A Study of Some Rectifying Sampling Plans With Inspection Errors Using Cost Minimization Model

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Abstract **—** In the manufacturing industries, the use of acceptance sampling plan plays an important role in the inspection of raw materials; semi-finished products and finished products to either accept or reject the Lot based on the sample results. However, error-free inspection is often assumed leading to unrealistic sampling parameters with uneconomic cost. This study addresses this gap by developing an economic model for Rectifying Single Sampling (RSS) and Rectifying Double Sampling (RDS) plans, incorporating inspection errors into Kumar's cost model. The model optimizes by maximizing acceptance probability at the Acceptable Quality Level (AQL) and minimizing it at the Lot Tolerant Percent Defective (LTPD). Comparative analysis of the adjusted model with inspection errors against Kumar's error-free model reveals significant advantages. The adjusted model minimizes total costs while significantly bolstering protection for both producers and consumers against losses in RSS and RDS Plans. These findings underscore the practical importance of considering inspection errors in acceptance sampling plan, providing cost effective solutions for industry stakeholders.

Keywords – *Average Total Inspection (ATI), Average Outgoing Quality (AOQ), Average Outgoing Quality Limit (AOQL), Acceptable Quality Level (AQL), Lot Tolerant Percent Defective (LTPD).*

I. Introduction

Acceptance Sampling is a method of statistical quality control used by inspectors for lots sentencing. A random sample is taken from the submitted lots and the results of the sample is used to either accepted or rejected the lots. Rectifying inspection is a form of acceptance sampling where the rejected lots are inspected 100% and all the defective units are replaced with non-defective units. In the manufacturing industries, this form of acceptance sampling plan plays an important role on the inspection of raw materials; semi-finished products and finished products. Amitava (2016) also state that this form of acceptance sampling can be used as form of product inspection between companies and their vendors, between manufacturers and their customers.

For the purpose of this work, Rectifying Single Sampling (RSS) and Rectifying Double Sampling (RDS) plans are considered because they are widely used for product inspection.

In Rectifying Single Sampling (RSS) Plan a decision to accept or reject a lot is based on the result of a single random sample taken from the lot. If the lot is rejected the entire lot is inspected 100% all the defective units are removed and replaced with non-defective units.

Rectifying double sampling (RDS) plan allows for second sample to be taken from a lot before the decision is taken to either accept or reject the lot. If the lot is rejected, the entire lot is inspected 100% and all the defective units are removed and replaced with non-defective units.

Inspection errors occurs in every form of acceptance sampling where an inspected item is misclassified as defective when it is actually non-defective (Type I inspection error) or misclassified as non-defective when it is actually defective (Type II inspection error). It is therefore important to consider inspection errors in the design of acceptance sampling plan in order to obtain optimal sampling plan that reduce both the producer's and the consumer's losses at a minimum cost.

Mohammed et al. (2015) developed a mathematical model to design a single stage and double stage sampling plans used to determine the optimal tolerance limits and sample size. Mohammad and Abolghasem (2015) developed cost analysis models for acceptance sampling using dynamic programming and Bayesian inference in the presence of inspection error. First, the concept of Bayesian modeling was first used to determine the probability distribution of defective proportion of the lot and the optimal decision was determined using dynamic programming. Mohammad and Ahmad, (2016) proposed a new optimization model for designing an acceptance sampling plan based on cumulative sum of run length of conforming units. The model uses the concept of minimum angle method to minimize both the producer and the consumer losses. Fallahnezhad et al., (2018) developed an `optimization model using Maxima

Nomination Sampling (MNS) method for acceptance single sampling plan with inspection error. Results of the Maxima Nomination Sampling (MNS) method were compared with the classical method.

Kumar (2018) developed an economic cost model for achieving optimal single sampling plan that minimizes total cost and satisfies the producer and consumer quality risk. Iorkegh and Osanaiye (2022) adjusted the model developed by Kumar to obtain optimal sampling for Rectifying Single Sampling (RSS) plan with the introduction of inspection error. The adjusted model was therefore compared with the existing model. It was found that the sample size and the total cost in the improved model with inspection errors were smaller than in the Kumar's model.

In this study the adjusted model is adopted to obtained optimal sampling plan for RSS and RDS plans and the performance compared with the Kumar's model.

II. METHODOLOGY

Rectifying Single Sampling (RSS) Plan

In a single sample inspection plan, a random sample size (n) is drawn from a lot. If the sample contains defective(x) units equal to or less than acceptance number (c), the lot is accepted, the lot is rejected if more than c defective are observed. We assumed rectifying inspection where 100% inspection is carried out on all the rejected lots and all the defective units are replaced with non-defective units before the lot is accepted.

Rectifying Double Sampling (RDS) Plan

In double sampling a random sample of size n_1 is first taken from the lot and inspected for defective units. If the number of defective units in the first sample is less or equal to the first acceptance number c_1 the lot is accepted or if it is greater than c_2 the lot is rejected. However, if the number of defective units is between c_1 and c_2, a second sample of size n_2 is taken and the lot is accepted if the number of defectives in the two samples combined is less or equal to the second acceptance number c_2 the lot is rejected if otherwise. 100% inspection is carried out on the rejected lot where all the defective units are removed or replaced with non –defective units.

The performance evaluation measures for rectifying sampling inspection is the average outgoing quality (AOQ) and Average Total Inspection (ATI).

The Average Outgoing Quality (AOQ) is the average quality of the lot after rectifying inspection is carried out.

Therefore, the Average Outgoing Quality Limit (AOQL) represents the worst average quality that would leave the inspection station, assuming rectification, regardless of the incoming lot quality (Amitava 2016).

The average total inspection (ATI) is the average number of units inspected in a sample and in the rejected portion of a lot.

Average Outgoing Quality (AOQ) for RSS plan

Thus, when error-free inspection is assumed; the Average Outgoing Quality (AOQ) for RSS is given as:

$$AOQ_1 = \frac{(N-n)pP_a}{N} \qquad (1)$$

The probability of acceptance(P_a) assuming error-free inspection is:

$$P_a = p(x \le c) = \sum_{x=0}^{C} \binom{n}{x} p^x (1-p)^{n-x} \qquad (2)$$

N is the lot size and p is fraction defective unit

When inspection error is considered, the average outgoing quality (AOQ) is:

$$AOQ_{1e} = \frac{npe_2 + p(N-n)P_{a_e} + p(N-n)e_2(1-P_{a_e})}{N} \qquad (3)$$

The probability of acceptance with inspection error is given as :

$$P_{a_e} = \sum_{x=0}^{C} \binom{n}{x} {p_e}^x (1-p_e)^{n-x} \qquad (4)$$

Where p_e apparent fraction is defective while, e_1 and e_2 are type I inspection error and type II inspection error respectively.

Average Total Inspection (ATI) for RSS

$$ATI = n + (1-P_a)(N-n) \qquad (5)$$

When inspector error is considered, the amount of inspection is calculated as:

$$ATI_e = n + (1-P_{ae})(N-n) \qquad (6)$$

Average Outgoing Quality (AOQ) for RDS plan

The Average Outgoing Quality (AOQ) for RDS when error-free inspection is assumed is:

$$\mathrm{AOQ}_1 = \frac{\mathrm{p}[\mathrm{P}_{\mathrm{a}_1}(\mathrm{N}-\mathrm{n}_1) + \mathrm{P}_{\mathrm{a}_2}(\mathrm{N}-\mathrm{n}_1-\mathrm{n}_2)]}{\mathrm{N}} \qquad (7)$$

Where the probability of acceptance on the first sample and second samples are given as:

$$P_{a_1} = Pr\{x_{1\le} c_1\} = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \qquad (8)$$

$$P_{a_2} = \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \right\}$$

(9)

Average Outgoing Quality (AOQ_{1e}) for RDS when inspection error is considered is given below:

$$\mathrm{AOQ}_{1e} = \frac{\mathrm{pn_1e_2 + p(N-n_1)P_{a1_e} + p(N-n_1)(1-P_{a1_e})e_2 + p(n_2e_2) + p(N-n_1-n_2)P_{a2_e} + p(N-n_1-n_2)e_2}}{\mathrm{N}}$$

(10)

Where the probability of acceptance on the first sample and second samples are given as:

$P_{ae_1} = Pr\{x_1 \leq c_1\} = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p_e^{x_1}(1-p_e)^{n_1-x_1}$ (11)

$= \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p_e^{x_1}(1-p_e)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p_e^{x_2}(1-p_e)^{n_2-x_2} \right] \right\}$ (12)

Average Total Inspection (ATI) for RDS plan

The average total inspection (ATI) for RDS plan under error-free inspection assumption is:

ATI=$n_1P_{a_1} + (n_1+n_2)P_{a2} + N(1-P_{a1}-P_{a2})$ (13)

The Average Total Inspection (ATI) for RDS plan when inspector error is considered is:

$ATI_e = n_1P_{a_{1e}} + (n_1+n_2)P_{a2e} + N(1-P_{a_{1e}}-P_{a_{2e}})$ (14)

Defective Units in Rectifying Single Sampling (RSS) plan.

In single sampling plan, if a lot is accepted based on the sample, all observed defective units in the sample will be replaced with non-defective units. However, if the lot is rejected, it is assumed that all defective units in the sample and in the rejected lot are replaced with non-defective units and the lot will contain no defective units. Therefore, the number of defective units not detected per lot for RSS plan is given as:

$D_n = (N-n)pP_a$ (15)

The detected defective units in the rejected lots for RSS plan is stated below as:

$Dd = np + p(1-P_a)(N-n)$ (16)

In situations where inspection errors are taken into consideration, defective units can be of two forms: (i) defective units classified as not being defective but are actually defective (ii) defective units actually defective and are classified as being defective.

Therefore, the number of defective units not detected in accepted lot for RSS plan when inspection error is considered is:

$Dn_e = npe_2 + p(N-n)P_{a_e} + p(N-n)\left(1-P_{a_e}\right)e_2$ (17)

It is also assumed that during 100% inspection of the rejected lots, some defective units in the sample and in the remaining portion of the screened rejected lot are correctly classified as defective. Detected defective unit is given as:

$Dd_e = np(1-e_2) + p(N-n)(1-e_2)(1-P_{a_e})$ (18)

Defective units in Rectifying Double Sampling (RDS) plan

.If the lot is accepted based on the first sample n_1, all the defective units in the sample are replaced with non-defective units and the remaining portion of the lots with fraction defective units p will be accepted with a probability P_{a1}. On the other hand, if the lot is accepted based on the second sample n_2, all defective units in the sample are replaced with non-defective units and the remaining portion of the un-inspected lots $(N-n_1-n_2)$ with fraction defective units p is accepted with probability P_{a2}.

The number of defective units not detected for RDS plan is stated below as:

$Dn = p\left[P_{a_1}(N-n_1) + P_{a_2}(N-n_1-n_2)\right]$ (19)

The number of detected defective units in the rejected lots for RDS plan is therefore stated as:

$Dd = pn_1 + p(N-n_1)(1-Pa_1) + pn_2 + p(N-n_1-n_2)(1-P_{a2})$ (20)

When inspection error is considered, a situation occurs where (i) defective units are classified as not being defective but are actually defective (ii) and defective units that are actually defective and are classified as being defective. If defective units in the first sample n_1 is misclassified as non-defective with probability e_2 ,

The number of defective units not detected in the accepted lots for RDS plan with inspection error is :

$Dn_e = pn_1e_2 + p(N-n_1)P_{a1_e} + p(N-n_1-n_2)P_{a2_e} + pn_2e_2 + p(N-`n_1)\left(1-P_{a_{1e}}\right)e_2 + p(N-n_1-n_2)e_2\left(1-P_{a_{2e}}\right)$ (21)

Therefore, the number of detected defective units in the rejected lots for RDS plan with inspection error is:

$Dd_e = pn_1(1-e_2) + p(N-n_1)(1-P_{a1e})(1-e_2) + pn_2(1-e_2) + p(N-n_1-n_2)(1-P_{a2e})(1-e_2)$ (22)

Design of Acceptance Sampling Plans

In the design of Single and Double sampling plans we considered two quality levels: Acceptable Quality Level (AQL) and Lot Tolerant Percent Defective (LTPD) with or with no inspection errors. Acceptable Quality Level (AQL) is the small percentage of defective units in a lot that is acceptable by the consumer. Rejecting a lot with this quality level results to producer risk (α) . We consider AQL=0.02 with $\alpha = 0.05$We want the probability of rejection of the lot given acceptable quality level(AQL) to be less than or equal to the producer's risk (α) .Also lot tolerant percent defective (LTPD) is the highest percentage of defective units a consumer can tolerate in a lot .Accepting lot with this defect level results to consumer's risk (β).We want LTPD=0.07 with consumer's risk(β)=0.1 and .the probability of acceptance of the lot given lot Tolerant Proportion Defective (LTPD) to be less than or equal to consumer's risk(β)

Probability of Acceptance in Single Sampling Plan with specified AQL and LTPD Quality Levels

The probability of rejection at $p_1 = AQL$ or producer's risk is:

$$1 - \sum_{x=0}^{C} \binom{n}{x} AQL^x (1 - AQL)^{n-x} \leq \alpha \qquad (23)$$

When inspection error is considered, The probability of lot rejection at $p_1 = AQLe$ in the adjusted model is thus

$$1 - \sum_{x=0}^{C} \binom{n}{x} \{1 - (1 - AQL)^n (1 - e_2) + e_1 (1 - AQL)^n\}^x \{(1 - AQL)^n (1 - e_2) + e_1 (1 - AQL)^n\}^{n-x} \leq \alpha$$
(24)

On the other hand, if the lot with quality level $p_2 = LTPD$ is accepted, then the probability of accepting lot with quality level $p_2 = LTPD$ under no inspection error assumed is stated below:

$$\sum_{x=0}^{c} \binom{n}{x} LTPD^X (1 - LTPD)^{n-x} \leq \beta \qquad (25)$$
(26)

When inspection error is considered, the probability of lot acceptance at $p_2 = LTPD_e$ or consumer's risk (β) is thus:

$$\sum_{x=0}^{c} \binom{n}{x} [\{1 - (1 - LTPD)^n\}(1 - e_2) + e_1 (1 - LTPD)^n]^x [\{1 - [(1 - LTPD)^n]\}(1 - e_2) + e_1 (1 - LTPD)^n]^{n-x} \leq \beta \qquad (26)$$

Probability of Acceptance in Double Sampling Plan with specified AQL and LTPD quality levels

Given the quality level $p_1 = AQL$ the probability of lot rejection is:

$$\sum_{x_1=0}^{c_1} \binom{n_1}{x_1} AQL^{x_1} (1 - AQL)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} AQL^{x_1} (1 - AQL)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} AQL^{x_2} (1 - AQL)^{n_2 - x_2} \right] \right\} \leq \alpha \qquad (27)$$

The Probability of lot acceptance $(1 - \alpha)$ in the presence of inspection error is

$$= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} AQL_e^{\ x_1} (1 - AQL_e)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} AQL_e^{\ x_1} (1 - AQL_e)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} AQL_e^{\ x_2} (1 - AQL_e)^{n_2 - x_2} \right] \right\} \leq \alpha \qquad (28)$$

On the other hand, the probability of accepting the lot with quality level $p_2 = LTPD$ under error-free as

$$\sum_{x_1=0}^{c_1} \binom{n_1}{x_1} LTPD^{x_1} (1 - LTPD)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} LTPD^{x_1} (1 - LTPD)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} LTPD^{x_2} (1 - LTPD)^{n_2 - x_2} \right] \right\} \leq \beta \qquad (29)$$

When inspection error is considered, the probability of acceptance of the lot is formulated as shown below with $p_2 = LTPD$ is replaced with $p_2 = LTPDe$:

Thus the probability of acceptance at $p_2 = LTPDe$ is:

$$\sum_{x_1=0}^{c_1} \binom{n_1}{x_1} LTPD_e^{\ x_1} (1 - LTPD_e)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} LTPD_e^{\ x_1} (1 - LTPD_e)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} LTPD_e^{\ x_2} (1 - LTPD_e)^{n_2 - x_2} \right] \right\} \leq \beta \qquad (30)$$

Applications

In this section, optimal values of RSS and RDS plans obtained using Kumar's model and the adjusted model with inspection error are compared and discussed as shown in tables and figures below.

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Table 1: Rectifying Single Sampling (RSS) Plans using Kumar's model with no inspection error assumed satisfying the parameters ($N = 1000, AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1, with$n \leq 250$)

n	c	AOQ	ATI	D_n	D_d	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$	TC
196	7	0.0184	386.83	18.40	11.60	0.0448	0.0322	0.7627	593.99
196	8	0.0208	306.47	20.81	9.19	0.0180	0.0642	0.8626	532.92
197	7	0.0183	390.88	18.27	11.73	0.0459	0.0309	0.7586	597.07
197	8	0.0207	309.74	20.71	9.29	0.0185	0.0620	0.8596	535.41
198	7	0.0182	394.95	18.15	11.85	0.0470	0.0297	0.7544	600.16
198	8	0.0206	313.04	20.61	9.39	0.0190	0.0598	0.8566	537.91
199	7	0.0180	399.03	18.03	11.97	0.0482	0.0285	0.7503	603.26
199	8	0.0205	316.35	20.51	9.49	0.0196	0.0577	0.8535	540.42
200	7	0.0179	403.12	17.91	12,09	0.0493	0.0274	0.7461	606.37
200	8	0.0204	319.68	20.41	9.59	0.0202	0.0556	0.8504	542.95
201	8	0.0203	323.03	20.31	9.69	0.0208	0.0537	0.8473	545.50
201	**9**	**0.0220**	**267.19**	**21.98**	**8.02**	**0.0077**	**0.0979**	**0.9172**	**503.07**
202	8	0.0202	326.40	20.21	9.79	0.0214	0.0518	0.8441	548.06
202	9	0.0219	269.78	21.91	8.09	0.0080	0.0947	0.9151	505.03
203	8	0.0201	329.79	20.11	9.89	0.0220	0.0499	0.8409	550.64
203	9	0.0218	272.39	21.83	8.17	0.0083	0.0917	0.9129	507.02
204	8	0.0200	333.19	20.00	10.00	0.0226	0.0481	0.8377	553.23
204	9	0.0217	275.03	21.75	8.25	0.0085	0.0888	0.9108	509.02
205	8	0.0199	336.62	19.90	10.10	0.0232	0.0464	0.8344	555.83
205	9	0.0217	277.68	21.67	8.33	0.0088	0.0859	0.9086	511.04

Table 2: Rectifying Single Sampling (RSS) Plans using the Adjusted model with inspection error satisfying the given parameters (N=1000,$AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1, $AOQL = 0.03$ with $n \leq 250$)

n	c	AOQ_e	ATI_e	D_{ne}	D_{de}	$1 - P_{ae}(AQL_e)$	$P_{ae}(LTPD_e)$	$P_{ae}(p)$	TC
13	5	0.0237	212.67	23.68	6.32	0.0471	0.0967	0.7977	462.14
19	11	0.0276	79.26	27.65	2.35	0.0037	0.0974	0.9386	360.43
20	10	0.0225	253.65	22.47	7.53	0.0295	0.0117	0.7616	493.38
21	13	0.0280	68.52	27.96	2.04	0.0020	0.0838	0.9515	352.24
22	11	0.0195	352.56	19.53	10.47	0.0462	0.0027	0.6620	568.79
23	12	0.0201	332.46	20.13	9.87	0.0371	0.0023	0.6833	553.47
23	13	0.0239	204.40	23.83	6.07	0.0138	0.0084	0.8143	455.84
23	14	0.0266	115.21	26.58	3.42	0.0044	0.0261	0.9056	387.83
23	**15**	**0.0279**	**71.90**	**27.86**	**2.14**	**0.0012**	**0.0687**	**0.9590**	**348.05**
24	13	0.0206	316.24	20.61	9.39	0.0302	0.0020	0.7006	541.10
25	14	0.0210	303.28	20.99	9.01	0.0249	0.0017	0.7146	531.22
25	15	0.0245	186.82	24.45	5.55	0.0091	0.0062	0.8341	442.43
26	15	0.0213	293.11	21.29	8.71	0.0209	0.0014	0.7258	523.46
27	15	0.0175	422.00	17.47	12.53	0.0431	0.0003	0.5940	621.73

In tables 1 and 2 above, it can be seen that the optimal values of RSS in the adjusted model with inspection errors have smaller sample size (n) of 23, smaller value of the producer's risk $1 - P_{ae}(AQL_e) = 0.0012$, consumer's risk (β) $P_{ae}(LTPD_e) = 0.0687$ and smaller total cost $(TC) = 348.05$ than in the existing model with optimal sample size(n) of 201 , producer risk(α) of $1 - P_a(AQL) = 0.0077$, consumer's risk (β) $P_a(LTPD) = 0.0979$ with total cost (TC) of 503.07

Table 3: Rectifying Double Sampling (RDS) Plans using Kumar's model with no Inspection error assumed satisfying the parameters ($N = 1000, AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1 with$n_1$ and $n_2 \leq 250$)

n_1	n_2	c_1	n_2	AOQ	ATI	D_n	D_d	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$	TC
66	132	1	8	0.0226	247.16	22.58	35.43	0.0168	0.0941	0.8717	543.88
67	134	1	8	0.0223	256.78	22.30	35.69	0.0183	0.0867	0.8632	551.13
68	136	1	8	0.022	266.57	22.00	35.96	0.0199	0.0799	0.8546	558.51
69	138	1	8	0.0217	276.53	21.70	36.23	0.0216	0.0736	0.8456	566.03
70	140	1	8	0.0214	286.66	21.40	36.50	0.0235	0.0678	0.8365	573.66
71	142	1	8	0.0211	296.94	21.09	37.78	0.0254	0.0625	0.8270	581.41
72	144	1	8	0.0208	307.35	20.78	37.06	0.0274	0.0576	0.8173	589.27
73	146	1	8	0.0205	317.90	20.46	37.35	0.0295	0.0530	0.8075	597.23
74	148	1	8	0.0201	328.57	20.14	37.64	0.0318	0.0489	0.7973	605.27
75	150	1	8	0.0198	339.35	19.82	37.93	0.0342	0.0450	0.7870	613.41
76	152	1	8	0.0195	350.23	19.49	38.23	0.0367	0.0415	0.7465	621.61
77	154	1	8	0.0192	361.19	19.16	38.53	0.0393	0.0383	0.7658	629.88
77	154	2	8	0.0215	282.40	21.53	36.16	0.0300	0.0953	0.8146	570.02
78	156	1	8	0.0188	372.22	18.83	38.83	0.0420	0.0353	0.7550	638.21
78	156	2	8	0.0212	291.74	21.25	36.41	0.0321	0.0901	0.8057	577.06
79	158	1	8	0.0185	383.32	18.50	39.13	0.0448	0.0326	0.7439	646.59
79	158	2	8	0.0210	301.17	20.96	36.67	0.0343	0.0852	0.7966	584.11
80	160	1	8	0.0182	394.48	18.17	39.43	0.0478	0.0301	0.7328	655.00
80	160	2	8	0.0207	310.68	20.68	36.92	0.0366	0.0805	0.7874	591.32
81	162	2	8	0.0204	320.27	20.39	37.18	0.0390	0.0761	0.7780	598.54
82	164	2	8	0.0201	329.92	20.10	37.44	0.0415	0.0719	0.7686	605.82
83	166	2	8	0.0198	339.63	19.81	37.70	0.0441	0.0680	0.759	613.14
84	168	2	8	0.0195	349.39	19.52	37.96	0.0469	0.0643	0.7493	620.49
85	170	2	8	0.0192	359.18	19.22	38.23	0.0497	0.0608	0.7395	627.88
95	190	3	10	0.0221	263.69	22.09	35.06	0.0175	0.0972	0.8488	554.70
96	192	3	10	0.0219	271.07	21.87	35.25	0.0187	0.0926	0.8419	560.26
96	**192**	**3**	**11**	**0.0229**	**237.32**	**22.88**	**34.24**	**0.0093**	**0.0971**	**0.8891**	**534.60**
97	194	3	10	0.0216	278.10	21.64	35.45	0.0200	0.0882	0.8349	565.88
97	194	3	11	0.0227	244.10	22.68	34.41	0.0100	0.0923	0.8835	539.69
98	196	3	10	0.0214	286.10	21.42	35.64	0.0213	0.0841	0.8277	571.56
98	196	3	11	0.0225	250.98	22.47	34.59	0.0108	0.0878	0.8775	544.87

Table 4: Rectifying Double Sampling (RDS) Plans using the Adjusted model with inspection error satisfying the parameters $N = 1000, AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1, $AOQL = 0.03$, withn_1 and $n_2 \leq 250$

n_1	n_2	c_1	c_2	AOQ_e	ATI_e	D_{ne}	D_{de}	$1 - P_{ae}(AQL_e)$	$P_{ae}(LTPD_e)$	$P_{ae}(p)$	TC
9	18	1	8	0.0257	153.68	25.73	34.00	0.0164	0.0714	0.8633	479.01
10	20	2	8	0.0224	266.31	22.39	37.31	0.0437	0.0539	0.7457	564.81
10	**20**	**2**	**10**	**0.0269**	**113.69**	**26.92**	**32.78**	**0.0076**	**0.0800**	**0.9030**	**448.45**
11	22	2	10	0.0225	262,44	22.50	17.17	0.0302	0.0211	0.7541	561.79
11	22	3	10	0.0245	196.46	24.46	35.21	0.0214	0.0717	0.8163	511.49
11	22	3	11	0.0262	137.86	26.20	33.47	0.0098	0.0763	0.8769	466.82
12	24	3	11	0.0216	291.88	21.63	38.01	0.0353	0.0258	0.7220	584.18
13	26	4	13	0.0231	241.35	23.13	36.48	0.0210	0.0320	0.7731	545.59
14	28	4	14	0.0198	353.55	19.80	39.78	0.0365	0.0099	0.6613	631.07

In tables 3 and 4 above, it can be seen that the optimal values of RDS in the adjusted model with inspection errors have smaller sample size (n) of 10 and 20 smaller value of the producer's risk $1 - P_{ae}(AQL_e) = 0.0076$, consumer's risk (β) $P_{ae}(LTPD_e) = 0.0800$ and smaller total cost $(TC) = 448.45$ than in the existing model with optimal sample size(n) of 96 and 192, producer risk(α) of $1 - P_a(AQL) = 0.0093$, consumer's risk (β) $P_a(LTPD) = 0.0971$ with total cost (TC) of 534.60.

Comparison of the Total Cost (TC) in Optimal RSS Plans under the existing model and the adjusted model with inspection error

The effect of fraction defective unit (p) on the optimal Total Cost (TC) for optimal RSS and RDS plans using Kumar's model and the adjusted model with inspection errors are compared as shown below:

Table 5: Total Cost (TC) for Optimal RSS plans in Kumar's model and the adjusted model

	Optimal RSS Plan using Kumar's model $(n = 201, c = 9)$	Optimal RSS plan using the Adjusted Model $n = 23, c = 15)$	
p	TC	TC	% Difference in TC
0.01	284.95	121.18	57.47%
0.02	374.03	220.31	41.09%
0.03	**503.07**	**348.05**	30.81%
0.04	692.00	571.92	17.35%
0.05	885.07	836.47	5.49%
0.06	1024.84	1022.57	0,22%
0.07	1105.62	1115.68	-0.91%
0.08	1149.69	1159.35	-0.84%
0.09	1177.42	1185.82	-0.71%
0.10	1199.47	1207.77	-0.69%
0.11	1219.92	1221.04	-0.09%
0.12	1239.99	1242.08	-0.16%

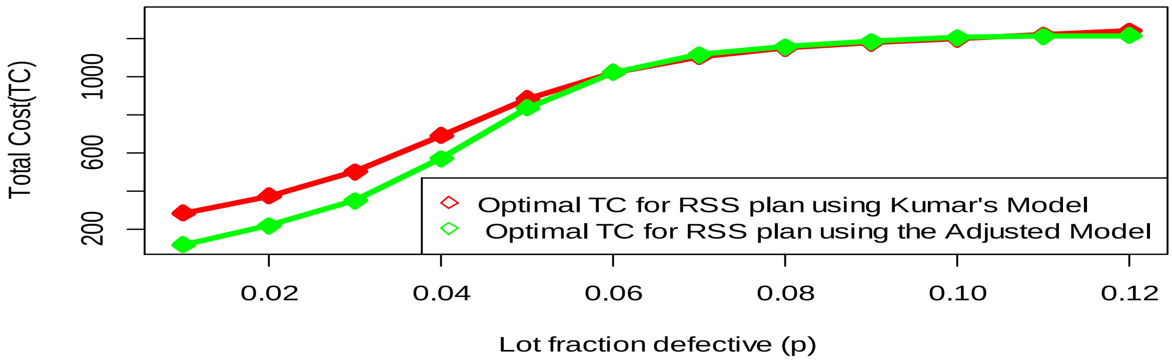

Figure 1: Optimal Total Cost(TC) for RSS plans

The optimal Total Cost (TC) for RSS plans using Kumar's model and the Adjusted Model with inspection error is 503.07 and 348.05 respectively. The total cost in all the models increased as the fraction defective (p) increased. However, the optimal total cost in the adjusted model with inspection errors became higher than the optimal cost in Kumar's model at $p \geq 0.07$. This is because as the fraction defective units(p) increased the probability of lots rejection in the adjusted model became higher than in the Kumar's model leading to increase in the Average Total Inspection of the rejected lots hence the increase in total cost at ≥ 0.07 .

Comparison of the Total Cost (TC) in Optimal RDS Plans under the existing model and the adjusted model with inspection error

Table 6: Total Cost (TC) for optimal RDS plans in Kumar's model and the adjusted model.

	Optimal RDS plan using Kumar's Model $n_1 = 96, c_1=3, n_2 = 192, c_2 = 11,$	Optimal RDS plan using the Adjusted Model $n_1 = 10, c_1=2, n_2 = 20, c_2 = 10$	
p	TC	TC	%Difference in TC
0.01	209.25	130.60	37.59%
0.02	342.83	259.86	24.20%
0.03	**534.60**	**448.45**	16.11%
0.04	781.18	720.69	7.74%
0.05	996.56	979.15	1.75%
0.06	1139.58	1153.03	-1.18%
0.07	1228.56	1254.74	-2.13%
0.08	1289.28	1319.07	-2.31%
0.09	1336.95	1368.51	-2.36%
0.10	1378.86	1412.65	-2.45%
0.11	1418.35	1454.96	-2.58%
0.12	1456.88	1496.64	-2.73%

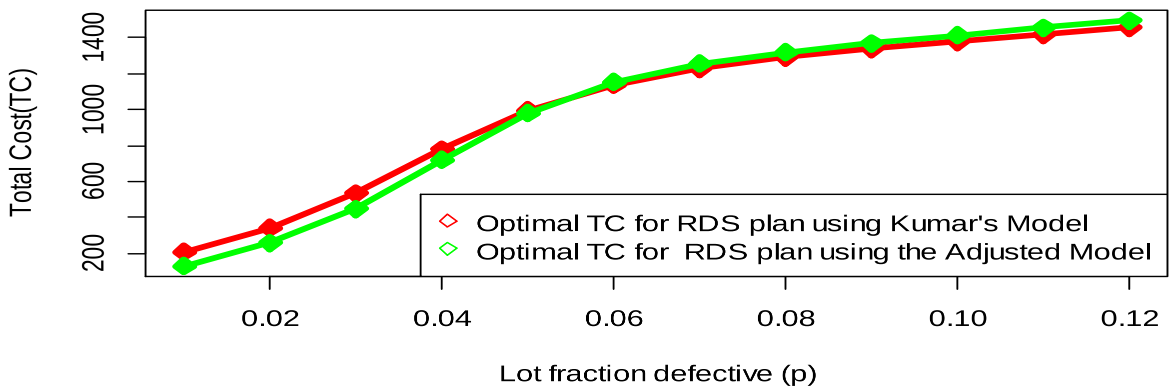

Figure 2:Optimal Total Cost(TC) for RDS plans

The optimal Total Cost (TC) for RDS plans using Kumar's model and the Adjusted Model with inspection error is 534.43 and 448.45 respectively. The total cost in all the models increased as the fraction defective (p) increased. However, the optimal total cost in the adjusted model with inspection errors became higher than the optimal cost in Kumar's model at $p \geq 0.07$. This is because as the fraction defective units(p) increased the probability of lots rejection in the adjusted model became higher than in the Kumar's model leading to increase in the Average Total Inspection of the rejected lots hence the increase in total cost at ≥ 0.07 .

III. Conclusion

Consequent upon the findings above, it is noted that RSS in the adjusted model has a smaller Sample size (n)and smaller total cost (TC) of 23 and 348.5 as against sample size (n) of 201 and Total Cost (TC) of 503.07 in Kumar's model. In the same manner, RDS in the adjusted model has smaller sample sizes of $n_1 = 10, n_2 = 20$ and smaller Total Cost (TC) of 448.05 as against sample sizes of $n_1 = 96, n_2 = 192$ and Total Cost (TC) of 534.60 using Kumar's model.
Similarly, RSS in the adjusted model has lower producer's risk of 0.0012 and consumer's risk of 0.0687 as against producer's risk of 0.0077 and consumer's risk of 0.0979 using Kumar's model. RDS in the adjusted model also has lower producer's risk of 0.0076 and consumer's risk of 0.0800 as against producer's risk of 0.0093 and consumer's risk of 0.0971 using Kumar's model. It can therefore be concluded that the adjusted model with inspection error is more economical and provides better protection for both the producer's and the consumer's risk requirements than the Kumar's model with no inspection error assumed.
This model is recommended to be used in a two-stage supply chain involving manufacturers and vendors to satisfy their quality requirements and provide protection against losses.

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