A Generalized Class of Factor-type Exponential Estimator for Population Mean under Simple Random Sampling

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***Abstract*— This paper investigates the estimation of the population mean with improved exponential factor-type estimator for the variable under study using some values of alpha (constants) ranging from -1 to +1. The expressions for the bias and mean square error (MSE) of the proposed estimator were derived to the first degree of approximation. A comparison was made with different existing exponential estimators in literature. The improvement of this estimator over the existing exponential estimators was shown through an empirical study.**

Keywords: *Auxiliary Variable, Bias, Mean Square Error, Exponential Estimator, Factor-type Estimator.*

I. INTRODUCTION

It is well known fact that the use of auxiliary information at the estimation stage improves the precision of estimates of the population mean of characteristic under study. Classical ratio, product and linear regression estimators are good examples in this context. If the study variable Y is positively correlated with auxiliary variable X, the ratio method of estimation introduced by Cochran (1940) is more applicable in practice while the product estimator advocated by Murthy (1964) is more useful when the study variable Y is negatively correlated with auxiliary variable X. later on, statisticians concentrated their attention to develop modified ratio and product estimators. Such modified estimators are generally developed either using one and more unknown constants or introducing a convex linear combination of sample and population means of auxiliary characteristic with unknown weights. In both the cases, optimum choices of unknown parameters are made by minimizing the mean square error of modified estimators so that they become more efficient than the conventional ones. In defining modified estimators based on unknown parameters, actually a class of estimators were developed which include a number of classical estimators. Hartley and Ross (1954) Goodman and Hartley (1958), Williams (1963), Tin (1965), Srivastava (1966), Walsh (1970), Ray et al (1979), Srivenkataramana and Tracy (1979), Vos (1980) and Srivenkataramana(1980), Isaki (1983), Singh and Singh (2007),Singh (2015), Singh et al (2015) did some other remarkable works in this direction.

In the sequence of suggesting modification over classical ratio and product estimators, the suggestion of exponential methods of estimation of population mean of study variable was first proposed by Bahl and Tuteja (1991), when the linear relationship between study variable and auxiliary variable is not strong.

Further, their work was extended; the survey statisticians have developed and are developing the exponential type estimators for different sampling situations. Exponential type estimators were discussed by Sharma and Tailor (2010), Upadhyaya et al. (2011), Solanki et al. (2012), Singh et al (2014),
Monika and Kumar (2015), and Kadilar (2016) and so on.

In this paper, a generalized class of factor-type exponential estimator has been suggested for estimating finite population mean of characteristic under study.

II. NOTATION

Let Y_i denotes the value of characteristic under study for the i^{th} unit in population of size N (i=1, 2. . . N). and X_i denotes the value of auxiliary characteristics for the i^{th} unit in population.

Then;

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of characteristic under study.
$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$; The population mean of auxiliary variable.
$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$; The population mean square of characteristic under study.
$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$; The population mean square of auxiliary variable.
$C_Y = \frac{S_Y}{\bar{Y}}$; The coefficient of variation of characteristic under study.

$C_X = \frac{S_X}{\bar{X}}$; The coefficient of variation of auxiliary variable.
$\rho = \frac{S_{XY}}{S_X S_Y}$; The correlation coefficient between the value of auxiliary variable and value of characteristic under study.

Let a sample of size n been drawn by method of simple random sampling without replacement. Further let y_i and x_i denote the values of characteristic under study and auxiliary characteristic respectively which is included in the sample at i^{th} draw (i=1, 2. . . n).
Now,
$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$; The sample mean of study characteristic. $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$; The sample mean of auxiliary characteristic.

III. EXISTING ESTIMATORS

Singh and Shukla (1987) suggested conventional factor-type estimator which is applicable when correlations between the study and auxiliary variables are either positive or negative, as;

$$t_{ss} = \bar{y}\left[\frac{(A+C)\bar{X} + fB\bar{x}}{(A+fB)\bar{X} + C\bar{x}}\right] \quad (1)$$

where;
$f = \frac{n}{N}$

$A = (d-1)(d-2)$
$B = (d-1)(d-4)$
$C = (d-2)(d-3)(d-4)$
Therefore $d = 1, 2, 3, and\ 4$.

$$Bias[t_{ss}] = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[(b^2 - ab)C_X^2 + (a-b)\rho C_X C_Y\right] \quad (2)$$

$$MSE[t_{ss}] = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_Y^2 + (a^2+b^2)C_X^2 + 2(a-b)\rho C_X C_Y - 2abC_X^2\right] \quad (3)$$

Bahl and Tuteja (1991) modified classical ratio and product estimators and were the first who proposed the exponential ratio and product type estimators as;

$$\bar{y}_{Re} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (4)$$

The Bias and MSE of the estimator $\bar{y}_{Re}$ is given by;

$$Bias[\bar{y}_{Re}] = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}C_X^2 - \frac{1}{2}\rho C_X C_Y\right] \quad (5)$$

$$MSE[\bar{y}_{Re}] = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_Y^2 + \frac{1}{4}C_X^2 - 2\rho C_X C_Y\right] \quad (6)$$

And the exponential product- type estimator as;

$$\bar{y}_{Pe} = \bar{y}\exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) \quad (7)$$

The Bias and MSE of the estimator $\bar{y}_{Pe}$ is given thus;

$$Bias[\bar{y}_{Pe}] = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}C_X^2 + \frac{1}{2}\rho C_X C_Y\right] \quad (8)$$

$$MSE[\bar{y}_{Pe}] = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_Y^2 + \frac{1}{4}C_X^2 + 2\rho C_X C_Y\right] \quad (9)$$

Respectively.
Sharma and Tailor (2010) considered the exponential dual to ratio type estimator of finite population mean in simple random sampling as;

$$t_{ST} = \bar{y}\exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \quad (10)$$

where;
$\bar{x}^* = (1+g)\bar{X} - g\bar{x}$

$$Bias[t_{ST}] = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}gC_X^2 - \frac{1}{2}g\rho C_X C_Y\right] \quad (11)$$

$$MSE[t_{ST}] = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_Y^2 + \frac{g^2}{4}C_X^2 - g\rho C_X C_Y\right] \quad (12)$$

where; $g = \frac{n}{N-n}$.

Upadhyaya et al. (2011) proposed a modified exponential ratio type estimator and shown that the mean square error of t_{Re} is equal to the MSE of the usual linear regression estimator. The proposed estimator is given as;

$$t_{Re} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+(\alpha-1)\bar{x}}\right) \quad (13)$$

$$Bias[t_{Re}] = \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{1}{\alpha}C_X^2 - \frac{1}{\alpha}2\rho C_X C_Y - \frac{1}{2\alpha^2}C_X^2\right] \quad (14)$$

$$MSE[t_{Re}] = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2 + \frac{1}{\alpha^2}C_X^2 - \frac{1}{\alpha}2\rho C_X C_Y\right] \quad (15)$$

Singh et al. (2014) suggested ratio and product type exponential estimators of population mean $\bar{Y}$ using Bahl and Tuteja (1991) as the predictive estimators of $\bar{Y}$. The proposed estimators are respectively given thus;

$$t_{Re}^* = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\bar{X}-\bar{x})}{N(\bar{X}-\bar{x})-2n\bar{x}}\right)\right] \quad (16)$$

$$Bias[t_{Re}^*] = \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{3}{8}C_X^2 - \frac{1}{2}\rho C_X C_Y\right] \quad (17)$$

$$MSE[t_{Re}^*] = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2 + \frac{C_X^2}{4}(1-4C)\right] \quad (18)$$

And,

$$t_{Pe}^* = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\bar{X}-\bar{x})}{N\bar{X}+(N-2n)\bar{x}}\right)\right] \quad (19)$$

$$Bias[t_{Pe}^*] = \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{3}{8}C_X^2 + \frac{1}{2}\rho C_X C_Y\right] \quad (20)$$

$$MSE[t_{Pe}^*] = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2 + \frac{C_X^2}{4}(1+4C)\right] \quad (21)$$

where $C = \rho\frac{C_Y}{C_X}$ in (18) and (21)

Monika and Kumar (2015) suggested an unbiased exponential product type estimator for population mean under the framework of simple random sampling using the knowledge of single auxiliary variable as;

$$\bar{y}_{SKPe} = [\bar{y} - k(t-1)] \quad (22)$$

where; $\boldsymbol{t} = exp\left(\frac{N\bar{X}-n\bar{x}}{N-n} - \bar{X}\right)$.

Therefore;

$$\bar{y}_{SKPe} = \bar{y} - k\left\{\exp\left(\frac{N\bar{X}-n\bar{x}}{N-n} - \bar{X}\right) - 1\right\} \quad (23)$$

The MSE of the estimator $\bar{y}_{SKPe}$ is given by;

$$MSE[\bar{y}_{SKPe}] = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{matrix} C_Y^2 + k^2\left(\frac{n}{N-n}\right)^2 C_X^2 \\ +2\left(\frac{n}{N-n}\right)k\rho C_X C_Y \end{matrix}\right] \quad (24)$$

Where; $k = -4, -2, 0, 2, 4$.

Following Bahl and Tuteja (1991) and Singh et al. (2009), a modified exponential type estimator for estimating finite population mean using single auxiliary information in simple random sampling was proposed by Kadilar (2016) as;

$$\bar{y}_{kPR} = \bar{y}\left[\frac{\bar{x}}{\bar{X}}\right]^{\alpha}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad (25)$$

$$Bias[\bar{y}_{kPR}] = \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{matrix} \frac{3}{8}C_X^2 + \frac{\alpha(\alpha-1)}{2}C_X^2 - \frac{\alpha}{2}C_X^2 \\ +\alpha\rho C_X C_Y - \frac{1}{2}\rho C_X C_Y \end{matrix}\right] \quad (26)$$

$$MSE[\bar{y}_{kPR}] = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2 + \frac{C_X^2}{4} + 2\alpha\rho C_X C_Y + \rho C_X C_Y\,\alpha^2 C_X^2 + \alpha C_X^2\right] \quad (27)$$

IV. Proposed Estimator

In follow up of the above works and motivated with the generic nature of exponential estimators and factor-type estimators, a generalized class of factor-type exponential estimator for population mean has been proposed as;

$$t_{FT}^* = \bar{y}\left[\frac{(A+C)\bar{X}+fB\bar{x}}{(A+fB)\bar{X}+C\bar{x}}\right]^{\alpha}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad (28)$$

where;

f, A, B, C and D are as defined in (1) and α is constant.

V. Bias and Mean Square Error (MSE) of t_{FT}^*

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To obtain the bias and mean square error, let us suppose;

$\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$,

Thus;

$E(e_0) = E(e_1) = 0$, $E(e_0^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_Y^2$,

$E(e_1^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_X^2$, $E(e_0e_1) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho C_X C_Y$

By substituting in (28) and simplifying, we have;

$$t_{FT}^* - \bar{Y} = \bar{Y}\left[\begin{matrix} 1 + e_0 + \alpha(a-b)e_1 - \frac{e_1}{2} + \alpha(a-b)e_0e_1 - \\ \frac{e_0e_1}{2} - \alpha(ab-b^2)e_1^2 + \frac{\alpha(\alpha-1)}{2}(a-b)^2e_1^2 - \frac{\alpha(a-b)}{2}e_1^2 + \frac{3}{8}e_1^2 \end{matrix}\right] \quad (29)$$

The bias of the proposed estimator t_{FT}^* to terms of first degree of approximation can be obtained by taking the expectation of (29) and substituting,

$$Bias[t_{FT}^*] = \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)$$

$$\begin{bmatrix} \frac{3}{8}C_X^2 + \frac{\alpha(\alpha-1)}{2}(a-b)^2C_X^2 - \frac{\alpha(a-b)}{2}C_X^2 \\ -\alpha(ab-b^2)C_X^2 + \alpha(a-b)\rho C_X C_Y - \frac{1}{2}\rho C_X C_Y \end{bmatrix} \quad (30)$$

Squaring both sides of (30), then taking expectation and substituting, we obtain the MSE of the estimator t_{FT}^* to terms of first degree of approximation as;

$$MSE[t_{FT}^*] = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)$$
$$\begin{bmatrix} C_Y^2 + \frac{1}{4}C_X^2 + \alpha^2(a-b)^2C_X^2 - \alpha(a-b)C_X^2 \\ +2\alpha(a-b)\rho C_X C_Y - \rho C_X C_Y \end{bmatrix} \quad (31)$$

By differentiating (31) with respect to α and equate to zero, we get the optimum value of α, as;

$$\alpha = \frac{C_X - 2\rho C_Y}{2(a-b)C_X} \quad (32)$$

By substituting the optimum value of α in (31), we obtain the minimum MSE of t_{FT}^* as;

$$MSE[t_{FT}^*]_{min} = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)C_Y^2(1-\rho^2) \quad (33)$$

This is the MSE of the linear regression estimator for population mean. Hence for optimum value of α, the proposed estimator t_{FT}^* is equally efficient as the linear regression estimator.

Table 1: Members of the proposed generalized class of factor-type exponential estimator.

Estimators	**d**
$\bar{y}\left[\frac{\bar{X}}{\bar{x}}\right]^\alpha \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	1
Kadilar (2016) $\bar{y}\left[\frac{\bar{x}}{\bar{X}}\right]^\alpha \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	2
Srivenkataramana (1980) exponential-type $\bar{y}\left[\frac{\bar{X}-\bar{x}f}{\bar{X}-\bar{X}f}\right]^\alpha \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	3
Bahl and Tuteja (1991) $\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	4

VI. Efficiency Comparison

In this section, we have found some theoretical efficiency conditions under which the proposed estimator performs better than the other relevant existing estimators by comparing the generalized class of factor-type exponential estimator with other existing estimators.

(i) By (3) and (31), $MSE[t_{ss}] \geq MSE[t_{FT}^*]$ if

$$\min\left(-1 + \frac{C_X - 4\rho C_Y + 2\sqrt{2\rho C_X C_Y}}{-2C_X}\right) \leq \alpha$$
$$\leq \max\left(1 + \frac{C_X + 2\sqrt{2\rho C_X C_Y}}{-2C_X}\right) \quad (34)$$

For the range of α given in (34), the proposed estimator t_{FT}^* is better than t_{ss}

(ii) By (6) and (31), $MSE[\bar{y}_{Re}] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{C_X - \sqrt{\rho C_X C_Y}}{-C_X}\right) \leq \alpha \leq \max\left(\frac{-2\rho C_Y + \sqrt{\rho C_X C_Y}}{-C_X}\right) \quad (35)$$

For the range of α given in (35), the proposed estimator t_{FT}^* is better than $\bar{y}_{Re}$

(iii) By (9) and (31), $MSE[\bar{y}_{Pe}] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{-2\rho C_Y - \sqrt{\rho C_X C_Y}}{-C_X}\right) \leq \alpha \leq \max\left(\frac{C_X + \sqrt{\rho C_X C_Y}}{-C_X}\right) \quad (36)$$

For the range of α given in (36), the proposed estimator t_{FT}^* is better than $\bar{y}_{Pe}$

(iv) By (12) and (31), $MSE[t_{ST}] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{(1-g)C_X + 2\sqrt{g\rho C_X C_Y} - 4\rho C_Y}{-2C_X}\right) \leq \alpha \leq$$
$$\max\left(\frac{(1+g)C_X - 2\sqrt{g\rho C_X C_Y}}{-2C_X}\right) \quad (37)$$

For the range of α given in (37), the proposed estimator t_{FT}^* is better than t_{ST}

(v) By (15) and (31), $MSE[t_{Re}^*] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{C_X - \sqrt{cC_X^2}}{-C_X}\right) \leq \alpha \leq \max\left(\frac{-2\rho C_Y + \sqrt{cC_X^2}}{-C_X}\right) \quad (38)$$

For the range of α given in (38), the proposed estimator t_{FT}^* is better than t_{Re}^*

(vi) By (21) and (31), $MSE[t_{Pe}^*] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{-2\rho C_Y - \sqrt{cC_X^2}}{-C_X}\right) \leq \alpha \leq \max\left(\frac{C_X + \sqrt{cC_X^2}}{-C_X}\right) \quad (39)$$

For the range of α given in (39), the proposed estimator t_{FT}^* is better than t_{Pe}^*

(vii) By (24) and (31), $MSE[\bar{y}_{SKPe}] \geq MSE[t_{FT}^*]$ if

$$\min\left(\frac{C_X+2\left(k\left(\frac{n}{N-n}\right)C_X+\sqrt{2\left(\frac{n}{N-n}\right)k\rho C_X C_Y}\right)}{2(a-b)C_X}\right)\leq\alpha\leq$$

$$\max\left(\frac{C_X-2\left(k\left(\frac{n}{N-n}\right)C_X+\sqrt{2\left(\frac{n}{N-n}\right)k\rho C_X C_Y+2\rho C_Y}\right)}{2(a-b)C_X}\right) \qquad (40)$$

For the range of α given in (40), the proposed estimator t_{FT}^{*} is better than $\bar{y}_{SKPe}$

VII. Empirical Study

We consider two different real data sets to numerically evaluate the performances of the proposed and the existing estimators considered in this paper.

Population I. Source: Sukhatme and Chand (1977); the study variable Y is the apple trees of bearing age in 1964 and the auxiliary Variable X is the bushels harvested in 1964.

Population II. Source: Steel and Torrie (1960); the study variable Y is the log of Leaf burn in sacks and the auxiliary variable X is the chlorine percentage.

Table 2: Statistics of Populations

Parameters	Population I	Population II
N	200	30
n	20	6
$\bar{Y}$	1031.82	0.6860
$\bar{X}$	2934.58	0.8077
ρ	0.93	-0.4996
C_Y	1.59775	0.7001
C_X	2.00625	0.7493
f	0.1	0.2

* $f=\frac{n}{N}$ (sampling fraction).

Table 3: Percent Relative Efficiency of Proposed Estimator at different values of α and d

α	Population I				Population II			
	d=1	d=2	d=3	d=4	d=1	d=2	d=3	d=4
-1	39.03272	95.75998	302.8615	---	133.0384	19.2995	74.6855	---
-0.6	80.04341	295.2318	352.1128	---	110.5567	28.07093	66.02319	---
-0.2	226.6345	726.1952	409.7609	---	70.22867	43.29222	58.37028	---
0	441.6901	441.6901	441.6901	441.6901	54.9124	54.9124	54.9124	54.9124
0.2	726.1952	226.6345	475.469	---	43.29222	70.22867	51.68783	---
0.6	295.2318	80.04341	546.9816	---	28.07093	110.5567	45.89067	---
1	95.75998	39.03272	618.9906	---	19.2995	133.0384	40.87634	---

Table 4: PRE of existing and proposed estimators of population I and II

Existing Estimators		Population I PRE	Population II PRE
$\bar{y}$		100	100
$\bar{y}_{Re}$		441.6901	54.9124
$\bar{y}_{Pe}$		39.03272	133.0384
t_{ST}		112.3958	82.96892
t_{Re}^{*}		441.6901	54.9124
t_{Pe}^{*}		39.03272	133.0384
$\bar{y}_{SKPe}$	k=-4	365.7282	31.10507
	k=-2	178.9385	54.9124
	k=0	100	100
	k=2	62.62237	133.0384
	k=4	42.56276	92.93071
t_{SS}	d=1	20.35722	92.930
	d=2	414.6585	31.10
	d=3	78.18785	124.3
	d=4	100	100
Proposed Estimator t_{FT}^{*}		***740.1925***	***133.2623***

VIII. Conclusion

In the paper, we have succeeded in proposing a generalized class of factor-type exponential estimator for positively and negatively correlated data, for estimating population mean of study variable y, when information is available on single auxiliary variable under the framework of simple random sampling scheme. Upon drawing the observations from the efficiency comparison in section VI and the results in tables 3 and 4, it is evident that the proposed estimator t_{FT}^{*} is better than the existing estimators t_{ss}, $\bar{y}_{Re}$, $\bar{y}_{Pe}$, t_{ST}, t_{Re}^{*}, t_{Pe}^{*}, and $\bar{y}_{SKPe}$ as it has larger percent relative efficiency (PRE) than all these estimators. Therefore the proposed estimator t_{FT}^{*} should be preferred for the estimation of population mean.

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