)

$$Bias\left(\hat{\bar{Y}}_p\right)=\gamma\bar{Y}\left(\begin{aligned}&\frac{\left(\alpha^2-\alpha\right)}{2}C_x^2+\frac{\alpha}{2}C_x^2\\&-\alpha\rho C_yC_x\end{aligned}\right) \qquad (4.3)$$

Squaring and taking the expectation of (4.1), gives

$$MSE\left(\hat{\bar{Y}}_p\right)=\bar{Y}^2E\left(e_0-\alpha e_1\right)^2 \qquad (4.4)$$

Expanding (4.4) gives

$$MSE\left(\hat{\bar{Y}}_p\right)=\bar{Y}^2E\left(e_0^2+\alpha^2e_1^2-2\alpha e_0e_1\right) \qquad (4.5)$$

Applying the results of (3.4) to (4.5) gives

$$MSE\left(\hat{\bar{Y}}_p\right)=\gamma\bar{Y}^2\left(C_y^2+\alpha^2C_x^2-2\alpha\rho C_yC_x\right) \qquad (4.6)$$

Obtaining the expression for the value of α, differentiate $MSE\left(\hat{\bar{Y}}_p\right)$ partially with respect to α and equate to zero then simplifying for α, obtaining an optimum value of α and Substitute in (4.1) gives:

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min}=\gamma\bar{Y}^2C_y^2\left(1-\rho^2\right) \qquad (4.7)$$

where $\alpha=\dfrac{\rho C_y}{C_x}$

3.1 Efficiency Comparison

The proposed estimator $\left(\hat{\bar{Y}}_p\right)$ of the population mean is more efficient than the sample mean $\left(\bar{y}\right)$ if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < V\left(\bar{y}\right)$$

$$\gamma\bar{Y}^2C_y^2\left[1-\rho^2\right] < \gamma\bar{Y}^2C_y^2 \quad (4.8)$$

$$\rho^2 > 0 \quad (4.9)$$

The proposed estimator $\left(\hat{\bar{Y}}_p\right)$ of the population mean is more efficient than $\left(\hat{\bar{Y}}_R\right)$ if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < MSE\left(\hat{\bar{Y}}_R\right)$$

$$\gamma\bar{Y}^2C_y^2\left[1-\rho^2\right] < \gamma\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_y C_x\right) \quad (5.0)$$

$$\rho^2 > \frac{C_x\left(C_x - 2\rho C_y\right)}{C_y^2} \quad (5.1)$$

The proposed estimator $\left(\hat{\bar{Y}}_p\right)$ of the population mean is more efficient than $\hat{\bar{Y}}_i$ if,

$$MSE\left(\hat{\bar{Y}}_p\right)_{\min} < MSE\left(\hat{\bar{Y}}_i\right) \qquad i = 1, 2, \ldots, 6$$

$$\gamma\bar{Y}^2C_y^2\left[1-\rho^2\right] < \gamma\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i\rho C_y C_x\right) \quad (5.2)$$

$$\rho^2 > \frac{C_x\left(2\theta_i\rho C_y - \theta_i^2 C_x\right)}{C_y^2} \quad (5.3)$$

When conditions (4.9), (5.4) and (5.3) are satisfied, we conclude that the proposed estimator is more efficient than the sample mean, the ratio estimator, and other existing estimators considered in the study.

3.2 Empirical Study

To assess the performance of the proposed estimator, we considered the two populations as: Source: [Population I: Raja and Maqbool (2021). Population II: Subzar *et al*. (2018)] Auxiliary variable (X) = Fixed Capital Study variable (Y) = Output of 80 factories

Table 1: Parameters of the Populations

Parameter	Population I	Population II
N	80	80
n	20	20
$\bar{Y}$	51.8264	51.8264
$\bar{X}$	11.2646	11.2646
ρ	0.9413	0.9413
S_y	18.3569	733.1407
C_y	0.3542	0.8561
S_x	8.4542	150.2150
C_x	0.75	0.7531
$\beta_{2(x)}$	2.866	1.0445
$\beta_{1(x)}$	1.05	1.1823
TM	9.318	165.562
M_d	7.575	142.5

Table 1 shows the descriptive statistics of the two populations

Table 2: The Bias and Mean Square Error (MSE) of the Suggested and other Estimators

Estimator	Population I		Population II	
	Bias	MSE	Bias	MSE
Sample Mean $\left(\bar{y}\right)$	0	332.764	0	1943.964
Ratio Estimator $\left(\hat{\bar{Y}}_R\right)$	15.9904	498.240	-2.0333	228.9027
Yan and Tian (2010) $\left(\hat{\bar{Y}}_1\right)$	6.4950	174.130	-4.2558	260.6866
Yan and Tian (2010) $\left(\hat{\bar{Y}}_2\right)$	14.9886	461.379	-3.8425	252.1834
Subramani and Kumarapandiyan (2012) $\left(\hat{\bar{Y}}_3\right)$	1.2554	48.1266	-2.2001	1707.171
Subramani and Kumarapandiyan (2013) $\left(\hat{\bar{Y}}_4\right)$	13.3073	400.589	-4.1145	257.6161
Jerajuddin and Kishun (2016) $\left(\hat{\bar{Y}}_{JK}\right)$	-0.5679	270.1689	-7.4226	979.3048
Raja and Maqbool (2021) $\left(\hat{\bar{Y}}_{RM}\right)$	-6.0136	39.9199	1.7073	1736.977
Proposed Estimator $\left(\hat{\bar{Y}}_p\right)$	-2.8445	37.9199	-16.6174	221.5231

The Values of Bias and MSE of the Existing and Suggested Estimators

IV. Result and Discussion

A logarithmic-type ratio estimator for the estimation of the population mean of the study variable is suggested. The bias and mean square error (MSE) of the suggested estimator are derived up to the first order of appreciation. A theoretical comparison of the suggested logarithmic-type ratio estimator of the population mean with sample mean, ratio estimator and other existing estimators considered in the study was established. The values of mean square errors (MSE) of the suggested estimator are smaller than the sample mean, ratio estimator, and other estimators considered in the study. The performance of the suggested estimator over the sample mean, ratio estimator, and other selected existing estimators using two real populations was obtained. The results show that the suggested estimator is more efficient than the sample mean, ratio estimator, Yan and Tian (2010), Subramani and Kuranpadiyan (2012, 2013), Jerajuddin, and Kishun (2016) and Raja and Maqbool (2021) estimators.

V. Conclusion

The results in Table 1 clearly showed that the suggested logarithmic-type ratio estimator performed better than the sample mean, ratio estimator, Yan and Tian (2010), Subramani and Kuranpadiyan (2012, 2013), Jerajuddin, and Kishun (2016) and Raja and Maqbool (2020) estimators considered in the study having the Least Mean Square Error (MSE). We hereby recommend the suggested estimator for use in estimating the finite population mean.

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Appendix

Population I: R-software code for Population I

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N=80;n=20;ybar=51.8264;xbar=11.2646;rho=0.9413;cy=0.3542;cx=0.75;b2=0.75;b1=2.866;tm=9.318;md=7.575;
f=n/N;g=1-f/n;
teta1=xbar*b2/(xbar*b2+b1);
teta2=xbar*b1/(xbar*b1+b2);
teta3=xbar*b2/(xbar*b2+md);
teta4=xbar/(xbar+b2);
teta5=xbar/(xbar*n);
teta6=xbar*b2/(xbar*b2+tm);
t=rho*cy/cx;
v=g*ybar^2*cy^2;
bratio=g*ybar*(cx^2-rho*cy*cx);
mseratio=g*ybar^2*(cy^2+cx^2-2*rho*cy*cx);
bias1=g*ybar*(teta1^2*cx^2-teta1*rho*cy*cx);
bias2=g*ybar*(teta2^2*cx^2-teta2*rho*cy*cx);
bias3=g*ybar*(teta3^2*cx^2-teta3*rho*cy*cx);
bias4=g*ybar*(teta4^2*cx^2-teta4*rho*cy*cx);
biasjk=g*ybar*(teta5^2*cx^2-teta5*rho*cy*cx);
biasrm=g*ybar*(teta5^2*cx^2-teta6*rho*cy*cx);
biasp=g*ybar*((t^2-t)/2*cx^2+t/2*cx^2-t*rho*cy*cx);
mse1=g*ybar^2*(cy^2+teta1^2*cx^2-2*teta1*rho*cy*cx);
mse2=g*ybar^2*(cy^2+teta2^2*cx^2-2*teta2*rho*cy*cx);
mse3=g*ybar^2*(cy^2+teta3^2*cx^2-2*teta3*rho*cy*cx);
mse4=g*ybar^2*(cy^2+teta4^2*cx^2-2*teta4*rho*cy*cx);
msejk=g*ybar^2*(cy^2+teta5^2*cx^2-2*teta5*rho*cy*cx);
mserm=g*ybar^2*(cy^2+teta6^2*cx^2-2*teta6*rho*cy*cx);
msep=g*ybar^2*cy^2*(1-rho^2);
bratio;bias1;bias2;bias3;bias4;biasjk;biasrm;biasp;
v;mseratio;mse1;mse2;mse3;mse4;msejk;mserm;msep;
```

Population II: R-software code for Population II

```
N=80;n=20;ybar=51.8264;xbar=11.2646;rho=0.9413;cy=0.8561;cx=0.7531;b2=1.0445;b1=1.1823;tm=165.562;md=142.5;
f=n/N;g=1-f/n;
teta1=xbar*b2/(xbar*b2+b1);
teta2=xbar*b1/(xbar*b1+b2);
teta3=xbar*b2/(xbar*b2+md);
teta4=xbar/(xbar+b2);
teta5=xbar/(xbar+n);
teta6=xbar*b2/(xbar*b2+tm);
t=rho*cy/cx;
v=g*ybar^2*cy^2;
bratio=g*ybar*(cx^2-rho*cy*cx);
mseratio=g*ybar^2*(cy^2+cx^2-2*rho*cy*cx);
bias1=g*ybar*(teta1^2*cx^2-teta1*rho*cy*cx);
bias2=g*ybar*(teta2^2*cx^2-teta2*rho*cy*cx);
bias3=g*ybar*(teta3^2*cx^2-teta3*rho*cy*cx);
bias4=g*ybar*(teta4^2*cx^2-teta4*rho*cy*cx);
biasjk=g*ybar*(teta5^2*cx^2-teta5*rho*cy*cx);
biasrm=g*ybar*(teta5^2*cx^2-teta6*rho*cy*cx);
biasp=g*ybar*((t^2-t)/2*cx^2+t/2*cx^2-t*rho*cy*cx);
mse1=g*ybar^2*(cy^2+teta1^2*cx^2-2*teta1*rho*cy*cx);
mse2=g*ybar^2*(cy^2+teta2^2*cx^2-2*teta2*rho*cy*cx);
mse3=g*ybar^2*(cy^2+teta3^2*cx^2-2*teta3*rho*cy*cx);
mse4=g*ybar^2*(cy^2+teta4^2*cx^2-2*teta4*rho*cy*cx);
msejk=g*ybar^2*(cy^2+teta5^2*cx^2-2*teta5*rho*cy*cx);
mserm=g*ybar^2*(cy^2+teta6^2*cx^2-2*teta6*rho*cy*cx);
msep=g*ybar^2*cy^2*(1-rho^2);
bratio;bias1;bias2;bias3;bias4;biasjk;biasrm;biasp;
v;mseratio;mse1;mse2;mse3;mse4;msejk;mserm;msep;
```