Bayesian Analysis of Transmuted Inverse Rayleigh Distribution

T. M. Adegoke[1]; O. B. Sule[2]; I. I. Ibrahim[3]

[1]Department of Mathematics and Statistics,
First Technical University, Ibadan, Oyo State, Nigeria.

[2,3] Department of Mathematical Sciences,
Prince Abubakar Audu University, Anyigba, Kogi State, Nigeria.
Email: adegoketaiwom@gmail.com

***Abstract* — In this paper, Bayes estimators of the unknown two parameters of the Transmuted Inverse Rayleigh Distribution (TIRD) have been derived using both the frequentist and bayesian methods. The Bayes theorem was adopted to obtain the posterior distribution of the parameters of TIRD using both the conjugate and non-conjugate prior distributions under different loss functions. The posterior distributions derived for the parameters are intractable and a Lindley approximation was adopted to obtain the parameters of interest. The loss functions were employed to obtain the estimates for the parameters with an assumption that the parameters are unknown and independent. The Bayes estimates obtained under different loss functions are close to the true parameter value of the shape and scale parameters. The estimators are then compared in terms of their Mean Square Error (MSE). We deduce that the MSE reduces as the sample size (n) increases. All analysis were performed with R statistical software.**

Keywords: *Inverse Rayleigh Distribution, Transmutation Map, Hazard Rate Function, Reliability Function, Order Statistics, Parameter Estimation.*

I. Introduction

The Inverse Rayleigh distribution has many applications in the area of reliability studies. Voda (1972) mentioned that the distribution of lifetimes of several types of experimental units can be approximated by the inverse Rayleigh distribution. Ahmad *et al* (2014), uses transmutation map approach suggested by Shaw and Buckley (2007) to define a new model which generalizes the Inverse Rayleigh model. Many authors have studied the transmuted distribution of many distribution. Aryal and Tsokos (2009, 2011) proposed the transmuted extreme distributions. Merovci (2013) derived the transmuted Rayleigh distribution.

Ashouret and Eltehiwy (2013) derived the transmuted exponentiated Lomax distribution. Ahmad *et al* (2014), proposed the transmuted Inverse Rayleigh distribution. The probability density distribution (PDF) of transmuted inverse Rayleigh distribution is expressed as

$$f(x;\theta,\lambda) = \frac{2\theta}{x^3} e^{-\frac{\theta}{x^2}} \left(1 + \lambda - \lambda e^{-\frac{\theta}{x^2}}\right) \tag{1}$$

and its cumulative density function (CDF) is given as;

$$F(x;\theta,\lambda) = e^{-\frac{\theta}{x^2}} \left(1 + \lambda - \lambda e^{-\frac{\theta}{x^2}}\right) \tag{2}$$

where θ is a scale parameter and λ is a transmutted parameter. The Inverse Rayleigh distribution is clearly a special case for $\lambda = 0$. Figure 1 illustrates some of the possible shapes of the pdf of a transmuted inverse Rayleigh distribution for selected values of the parameters θ and λ.

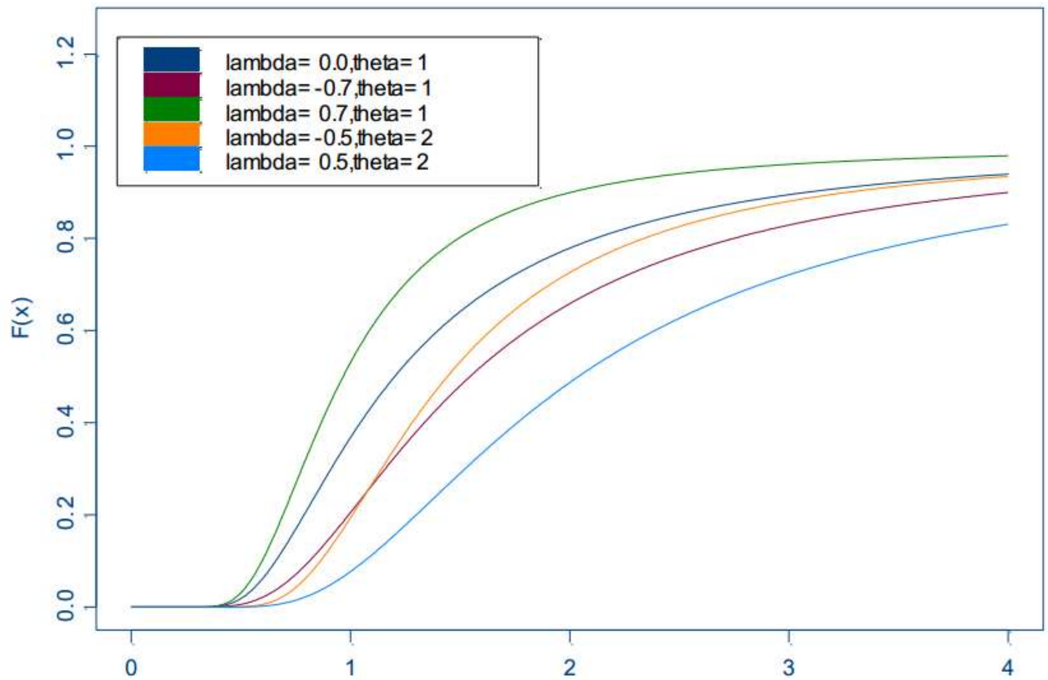

Figure 1: The cdf's of various transmuted inverse Rayleigh distributions

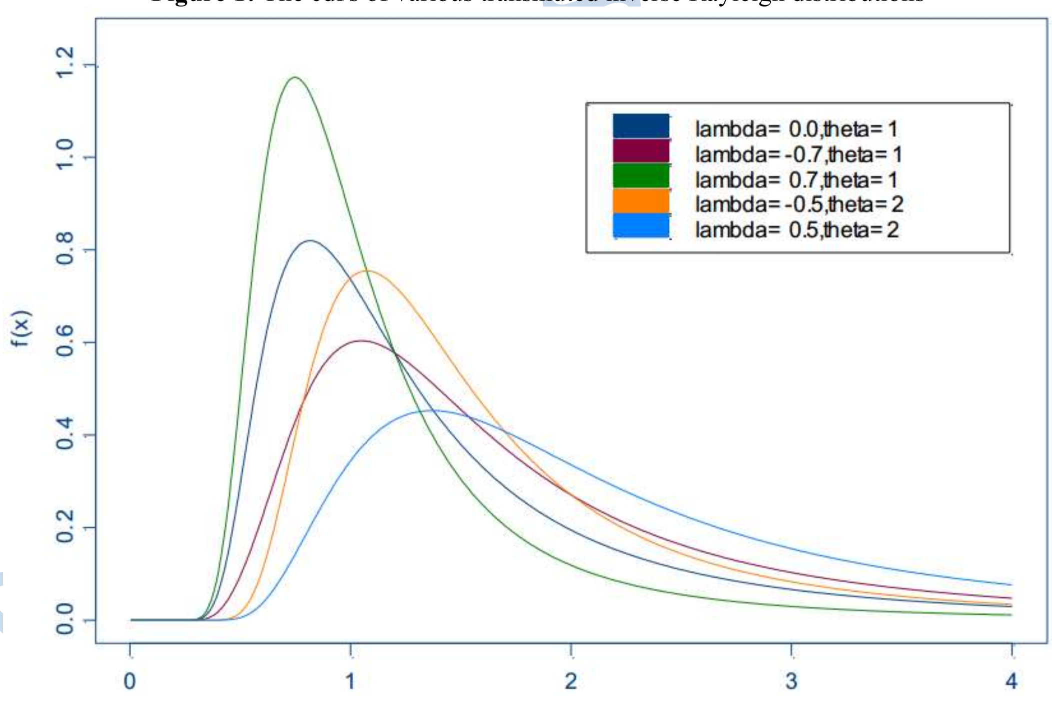

Figure 2: The pdf's of various transmuted inverse Rayleigh distributions

II. Methodology

A. Maximum Likelihood Method

Let x = $(x_1, x_2, \dots, x_n)$ be a random variable drawn from TIRD with size n. The likelihood function for the given random sample can be expressed as

$$L = \frac{(2\theta)^n}{\prod_{i=1}^{n} x^3} e^{\sum_{i=1}^{n}\left(-\frac{\theta}{x^2}\right)} \prod_{i=1}^{n}\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right) \quad (3)$$

The log-likelihood function of (3) is

$$\log L = n\log 2 + n\log\theta - \sum_{i=1}^{n}\log x_i^3 - \sum_{i=1}^{n}\frac{\theta}{x_i^2} + \sum_{i=1}^{n}\log\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right) \quad (4)$$

Therefore, MLE's of θ and λ which maximizes (4) must satisfy the following normal equations

$$\frac{\partial \log L}{\partial\theta} = \frac{n}{\theta} - \sum_{i=1}^{n}\frac{1}{x^2} + \sum_{i=1}^{n}\left[\frac{\frac{2\lambda}{x^2}e^{-\frac{\theta}{x^2}}}{\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right)}\right] \quad (5)$$

$$\frac{\partial \log L}{\partial\lambda} = \sum_{i=1}^{n}\left[\frac{1-2e^{-\frac{\theta}{x^2}}}{\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right)}\right] \quad (6)$$

The MLE of $\hat{\theta}$ and $\hat{\lambda}$ is obtained by solving this nonlinear system of equations. It is usually more convenient to use nonlinear optimization algorithms. In this study we are adopting the Newton Raphson Approach.

B. Bayesian Analysis

In the estimation of TIRD parameters under Bayesian method, three types of loss function were considered. The first is LINEX loss function (LLF) which is also known as linear-exponential loss function which is asymmetric. Varian (1975) introduced the LLF. The Entropy loss function (ELF) was also introduced by Calabria & Pulcini (1994). The third loss function is the scale invariant squared error loss function (SISLF) which was introduced by DeGroot (1970) and it is also known as De-Groot loss function.

C. *Linex Loss Function*

$$\hat{\theta} = -\frac{1}{\tau}\ln\left(E_\theta\left[e^{-\tau\theta}\right]\right) \quad (7)$$

provided that $E_\theta\left[e^{-\tau\theta}\right]$ exits.

D. *Entropy Loss Function*

The Bayes estimator of the ELF is the value $\hat{\theta}$ and can be expressed as

$$\hat{\theta}_{ELF} = \left[E(\theta^{-1})\right]^{-1} \quad (8)$$

E. *Scale invariant squared error loss function*

The Bayes estimate using SISLF is given by

$$\hat{\theta}_{SISLF} = \frac{E\left(\frac{1}{\theta}\right)}{E\left(\frac{1}{\theta^2}\right)} \quad (9)$$

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F. *Posterior Distribution*

Let x = (x_1, x_2, ,…,x_n) be a random variable with parameters Θ and λ, having size n. from the bayes' the posterior probability density function of the parameters Θ and λ given x can be expressed as

$$\Pr(\theta,\lambda|x) \propto \frac{L(\theta,\lambda|x)\pi(\theta,\lambda)}{\int_0^\infty\int_0^\infty L(\theta,\lambda|x)\pi(\theta,\lambda)\partial\theta\partial\lambda} \quad (10)$$

Where $L(\theta,\lambda|x)$ is the likelihood and $\pi(\theta,\lambda)$ is the prior probability distrbution

We adopted conjugate prior distribution for parameter $\theta \sim G(a,b)$ and a non-conjugare prior for $\lambda \sim U(0,\lambda)$

$$\pi(\theta,\lambda) = \frac{b^a}{\lambda\Gamma(a)}\theta^{a-1}e^{-b\theta} \qquad a>0, b>0, \lambda>0, \theta>0 \quad (11)$$

To obtain the posterior distribution, we substitute the likelihood (3) and prior distribution (11) into (10) to obtain the posterior distribution.

$$\Pr(\theta,\lambda|x) \propto \frac{\frac{(2\theta)^n}{\prod_{i=1}^{n}x^3}e^{\sum_{i=1}^{n}\left(-\frac{\theta}{x^2}\right)}\prod_{i=1}^{n}\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right)\frac{b^a}{\lambda\lambda\ (a)}\theta^{a-1}e^{-b\theta}}{\int_0^\infty\int_0^\infty\frac{(2\theta)^n}{\prod_{i=1}^{n}x^3}e^{\sum_{i=1}^{n}\left(-\frac{\theta}{x^2}\right)}\prod_{i=1}^{n}\left(1+\lambda-\lambda e^{-\frac{\theta}{x^2}}\right)\frac{b^a}{\lambda\lambda\Gamma(a)}\theta^{a-1}e^{-b\theta}\partial\theta\partial\lambda} \quad (12)$$

G. Lindley's Approximation

It's noted that the posterior distribution (12) takes a ratio form that involves an integration in the denominator and that the denominator can't be reduced to a closed form. Thus to estimate the posterior distribution (12) will be difficult. In other to estimate the posterior distribution we will adopt the Lindley's approximation suggested by Lindley(1980) which treats the ratio of the integrals as a whole which results to a single numerical result. In this work, we compute $E(\theta_i|x)$ and $E(\theta_i^2|x)$ in order to find the variance estimates given by

$$Var(\theta_i|x) = E(\theta_1^2|x) - \left(E(\theta_i|x)\right)^2 \qquad i = 1,2 \quad (13)$$

where $\theta_1 = \theta$ and $\theta_2 = \lambda$. If n is sufficiently large, according to Lindley(1980), any ratio of the integral of the form

$$I(x) = E[u(\theta,\lambda)] = \frac{\int\int u(\theta,\lambda)e^{l(\theta,\lambda)+\rho(\theta,\lambda)}d\theta d\lambda}{\int\int e^{l(\theta,\lambda)+\rho(\theta,\lambda)}d\theta d\lambda} \quad (14)$$

where $u(\theta,\lambda)$ is a function of θ and λ only, $l(\theta,\lambda)$ is the log-likelihood and $\rho(\theta,\lambda)$ is the log of the prior distribution $\pi(\theta,\lambda)$. Thus, for the unknown parameter θ, the Lindley's approximation is

$$E[u(\theta,\lambda)|\underline{x} = u(\hat{\theta},\hat{\lambda}) + \frac{1}{2}(u_{11}\phi_{11}) + \rho_1 u_1\phi_{11} + \frac{1}{2}(L_{30}u_1\phi_{11}^2) + \frac{1}{2}(L_{12}u_1\phi_{11}\phi_{22}) \quad (15)$$

where $u(\hat{\theta},\hat{\lambda}) = \frac{1}{\hat{\theta}}$

Also, for the unknown parameter λ, the Lindley's approximation is

$$E[u(\theta,\lambda)|\underline{x} = u(\hat{\theta},\hat{\lambda}) + \frac{1}{2}(u_{22}\phi_{22}) + \rho_2 u_2\phi_{22} + \frac{1}{2}(L_{03}u_2\phi_{22}^2) + \frac{1}{2}(L_{21}u_2\phi_{11}\phi_{22}) \quad (16)$$

where $u(\hat{\theta},\hat{\lambda}) = \frac{1}{\hat{\lambda}}$

All the quantities in the above expression of I(x) have the following representations:

$$L_{ij} = \frac{\partial^{i+j} l(\theta,\lambda)}{\partial\theta^i\partial\lambda^j} \qquad i,j = 0,1,2,3$$

$$\frac{\partial^2 l}{\partial\theta^2} = -\frac{n}{\theta^2} - \sum_{i=1}^n\left[\frac{2\lambda e^{-\frac{\theta}{x^2}}}{x^4\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)}\right] - \sum_{i=1}^n\left[\frac{4\lambda^2\left(e^{-\frac{\theta}{x^2}}\right)^2}{x^4\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2}\right] \quad (17)$$

$$\frac{\partial^2 l}{\partial\lambda^2} = -\sum_{i=1}^n\left[\frac{n\left(1-2e^{-\frac{\theta}{x^2}}\right)^2}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2}\right] \quad (18)$$

$$\frac{\partial^2 l}{\partial\lambda\partial\theta} = \sum_{i=1}^n\left[\frac{2e^{-\frac{\theta}{x^2}}}{x^2\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)}\right] - \sum_{i=1}^n\left[\frac{2\left(1-2e^{-\frac{\theta}{x^2}}\right)\lambda e^{-\frac{\theta}{x^2}}}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2 x^2}\right] \quad (19)$$

$$\frac{\partial^3 l}{\partial\lambda^3} = \sum_{i=1}^n\left[\frac{2\left(1-2e^{-\frac{\theta}{x^2}}\right)^3}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^3}\right] \quad (20)$$

$$\frac{\partial^3 l}{\partial\theta^3} = \frac{2n}{\theta^3} + \sum_{i=1}^n\left[\frac{2\lambda e^{-\frac{\theta}{x^2}}}{x^6\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)}\right] + \sum_{i=1}^n\left[\frac{12\lambda^2\left(e^{-\frac{\theta}{x^2}}\right)^2}{x^6\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2}\right] + \sum_{i=1}^n\left[\frac{16\lambda^3\left(e^{-\frac{\theta}{x^2}}\right)^3}{x^6\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^3}\right] \quad (21)$$

$$\frac{\partial^3 l}{\partial\theta\partial\lambda^2} = -\sum_{i=1}^n\left[\frac{4n\left(1-2e^{-\frac{\theta}{x^2}}\right)e^{-\frac{\theta}{x^2}}}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2 x^2}\right] + \sum_{i=1}^n\left[\frac{4n\left(1-2e^{-\frac{\theta}{x^2}}\right)^2\lambda e^{-\frac{\theta}{x^2}}}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^3 x^2}\right] \quad (22)$$

$$\frac{\partial^3 l}{\partial\theta^2\partial\lambda} = -\sum_{i=1}^n\left[\frac{2e^{-\frac{\theta}{x^2}}}{x^4\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)}\right] - \sum_{i=1}^n\left[\frac{8\left(e^{-\frac{\theta}{x^2}}\right)^2\lambda}{x^4\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2}\right] + \sum_{i=1}^n\left[\frac{2\left(1-2e^{-\frac{\theta}{x^2}}\right)\lambda e^{-\frac{\theta}{x^2}}}{x^4\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^2}\right] + \sum_{i=1}^n\left[\frac{8\left(1-2e^{-\frac{\theta}{x^2}}\right)\lambda^2\left(e^{-\frac{\theta}{x^2}}\right)^2}{\left(1+\lambda-2\lambda e^{-\frac{\theta}{x^2}}\right)^3 x^4}\right] \quad (23)$$

$$\phi_{11} = -\frac{1}{L_{20}}$$

and

$$\phi_{22} = -\frac{1}{L02}$$

$$\rho = \log\pi(\theta,\lambda) = a\log b - \log\lambda - \log\Gamma(a) + (a-1)\log\theta - b\theta$$

$$\rho_1 = \frac{\partial\rho}{\partial\theta} = \frac{a-1}{\theta} - b$$

$$\rho_2 = \frac{\partial\rho}{\partial\lambda} = -\frac{1}{\lambda}$$

The values of the Bayes estimates of parameters σ and α can now be obtained.

H. Case of the LINEX loss function (LLF)

1. For the parameter σ, let $u(\hat{\sigma},\hat{\alpha}) = e^{-k\hat{\sigma}}$ then $u_1 = -ke^{-k\hat{\sigma}}$ and $u_{11} = k^2e^{-k\hat{\sigma}}$, $u_2 = u_{22} = 0$

$$\hat{\sigma}_{LLF} = -\frac{1}{k}\left[e^{-k\hat{\sigma}} + \frac{1}{2}u_{11}\phi_{11} + \rho_1 u_1\phi_{11} + \frac{1}{2}L_{30}u_1\phi_{11}^2 + \frac{1}{2}L_{12}u_1\phi_{11}\phi_{22}\right] \quad (24)$$

2. For the parameter α, let $u(\hat{\sigma},\hat{\alpha}) = e^{-k\hat{\alpha}}$ then $u_2 = -ke^{-k\hat{\alpha}}$ and $u_{22} = k^2e^{-k\hat{\alpha}}$, $u_1 = u_{11} = 0$

$$\hat{\alpha}_{LLF} = -\frac{1}{k}\left[e^{-k\hat{\alpha}} + \frac{1}{2}u_{22}\phi_{22} + \rho_2 u_2\phi_{22} + \frac{1}{2}L_{03}u_2\phi_{22}^2 + \frac{1}{2}L_{21}u_2\phi_{11}\phi_{22}\right] \quad (25)$$

I. Case of the Entropy loss function (ELF)

1. For the parameter σ, let $u(\hat{\sigma},\hat{\alpha}) = \frac{1}{\sigma}$ then $u_1 = -\frac{1}{\hat{\sigma}^2}$ and $u_{11} = \frac{2}{\hat{\sigma}^3}$, $u_2 = u_{22} = 0$

$$\hat{\sigma}_{ELF} = \left[\frac{1}{\hat{\sigma}} + \frac{1}{2}u_{11}\phi_{11} + \rho_1 u_1\phi_{11} + \frac{1}{2}L_{30}u_1\phi_{11}^2 + \frac{1}{2}L_{12}u_1\phi_{11}\phi_{22}\right]^{-1} \quad (26)$$

2. For the parameter α, let $u(\hat{\sigma},\hat{\alpha}) = \frac{1}{\hat{\alpha}}$ then $u_2 = -\frac{1}{\alpha^2}$ and $u_{22} = \frac{2}{\alpha^3}$, $u_1 = u_{11} = 0$

$$\hat{\alpha}_{ELF} = \left[\frac{1}{\hat{\alpha}} + \frac{1}{2}u_{22}\phi_{22} + \rho_2 u_2\phi_{22} + \frac{1}{2}L_{03}u_2\phi_{22}^2 + \frac{1}{2}L_{21}u_2\phi_{11}\phi_{22}\right]^{-1} \quad (27)$$

J. Case of the scale invariant squared error loss function (SISLF)

1. For the parameter σ, let $u(\hat{\sigma},\hat{\alpha}) = \frac{1}{\sigma}$ then $u_1 = -\frac{1}{\hat{\sigma}^2}$ and $u_{11} = \frac{2}{\hat{\sigma}^3}$, $u_2 = u_{22} = 0$ and also let $u^*(\hat{\sigma},\hat{\alpha}) = \frac{1}{\hat{\sigma}^2}$ then $u_1^* = -\frac{2}{\hat{\sigma}^3}$ and $u_{11}^* = \frac{6}{\hat{\sigma}^4}$, $u_2^* = u_{22}^* = 0$

$$\hat{\sigma}_{SISLF} = \frac{\left[\frac{1}{\hat{\sigma}^2} + \frac{1}{2}u_{11}^*\phi_{11} + \rho_1 u_1^*\phi_{11} + \frac{1}{2}L_{30}u_1^*\phi_{11}^2 + \frac{1}{2}L_{12}u_1^*\phi_{11}\phi_{22}\right]}{\left[\frac{1}{\hat{\sigma}} + \frac{1}{2}u_{11}\phi_{11} + \rho_1 u_1\phi_{11} + \frac{1}{2}L_{30}u_1\phi_{11}^2 + \frac{1}{2}L_{12}u_1\phi_{11}\phi_{22}\right]} \quad (28)$$

2. For the parameter α, Let $u^*(\hat{\sigma},\hat{\alpha}) = \frac{1}{\hat{\alpha}^2}$ then $u_2^* = -\frac{2}{\hat{\alpha}^3}$ and $u_{22}^* = \frac{6}{\hat{\alpha}^4}$, $u_1^* = u_{11}^* = 0$ and also let $u(\hat{\sigma},\hat{\alpha}) = \frac{1}{\hat{\alpha}}$ then $u_2 = -\frac{1}{\alpha^2}$ and $u_{22} = \frac{2}{\alpha^3}$, $u_1 = u_{11} = 0$

$$\hat{\alpha}_{SISLF} = \frac{\left[\frac{1}{\hat{\alpha}^2} + \frac{1}{2}u_{22}^*\phi_{22} + \rho_2 u_2^*\phi_{22} + \frac{1}{2}L_{03}u_2^*\phi_{22}^2 + \frac{1}{2}L_{21}u_2^*\phi_{11}\phi_{22}\right]}{\left[\frac{1}{\hat{\alpha}} + \frac{1}{2}u_{22}\phi_{22} + \rho_2 u_2\phi_{22} + \frac{1}{2}L_{03}u_1\phi_{22}^2 + \frac{1}{2}L_{21}u_2\phi_{11}\phi_{22}\right]} \quad (29)$$

III. Applications

In this section, we simulated a random sample of sizes *n* = 30, 50, 100, 200 and 500 from TIR distribution with parameters ϴ =0.5, 0.8 and 1.0, λ = -0.5, 0.5 and 1.0. The results are replicated 1,000 times and the average result were presented in the tables. The estimate and mean square error (MSE) values obtained by the method of MLE, LLF, ELF and SISLF are shown in Tables 1.

Based on the results in Table 1, we can deduced that that the obtained estimated for oboth clasical and bayesian methods are close to the predefined values. Also, we observed that as n increases the MSE increases. Futhermore, we deduced that the Bayesian estimates of the scale and transmutted parameters under the Bayesian method performs better than that of the classical techniques because the have the small MSE . Finally, among theBayesian estimates, LLF seems to have have the best estimates because it has the smallest MSE among other Bayes estimates.

Table 1. Estimates of the parameters of the Four methods: MLE, LLF, ELF AND SISLF with their MSE with different parameter values

n	**Parameter**		**MLE**		**LLF**		**ELF**		**SISLF**	
20	Θ	λ	Θ	λ	Θ	λ	Θ	λ	Θ	λ
	1 **a = 1.3** **b = 1** **k = 2**	**0.5**	0.4757 (0.1810)	0.3274 (0.7424)	0.5086 (0.0075)	0.1285 (0.2653)	0.4941 (0.0076)	0.2028 (0.2019)	0.4757 (0.0081)	0.2501 (0.1948)
	0.5 **a = 1.3** **b = 1** **k = 0.5**	**-0.5**	0.6624 (0.7466)	-0.4893 1.6635	0.6460 (0.0223)	-0.1586 (0.1170)	0.6037 (0.0116)	-0.2200 (0.0784)	0.5733 (0.0062)	-0.2682 0.0537
	0.8 **a = 1** **b = 0.5** **k = 0.5**	**1**	1.1248 (0.2214)	1.3565 0.3165	1.2161 0.1731	1.0719 0.0052	1.2031 (0.1625)	1.0905 (0.0082)	1.1820 (0.1459)	1.2758 (0.0761)
	0.5 **a = 1.5** **b = 1** **k = 1**	**0.5**	0.6263 (0.1739)	0.5889 (0.5359)	0.5024 (0.0229)	0.0582 (0.2694)	0.4877 (0.0225)	0.1480 (0.1894)	0.4687 (0.0219)	0.1923 (0.0225)
50	**1** **a = 1.3** **b = 1** **k = 2**	**0.5**	0.5215 (0.0879)	0.4738 (0.3025)	0.5595 (0.0113)	0.6825 (0.1817)	0.5548 (0.0109)	0.6823 (0.1809)	0.5484 (0.0104)	0.7160 (0.2100)
	0.5 **a = 1.3** **b = 1** **k = 0.5**	**0.5**	0.4719 (0.18160	-0.3581 (0.57970	0.7022 (0.09100	0.0096 (0.2994)	0.6870 (0.0837)	0.0078 (0.2977)	0.6748 (0.0784)	0.0140 (0.3145)
	0.8 **a = 1 b** **= 0.5** **k = 0.5**	**1**	0.9651 (0.1197)	1.4699 (0.2036)	1.0037 (0.0415)	1.3466 (0.1201)	0.9995 (0.0398)	1.3462 (0.1199)	0.9936 (0.0375)	1.4445 (0.1976)
	0.5 **a = 1.5** **b = 1** **k = 1**	**0.5**	0.5117 (0.0818)	0.7732 (0.3118)	0.5213 (0.0015)	0.2127 (0.2503)	0.5153 (0.0014)	0.3143 (0.1154)	0.5075 (0.0012)	0.3390 (0.0990)
100	**1** **a = 1.3** **b = 1** **k = 2**	**0.5**	0.5610 (0.0531)	0.9195 (0.1592)	0.5011 (0.0026)	0.4651 (0.0635)	0.4984 (0.0026)	0.4568 (0.0637)	0.4947 (0.0026)	0.4608 (0.0615)
	0.5 **a = 1.3** **b = 1** **k = 0.5**	**0.5**	0.6789 (0.0983)	0.2466 (0.2590)	0.5692 (0.0178)	-0.2272 (0.1950)	0.5627 (0.0169)	-0.2168 (0.1964)	0.5572 (0.0162)	-0.2007 (0.2092)
	0.8 **a = 1** **b = 0.5** **k = 0.5**	**1**	0.8531 (0.0707)	1.1627 (0.1061)	0.8656 (0.0043)	1.0970 (0.0094)	0.8624 (0.0039)	1.0953 (0.0091)	0.8583 (0.0034)	1.1209 (0.0146)
	0.5 **a = 1.5** **b = 1** **k = 1**	**0.5**	0.5078 (0.0548)	0.6982 (0.1969)	0.5134 (0.0017)	0.3400 (0.0745)	0.5105 (0.0017)	0.3725 (0.0643)	0.5066 (0.0016)	0.3807 (0.0569)

n	Parameter		MLE		LLF		ELF		SISLF	
	Θ	**λ**	Θ	λ	Θ	λ	Θ	λ	Θ	λ
200	**1 a = 1.3 b = 1 k = 2**	**0.5**	0.5158 (0.0447)	0.5298 (0.1620)	0.4958 (0.0012)	0.4894 (0.0177)	0.4945 (0.0012)	0.4581 (0.0188)	0.4926 (0.0012)	0.4734 (0.0187)
	0.5 a = 1.3 b = 1 k = 0.5	**0.5**	0.5397 (0.1080)	-0.4244 (0.2919)	0.6120 (0.0175)	-0.2816 (0.0587)	0.6083 (0.0167)	-0.1802 (0.1377)	0.6051 (0.0160)	-0.1634 (0.1499)
	0.8 a = 1 b = 0.5 k = 0.5	**1**	0.7469 (0.0439)	0.9949 (0.0735)	0.7500 (0.0025)	0.9578 (0.0018)	0.7483 (0.0027)	0.9559 (0.0019)	0.7463 (0.0029)	0.9601 (0.0016)
	0.5 a = 1.5 b = 1 k = 1	**0.5**	0.4842 (0.0453)	0.3421 (0.1677)	0.4878 (0.0017)	0.3691 (0.0568)	0.4864 (0.0017)	0.3603 (0.0580)	0.4845 (0.0018)	0.3595 (0.0561)
500	**1 a = 1.3 b = 1 k = 2**	**0.5**	0.5412 (0.0243)	0.7416 (0.0776)	0.5179 (0.0012)	0.5334 (0.0194)	0.5173 (0.0011)	0.5275 (0.0197)	0.5166 (0.0011)	0.5232 (0.0199)
	0.5 a = 1.3 b = 1 k = 0.5	**0.5**	0.5523 (0.0806)	-0.2435 (0.2353)	0.5069 (0.0022)	-0.4261 (0.0487)	0.5056 (0.0021)	-0.4163 (0.0523)	0.5044 (0.0021)	-0.4094 (0.0544)
	0.8 a = 1 b = 0.5 k = 0.5	**1**	0.8382 (0.0302)	1.0262 (0.0388)	0.8398 (0.0016)	1.0000 (0.0000)	0.8391 (0.0015)	1.0000 (0.0000)	0.8282 (0.0015)	1.0116 (0.0001)
	0.5 a = 1.5 b = 1 k = 1	**0.5**	0.4759 (0.0261)	0.4902 (0.1011)	0.4883 (0.0002)	0.4950 (0.0024)	0.4878 (0.0002)	0.4886 (0.0025)	0.4870 (0.0003)	0.4838 (0.0027)

IV. Conclusion

In this work, both classical and bayesian estimation of TIR distribution was adopted to estimate the parameters of TIR distribution using different loss functions such as Entropy Loss Function, Linex Loss Function, Scale Invariant Squared Error Loss Function. Fig. 1-2 shows that the PDF and CDF of the TIR distribution at varying parameter values which shows that the distribution is positively skewed. Tables 1 shows the posterior estimates with MSE for different prior distribution under different loss functions for the simulated datasets. Based on the results displayed in Tables 1, we observed that all the posterior estimates for both parameters are close to the true values of parameters of TIR distribution. Also, we discovered the methods are consistent since the values of MSE decrease as sample size increases. It can be observed that the Bayesian estimates of the scale and transmutted parameters under the Bayesian techniques perform better than that of the classical techniques.

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