On Heteroscedastic Model for Consumer Price Index

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Abstract **— The purpose of this study is to provide an appropriate model for reliable output of Consumer Price Index (CPI) Nigeria data estimation for forecast accuracy. CPI is used to measure inflation which requires appropriate model for reliable forecast. CPI data is heteroscedastic in nature which require the best model among the heteroscedastic models [Autoregressive Conditional Heteroscedastic (ARCH), Generalized Autoregressive Conditional Heteroscedastic (GARCH), Threshold (GARCH), Exponential (GARCH) and Power (ARCH) with Normal (-N) and Student's *t* (-ST) distributions] used in this study. The least minimum information criteria (AIC, BIC and HQ) were revealed in EGARCH-ST which indicated the best model fit. EGARCH-ST has the highest value of log likelihood (LL) which indicated good distribution fit. The dynamic forecast evaluation revealed that EGARCH-ST has the minimum forecast error measures values of root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and geometric root of mean square error (GRMSE) among the heteroscedastic models. These were forecast error measures that determined the forecast accuracy when compared with other heteroscedastic models for forecasting. Therefore, EGARCH-ST is the best model fit for reliable estimation output and forecast accuracy using Nigeria CPI data for better decision that boost the nation's economy**.

Keywords: *Consumer Price Index, Heteroscendastic Model, Forcasting, Googness-of-fit.*

I. Introduction

The consideration of CPI in this study is that, it is the chief indicators of inflationary change and also regarded as the best gauge of inflation available to investors and others in the economies of the nation. CPI is created to signify a statistical estimate of inflation. Pure inflation is when there are no changes in product but there are changes in prices. The all-item CPI revealed the change in consumer prices from month to month and is used to measure inflation. Governments and central banks set inflation targets using CPI regardless of its limitations (Boskin et al., 1998; Feldstein, 2017).

Many researchers have work on price changes, pricing and inflation, but none revealed the right model for modelling inflation and CPI. White (1980) revealed the heteroscedastic error term in the model, and heteroscedastic errors posh serious challenges in forecasting models. Engle (1982) offered Autoregressive Conditional Heteroscedastic (ARCH) model to model the time varying volatility to have efficient estimation. ARCH model has limitations of fixed lag and abnormalities in some financial and economic sectors.Bollerslev (1986) presented generalized ARCH (GARCH) to the uplift of the weakness of ARCH model by allowing large lag structure with memory extension. Ewing and Malik (2013) observed that GARCH has weakness of excess kurtosis and volatility persistence (Vivian &Wohar, 2012). Engle (1982) and Bollerslev (1986) focused on the magnitude of returns in the heteroscedasticity.

With GARCH (Generalized ARCH) model counting the past volatility function as a determinant for volatility estimation and it successfully make simpler the calculation of onerous lagged values from accrued returns (Jiang & Xia, 2018).

The GARCH models rely on future volatility as a linear even function of squared returns and assume that future volatility reacts symmetrically to falling and rising markets which is the weakness of GARCH. The empirical indication has revealed that financial volatility is more liable to growing when negative shocks of returns is signifying that positive and negative returns of the same magnitude have a right-skewed effect on future volatility, that is there is increase in negative returns which is the leverage effect (Jiang & Xia, 2018).

However, ARCH and GARCH cannot grab the information on the volatility that affects the direction of returns (Nelson, 1991; Hentschel, 1995; Berument, Metin-Ozcan, &Neyapti, 2001). GARCH could not accommodate the asymmetry in volatility crowding. The reaction to news comes from the volatility shock. This news is when there is surprising drop in price (bad news) rises expectable volatility more than surprising rise in price (good news) of related magnitude (Engle & Ng, 1993; Engle 2001).

The asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in various ways are captured by threshold GARCH (TGARCH) and exponential GARCH (EGARCH). Leverage is a particular case of asymmetry (Harvey, 1993).

GARCH weaknesses of excess kurtosis and volatility persistence were modelled by the EGARCH. The conditional variance of EGARCH non-linear model is able to react to the asymmetric volatility behaviour (Harvey, 1993). Mutunga*et al.* (2015) stressed that the EGARCH forecast is more accurate when compared with TGARCH because EGARCH has the minimum mean square error and mean absolute error.

The TGARCH, EGARCH and power ARCH (PARCH) models revealed that there were positivity of conditional variances, stationarity, and existence of fourth-order moments; when the models are restricted. EGARCH estimates have larger percentage of the series at the finite kurtosis condition when compared with PARCH estimates which has a very small percentage of the series satisfy by the finite kurtosis restriction (Nelson, 1991; Hentschel, 1995; Rodríguez & Ruiz, 2012; McAleer, 2014).

The TGARCH asymmetry parameter promised stationary and finite kurtosis with restrictions and these restrictions are not tough on the leverage effect provided the persistence is small. The imposition of restrictions on TGARCH leverage effect is similar to EGARCH, nevertheless EGARCH has been more flexible in the asymmetric response of volatility. The EGARCH models enforced less restriction among the GARCH family, this permits it to be the best flexible model (Rodríguez & Ruiz, 2012). The parameters positivity restriction on the model made EGARCH to capture the asymmetry, but cannot model the leverage efficiently (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

Therefore, the right heteroscedastic model for modelling Nigeria CPI will be revealed.

II. Research Methodology

Thedata is retrieved from the website and the source is National Bureau of Statistics (NBS). The Consumer Price Index (CPI) monthly data from 1995 to 2015 is used to provide the appropriate heteroscedastic model for further computation to obtain reliable estimation and forecast accuracy. The heteroscedastic models used in this study are presented.

A. *The ARCH Model*

Engle (1982) presented Autoregressive Conditionally Heteroscedastic (ARCH) model to permit the conditional variance, which is the variance conditional on the past. The conditional variance is stated as a linear function of the squared past values of the series, leaving the unconditional variance constant.

$$y_t = \theta y_{t-1} + \epsilon_t \quad (1)$$

$$\epsilon_t \sim N(0, h_t)$$

$$h_t = \delta + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 \quad (2)$$

where,

$q > 0,\ \delta > 0$

$\alpha_i \geq 0,\quad i = 1, \ldots, q$

If q = 0 then ϵ_t is white noise.

The order of the ARCH process is q and α_i is the unknown parameters.

B. *The GARCH Model*

Bollerslev (1986) presented Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models to permit the conditional variance which is the variance conditional on the past. In the classical GARCH models, the conditional variance is expressed as a linear function of the squared past values of the series.

$$y_t = \theta y_{t-1} + \epsilon_t$$

$$\epsilon_t \sim N(0, h_t)$$

$$h_t = \delta + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p} \quad (3)$$

where,

$p \geq 0, q > 0,\ \delta > 0$

$\alpha_i \geq 0,\quad i = 1, \ldots, q$

$\beta_i \geq 0,\quad i = 1, \ldots, p$

If p = 0 then the process is ARCH (q) process and if p = q = 0 then ϵ_t is just white noise.

C. ***The TGARCH Model***

Zakoïan (1994) proposed TGARCH model which specifies as follows:
$\sigma_t^2 = \alpha + \beta|y_{t-1}| + \gamma\sigma_{t-1} + \delta_T y_{t-1}$(4)
If$\alpha > 0, \beta \geq 0, \gamma \geq 0$ and $|\delta_T| < \beta|$ then the conditional standard deviation is positive.

D. ***The EGARCH Model***

EGARCH model have been famous in modelling the heteroscedastic error. It can be written as:

$$logh_t = \alpha + \beta|z_{t-1}| + \delta z_{t-1} + \gamma\, logh_{t-1}, |\gamma| < 1 \quad (5)$$

where $z_t = \epsilon_t/\sqrt{h_t}$is the standardized shocks, $z_t \sim iid\,(0, A)$. $|\gamma| < 1$ is when there is stability. If $\delta \neq 0$ theimpact is asymmetric, while there is leverage existence if $\delta < 0$ and $\beta < -\delta$ (Nelson, 1991; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

E. ***The PARCH Model***

Ding, Granger and Engle (1993) presented Power ARCH model which specifies σ_t as of the form:
$\sigma_t^d = \alpha_0 + \sum_{i=1}^{p} \alpha_i\,(|\epsilon_{t-i}| + \gamma_i\epsilon_{t-i})^d + \sum_{i=1}^{q} \beta_i\, \sigma_{t-i}^d$(6)
where the α_i and β_i are the parameters of standard ARCH and GARCH, the parameter of the leverage is γ_i and the power term parameter is d.

III. Results And Discussion

The plot of time graph behaviour was trending and not stationary. There were excess kurtosis and skewness which indicated heteroscedasticity. The data were not normally distributed because Jarque-Bera of residual normality tests was significant. This revealed the assurance for further tests. Statistics and normality tests were conducted to confirm the heteroscedastic nature of the data sets.

Table 1: Statistical Summary and Normality Tests for Nigeria Consumer Price Index Data

	Standard deviation	Skewness	Kurtosis	Jarque-Bera
CPI	47.482	0.6245	2.1359	24.219 (0.0000)***

P-values ( ) *** significant at 1%, ** significant at 5%, * significant at 10%

The characteristics of the level data revealed the existence of heteroscedasticity and the level data were transformed which showed more stationary behaviour empirically as displayed in Table 2 (McAleer, 2014; McAleer & Hafner, 2014). This was also to observe whether the data exhibited volatility clustering, skewness and kurtosis, and the transformed data revealed that it was heteroscedastic in nature.

Table 2: Statistical Summary, Normality and ARCH Tests for Nigeria Consumer Price Index Data

	Standard deviation	Skewness	Kurtosis	Jarque-Bera	*F*-Statistic	Obs*R-squared
CPI	0.2442	-0.8738	3.3682	33.355 (0.0000)***	101.39 (0.0000)***	72.55 (0.0000)***

P-values ( ) *** significant at 1%, ** significant at 5%, * significant at 10%

ARCH effect was obtained by computation, and ARCH LM tests were executed to know the effect of heteroscedasticity. *F*-Statistic and Obs*R-squared were significant which revealed the existence of ARCH in the data. The existence of ARCH in the data was a justification of using GARCH model as GARCH is the generalization of ARCH. It also revealed the use of other GARCH family members.

These were evidence that CPI is heteroscedastic data and only heteroscedastic model is suitable for the analysis. Therefore, ARCH, GARCH, TGARCH, EGARCH and PARCH were heteroscedastic models considered to know the appropriate heteroscedastic model for Nigeria CPI data.

Specification of ARCH and GARCH Family Models Using Nigeria CPI Data

ARCH-Normal distribution description: When the mean of the variable is zero and autocovariances are zero, then ARCH is normally distributed. The value of alpha which was the coefficient of ARCH was positive and the coefficient was highly significant. The minimum information criteria and high log-likelihood value were displayed in Table 3.

The GARCH-Normal distribution description: When the mean value of the variable is zero and autocovariances are zero, then GARCH is normally distributed. The coefficient of mean equation and coefficient of variance equation were highly significant. The sum of the coefficients of mean and variance equations was less than one which revealed stationary (Bollerslev, 1987). The sum of ARCH and GARCH coefficients were close to one which revealed volatility persistence. The minimum information criteria and high log-likelihood value were displayed in Table 3.

GARCH-Student's *t* distribution description: An additional parameter called degrees of freedom, which changes its shape from standard normal distribution is known as GARCH is Student's *t* distributed. The coefficients of mean and variance equations were significant. The minimum information criteria and high value of log-likelihood were displayed in Table 3.

TGARCH-Student's *t* distribution specification: The TGARCH is Student's *t* distributed when an additional parameter, called degrees of freedom, which changes its shape from standard normal distribution. The coefficients of mean equations were highly significant. The coefficients of variance equations were highly significant. Threshold of the asymmetry of the variance equations were not significant. EGARCH-ST has the minimum information criteria and highest value of log-likelihood.

EGARCH-Student's *t* distribution description: An additional parameter called degrees of freedom, which changes its shape from standard normal distribution is known as EGARCH is Student's *t* distributed. The coefficient of mean equation was highly significant, and the coefficient of variance equation was significant. The coefficient of the past log term was less than one which revealed that the stability condition was met. The least minimum information criteria and highest value of log-likelihood were displayed in Table 3.

Table 3: Specification of ARCH, GARCH, TGARCH, EGARCH and PARCH Models Using CPI Data

	α	β	δ	γ	AIC	BIC	LL
ARCH	0.01 (0.00)				2.94	2.99	-364.4
GARCH- N	1.12 (0.00)	-0.83 0.00)			8.38	8.45	-1047.
GARCH- ST	1.01 (0.00)	-0.79 0.01)			2.44	2.53	-300.4
TGARCH-ST	0.34 (0.00)	0.77 (0.00)	0.05 (0.68)		2.97	2.89	-432.1
EGARCH-ST	1.01 (0.00)	0.51 (0.01)	0.20 (0.07)	0.87 (0.00)	2.28	2.38	-297.6
PARCH-ST	1.03 (0.00)	-0.57 (0.94)	-0.74 (0.39)	2.54 (0.65)	6.25	6.36	-776.6

P-values ( ), represented the values of ARCH, GARCH, TGARCH, EGARCH and PARCH with Normal (-N) and Student's t (-ST) distributions.

PARCH-Student's *t* distribution description: An additional parameter called degrees of freedom, which changes its shape from standard normal distribution is known as PARCH is Student's *t* distributed. The coefficient of mean equation was highly significant, and the coefficient of variance equation was not significant. There was existence of leverage effects since the value of delta (δ) was negative. The coefficient of the past log term was greater than one which revealed that the stability condition was not met. The minimum information criteria and high value of log-likelihood were displayed in Table 3.

ARCH, GARCT-ST, TGARCH-ST, EGARCH-ST, and PARCH-ST contributed 99.9% of R^2, while GARCH-N contributed 98.6%. EGARCH-ST has the minimum standard error. The results revealed that EGARCH-ST has the smallest SSR. The least minimum information criteria is revealed in EGARCH-ST. EGARCH-ST has the highest value of LL. ARCH, GARCH-ST, TGARCH-ST and EGARCH-ST have D-W value close to 2.

Table 4: Nigeria CPI

	ARCH	GARCH -N	GARCH -ST	TGARCH -ST	EGARCH -ST	PARCH -ST
R^2	0.9994	0.9860	0.9994	0.9994	0.9994	0.9990
STDE	0.0592	0.1977	0.2839	0.0765	0.0529	0.8611
SSR	327.44	7837.2	327.48	351.95	327.55	534.11
AIC	2.9352	8.3816	2.4414	2.9737	2.2833	6.2514
BIC	2.9913	8.4518	2.5257	2.8879	2.3816	6.3638
HQ	2.9578	8.4099	2.4753	2.8976	2.3228	6.2967
LL	-3643	-1046.9	-300.39	-432.09	-279.55	-776.56
D-W	2.1832	0.1025	2.1814	2.0295	2.1813	1.3638

EGARCH-ST has minimum RMSE value among the heteroscedastic models. EGARCH-ST has minimum MAE value. GARCH-ST and EGARCH-ST have the least RMSE values.

Table 5: Nigeria CPI Forecast

	ARCH	GARCH-N	GARCH-ST	TGARCH-ST	EGARCH-ST	PARCH-ST
RMSE	9.9949	15.554	4.3722	28.792	6.0653	40.219
MAE	8.6180	13.773	3.7208	21.636	4.5305	31.681
MAPE	12.043	19.109	6.4413	30.526	6.4305	49.971

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IV. Conclusion

EGARCH-ST has the highest log-likelihood values and indicated good distribution fit. EGARCH-ST has the minimum information criteria values which indicated best model fit among the models. EGARCH-ST have the least standard error, least minimum information criteria values and highest value of log-likelihood (Almeida & Hotta, 2014).

The Jarque-Bera of residual normality tests were significant and indicated non-normality of the data distribution which revealed the suitability of an heteroscedastic model. There was no first order serial correlation since D-W values were near 2.

The dynamic forecast evaluation revealed that EGARCH-ST has the minimum forecast error measures values of root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and geometric root of mean square error (GRMSE) among the heteroscedastic models. These were forecast error measures that determined the forecast accuracy when it was compared with other heteroscedastic models for forecasting. The model that has the minimum forecast error measure values revealed the best forecast accuracy (Mutunga et al. 2015) which were displayed in Table 5. Therefore, EGARCH-ST is the best model fit for reliable estimation output and forecast accuracy using Nigeria CPI data for better decision that boost the economy of the nation.

Acknowledgment

The authors are grateful to anonymous reviewers for their valuable comments on the original draft of this manuscript.

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