A New Tree-Parameter Inverse Exponential Distribution with Applications

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Abstract — **In this paper, we propose a new three-parameter Inverse Exponential Distribution called the Transmuted Exponential Inverse Exponential Distribution (TE-IED). Some statistical properties of this new distribution such as survival function, moment, moment generating function, quantile function, and order statistic were derived and presented in an explicit form. The maximum likelihood method is employed to estimate the distribution parameters. We prove empirically by using two real-life datasets that the new model provides adequate fits as compared to other competing models in modeling asymmetric datasets.**

Keywords: *Inverse Exponential distribution, Reliability function, Maximum Likelihood, Order statistics.*

I. Introduction

In recent years, there are numerous continuous distributions for modeling lifetime data such as Weibull, exponential, Lindley, gamma, and the likes. The quality of the procedures used in a statistical analysis depends heavily on the assumed probability distributions. Due to this, there have been different approaches used to generate a new family of classical distributions along with relevant statistical methods. However, some of the existing real-life data do not follow any of the existing models or distributions.

Exponential distribution has been widely used in modeling time between events and is considered an important distribution in the Poisson process. The Exponential distribution is memoryless and has a constant failure rate making it inappropriate for modeling real-world problems. Despite its common uses, a negative point of the distribution is the limited shape of its Hazard Rate Function (HRF) that can only be constant which makes the model unfit for real-life problems with bathtub failure rates (Singh et al., 2013).

Keller and Kamath (1982) proposed an alternative model to exponential distribution known as Inverse exponential distribution which is capable of modeling real-life phenomena with bathtub failure rates. However, a negative point of this model is that it is inappropriate in modeling a highly skewed dataset (Abouammoh and Alshingiti (2009)). Details about its applications can be found in (Oguntunde et al., 2014, Oguntunde and Adejumo 2015).

Attempts to increase the flexibility and skewness of the Inverse exponential distribution lead to the extensions of Inverse exponential distribution which are available in the literature such as Transmuted Kumaraswamy-inverse exponential distribution (Yahaya and Mohammed, 2017), Transmuted Kumaraswamy-inverse exponential distribution (Yahaya and Mohammed, 2017); On Exponentiated Kumaraswamy-Inverse Exponential Distribution (Abba et al., 2017), Exponentiated Transmuted Inverse Exponential Distribution with Application (Mohammed and Yahaya, 2019), properties and applications of a two-parameter inverse exponential distribution with a decreasing failure rate (Ieren and Abdullahi 2020).

In this paper, we present a new generalization of Inverse exponential distribution called the Transmuted Exponential-Inverse Exponential Distribution (TE-IED) by using the most recent family of distributions known as the Transmuted Exponential-G family proposed by (Mohammed and Ugwuowo 2020. The usefulness of this new TE-IE distribution was demonstrated using two real-life data to evaluate its flexibility over some of the existing models.

II. Research Methodology

The Definition of the new model

Based on the work of Mohammed and Ugwuowo (2020), the cumulative distribution function (CDF) and the probability density function (PDF) of the TE-G family are defined as;

$$F(x;\lambda,\theta) = [1-(1-G(x;\xi))^{\lambda}][1+\theta(1-G(x;\xi))^{\lambda}] \quad (1)$$

$$f(x;\lambda,\theta) = \lambda g(x;\xi)(1-G(x;\xi))^{\lambda-1}[1-\theta+2\theta(1-G(x;\xi))^{\lambda}] \quad (2)$$

where $G(x;\xi)$ and $g(x;\xi)$ are cdf and pdf of any continuous distribution respectively,

$$\lambda > 0 \text{ is the scale parameter and } -1 \leq \theta \leq 1 \text{ is the transmuted parameter.}$$

The CDF and PDF of Inverse Exponential distribution are respectively given as;

$$G(x) = e^{-\frac{\alpha}{x}} \quad (3)$$

$$g(x) = \frac{\alpha}{x^2} e^{-\frac{\alpha}{x}} \quad (4)$$

By substituting (3) and (4) in (1) and (2), we have the CDF and PDF of the TE-IED as;

$$F(x;\lambda,\theta) = \left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right] \quad (5)$$

$$f(x;\lambda,\theta) = \frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right] \quad (6)$$

where, $\lambda, \alpha > 0$ *are the scale parameter and* $-1 \leq \theta \leq 1$ *is the transmuted parameter.*

Model Validity Check

Proposition 1: The TE-WD is a well valid density function

$$\int_0^{\infty} f(x;\lambda,\theta)dx = 1 \quad (7)$$

Proof:

$$\int_0^{\infty} \frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]dx = 1$$

$let\ y = \left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}, \frac{dy}{dx} = -\lambda\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\frac{\alpha}{x^2}e^{-\frac{\alpha}{x}}; as\ x \to 0, y \to 1\ and\ x \to \infty, y \to 0$

$$-\int_1^0 \frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\frac{dy}{\lambda\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\frac{\alpha}{x^2}e^{-\frac{\alpha}{x}}}$$

$$\int_0^1 [1-\theta+2\theta y]dy = (y-\theta y+\theta y^2)_0^1 = 1-\theta+\theta = 1 \quad (prove)$$

Graphical representation of PDF and CDF of TE-IED

The graphs of the CDF and PDF of TE-IE distributions for different values of the parameters are shown in the Figure 1 and 2.

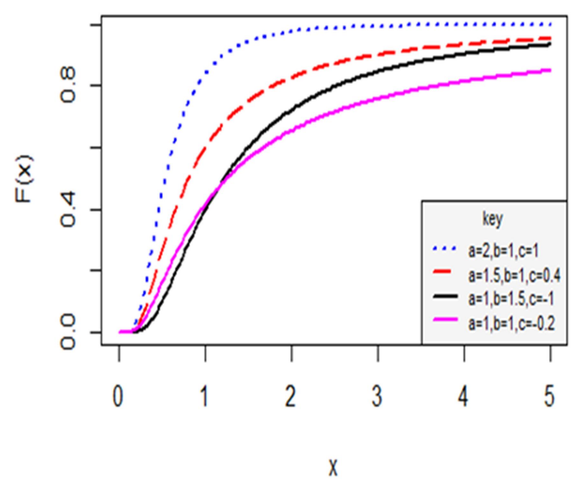

Figure 1: The cdf plot of TE-IED

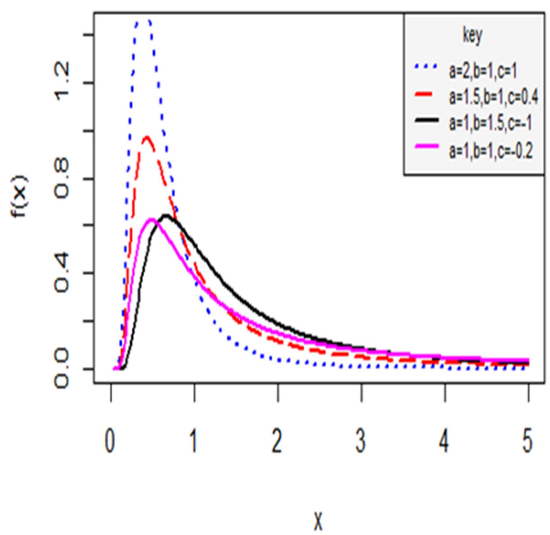

Figure 2: The pdf plot of TE-IED

Moments and Moment generating function

Definition 1. Let X be a random variable with the TE-IE density function (6).
Proposition 2: The r^th^ Moments of TE-IED is given by;

$$E(x^r) = \lambda\alpha^r(1+j)^{r-1}w_j\Gamma(1-r) \quad (8)$$

Where,

$$w_j = \sum_{j=0}^{\infty}(-1)^j\left((1-\theta)\binom{\lambda-1}{j} + 2\theta\binom{2\lambda-1}{j}\right)$$

Proof:

$$E(x^r) = \int_0^{\infty} x^r f(x)dx$$

But,

$$f(x;\lambda,\theta) = (1-\theta)\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1} + 2\theta\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda-1}$$

So, $\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1} = \sum_{j=0}^{\infty}(-1)^j\binom{\lambda-1}{j}e^{-\frac{\alpha j}{x}}$

And $\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda-1} = \sum_{j=0}^{\infty}(-1)^j\binom{2\lambda-1}{j}e^{-\frac{\alpha j}{x}}$

Therefore,

$$f(x;\lambda,\theta) = (1-\theta)\frac{\lambda\alpha}{x^2}\sum_{j=0}^{\infty}(-1)^j\binom{\lambda-1}{j}e^{-\frac{\alpha(1+j)}{x}} + 2\theta\frac{\lambda\alpha}{x^2}\sum_{j=0}^{\infty}(-1)^j\binom{\lambda-1}{j}e^{-\frac{\alpha(1+j)}{x}}$$

$$f(x;\lambda,\theta) = \sum_{j=0}^{\infty}(-1)^j\left((1-\theta)\binom{\lambda-1}{j} + 2\theta\binom{2\lambda-1}{j}\right)\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha(1+j)}{x}}$$

Where,

$$w_j = \sum_{j=0}^{\infty}(-1)^j\left((1-\theta)\binom{\lambda-1}{j} + 2\theta\binom{2\lambda-1}{j}\right)$$

$$f(x;\lambda,\theta) = w_j\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha(1+j)}{x}}$$

$$E(x^r) = w_j\int_0^{\infty} x^r\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha(1+j)}{x}}dx$$

$let, y = \frac{\alpha(1+j)}{x}, x = \frac{\alpha(1+j)}{y}, as x \to 0, y \to \infty \; and \; x \to \infty, y \to 0, \frac{dy}{dx} = \frac{-\alpha(1+j)}{x^2},$

$$dx = \frac{-x^2dy}{\alpha(1+j)}$$

$$E(x^r) = -w_j\int_{\infty}^{0}\left(\frac{\alpha(1+j)}{y}\right)^r\frac{\lambda\alpha}{x^2}e^{-y}\frac{x^2dy}{\alpha(1+j)}$$

$$E(x^r) = \lambda\alpha^r(1+j)^{r-1}w_j)\int_0^{\infty}(y)^{1-r+1}e^{-y}dy$$

$$\boldsymbol{E(x^r) = \lambda\alpha^r(1+j)^{r-1}w_j\Gamma(1-r)} \quad \text{(prove)}$$

Proposition 3: The Moment Generating Function of TE-IED is given as;

$$\boldsymbol{M_x(t) = \sum_{r=0}^{\infty}\frac{t^r}{r!}\left(w_j\lambda\alpha^r(1+j)^{r-1}\Gamma(1-r)\right)} \quad (9)$$

Where,

$$w_j = \sum_{j=0}^{\infty}(-1)^j\left((1-\theta)\binom{\lambda-1}{j} + 2\theta\binom{2\lambda-1}{j}\right)$$

Proof:

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx}f(x)dx$$

Using power series

$$e^{tx} = \sum_{r=0}^{\infty}\frac{(tx)^r}{r!}$$

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f(x)dx = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^{\infty} x^r f(x)dx = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(x^r)$$

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \left(w_j \lambda \alpha^r (1+j)^{r-1} \Gamma(1-r)\right)$$

Where,

$$w_j = \sum_{j=0}^{\infty} (-1)^j \left((1-\theta)\binom{\lambda-1}{j} + 2\theta\binom{2\lambda-1}{j}\right)$$

Survival function of TE-IED

The survival function of the model is given as;

$$S(x;\lambda,\theta) = 1 - F(x)$$

$$S(x;\lambda,\theta) = 1 - \left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]$$

$$S(x;\lambda,\theta) = 1 - \left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} - \left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} - \theta\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda}\right]$$

$$S(x;\lambda,\theta) = \left[-\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} + \left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} + \theta\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda}\right]$$

$$S(x;\lambda,\theta) = \theta\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda} - \theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} + \left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} \quad (10)$$

Hazard function of TE-IED

The hazard function of the model is given as;

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]}{\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{2\lambda} - \theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} + \left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}}$$

$$h(x) = \frac{\frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]}{\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} - \theta + 1}$$

$$h(x) = \frac{\frac{\lambda\alpha}{x^2} e^{-\frac{\alpha}{x}}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]}{\left(1-e^{-\frac{\alpha}{x}}\right)\left(\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda} - \theta + 1\right)} \quad (11)$$

The plots of the survival and hazard function of the TE-IED are respectively displayed in figure 3 and 4 for selected values $a = \lambda, b = \alpha \; and \; c = \theta$

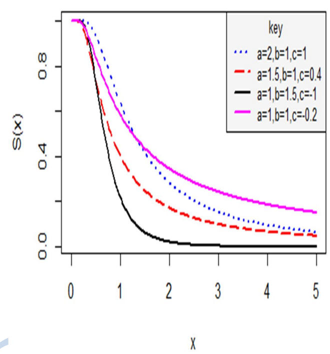

Figure 3: Survival function of TE-IED

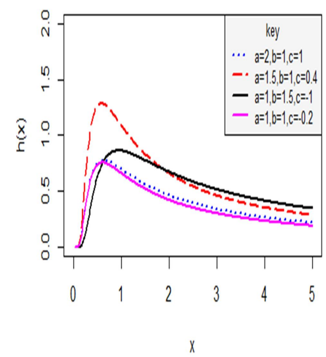

Figure 4: Hazard function of TE-IED

Quantile Function of TE-IED

Proposition 4: the Quantile Function of TE-IED is given as;

$$x_u = \frac{-\alpha}{ln\left\{1-\left(\frac{(\theta-1)+\sqrt{(\theta-1)^2+4\theta(1-u)}}{2\theta}\right)^{\frac{1}{\lambda}}\right\}} \quad (12)$$

Proof:

$$G(x) = e^{-\frac{\alpha}{x}}$$

$$u = e^{-\frac{\alpha}{x}}$$

$$lnu = -\frac{\alpha}{x}$$

$$x = -\frac{\alpha}{lnu}$$

But $u = 1 - \left(\frac{(\theta-1)+\sqrt{(\theta-1)^2+4\theta(1-u)}}{2\theta}\right)^{\frac{1}{\lambda}}$ (Mohammed and Ugwuowo 2020).

$$x_u = \frac{-\alpha}{ln\left\{1-\left(\frac{(\theta-1)+\sqrt{(\theta-1)^2+4\theta(1-u)}}{2\theta}\right)^{\frac{1}{\lambda}}\right\}}$$

Distribution of order statistic

Given that $X_1, X_2, \ldots, X_n$ is a random sample from TE-IED and $X_{1:n}, X_{2:n}, \ldots, X_{i:n}$ denote the associated order statistic from the same sample. We have the pdf of the ith order statistic as;

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\sum_{j=0}^{n-i}(-1)^j \binom{n-i}{j} f(x)F(x)^{i+j-1} \quad (13)$$

By replacing the pdf and cdf of TE-IED in (13) we have,

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\sum_{j=0}^{n-i}(-1)^j \binom{n-i}{j}\left(\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)\left(\left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)^{i+j-1} \quad (14)$$

Furthermore, the pdf of the minimum order statistic X_1 and the maximum order statistic X_n of the TE-IED are respectively given as;

$$f_{1:n}(x) = n\sum_{j=0}^{n-1}(-1)^j \binom{n-1}{j}\left(\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)\left(\left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)^{j} \quad (15)$$

and

$$f_{n:n}(x) = nf(x)F(x)^{n-1}$$

$$f_{1:n}(x) = n\left(\frac{\lambda\alpha}{x^2}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\left[1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)\left(\left[1-\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\left[1+\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right]\right)^{n-1}$$
(16)

Estimation of unknown parameters of TE-IED

Let $X_1, X_2, \ldots, X_n$ be a sample of size n independent and identically distributed random variables from the TE-IED with unknown parameters λ, θ and α.

The likelihood function of the TE-IED is given as;

$$L(X|\lambda,\theta,\alpha) = (\lambda\alpha)^n\prod_{i=0}^{n}\left(x^{-2}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}\right)e^{-\sum_{i=0}^{n}\frac{\alpha}{x}}\prod_{i=0}^{n}\left(1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right)$$

Let the log-likelihood function be $ll = L(X|\lambda,\theta,\alpha)$, we have,

$$ll = nlog\lambda + nlog\alpha - \alpha\sum_{i=0}^{n}\frac{1}{x} - 2logx + (\lambda-1)\sum_{i=0}^{n}log\left(1-e^{-\frac{\alpha}{x}}\right)+\sum_{i=0}^{n}log\left(1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right) \quad (17)$$

By differentiating (17) with respect to each of the parameters and equate the result to zero we have,

$$\frac{\delta ll}{\delta\lambda} = \frac{n}{\lambda}+\sum_{i=0}^{n}log\left(1-e^{-\frac{\alpha}{x}}\right)+2\theta\sum_{i=0}^{n}\frac{\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}ln\left(1-e^{-\frac{\alpha}{x}}\right)}{\left(1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right)} = 0 \quad (18)$$

$$\frac{\delta ll}{\delta\alpha} = \frac{n}{\alpha}-\sum_{i=0}^{n}\frac{1}{x}+(\lambda-1)\sum_{i=0}^{n}\frac{\frac{1}{x}e^{-\frac{\alpha}{x}}}{\left(1-e^{-\frac{\alpha}{x}}\right)}+ 2\theta\lambda\sum_{i=0}^{n}\frac{\frac{1}{x}e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda-1}}{\left(1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right)} = 0 \quad (19)$$

$$\frac{\delta ll}{\delta\theta} = \sum_{i=0}^{n}\frac{2\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}-1}{\left(1-\theta+2\theta\left(1-e^{-\frac{\alpha}{x}}\right)^{\lambda}\right)} = 0 \quad (20)$$

Solving for the solution of the non-linear system of equations in (18), (19) and (20) will give the maximum likelihood estimates (MLEs) of parameters λ, θ and α.

III. Analysis and Results

In this section, the goodness-of-fit of these distributions is To assess the flexibility of TE-IED, two real-life datasets were used. Some goodness-of-fit statistics for this distribution are compared with other competitive models and the MLEs of the model parameters are equally determined.

The model selection is carried out based upon the value of the log-likelihood function calculated at the MLEs, Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan Quin Information Criterion (HQIC). Furthermore, we equally compute other measures such as Anderson- Darling (A*), Cramer- Von Mises (W*) Statistic, and Kolmogorov-Smirnov (K-S) statistics. The details about the statistics A*, W*, and K-S are discussed in Chen and Balakrishnan (1995). Note: The model with the smallest value of these measures is considered to be the best among the competing models. The required computations are carried out using the R package "AdequacyModel" which is freely available from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf.

Dataset I: The first data set represents the times of failures and running times for a sample of devices from an eld-tracking study of a larger system. The data set has been previously studied by Meeker and Escobar (1988); Merovci and Elbatal (2015). The data set has thirty (30) observations and they are as follows:

2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66.

Dataset II: The data set represents the life of fatigue fracture of Kevlar 373/epoxy subjected to constant pressure at 90 percent stress level until all had failed. The dataset has been previously studied by Barlow et al. (1984); Andrews and Herzberg (1985); Abdul-Moniem and Seham (2015); Mohammed and Ugwuowo (2020). It has seventy-six (76) observations and they are as follows:

0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960.

Table 1: Summary of the dataset I

Min.	Max.	Mean	Variance	Skewness	Kurtosis
0.02	3.00	1.77	1.3233	-0.2840	1.4536

Table 2: MLEs with standard errors in parenthesis for the DatasetI

No.	Distribution	λ	α	θ	β
1	TE-IE	0.6159 (0.1176)	0.1367 (0.0474)	−0.8353 (0.1634)	–
2	GIE	–	0.1784 (0.0555)	–	0.4953 (0.1113)
3	IE	–	0.3122 (0.0570)	-	–

Table 3: Goodness-of-Fit Statistics for the Dataset I

No.	Distribution	$-ll$	AIC	BIC	$CAIC$	$HQIC$
1	TE-IE	61.4734	128.9468	133.1504	129.8699	130.2916
2	GIE	65.0736	134.1471	136.9495	134.5916	135.0436
3	IE	70.6309	143.2617	144.6629	143.4046	143.71

Table 4: The A*, W*, K-S Statistic for Dataset I

No.	Distribution	w^*	A^*	KS
1	TE-IE	0.5872	3.1917	0.2572
2	GIE	0.6419	3.5149	0.2874
3	IE	0.6357	3.4709	0.4372

Table 5: Summary of dataset II

Min.	Max.	Mean	Variance	Skewness	Kurtosis
0.0251	9.0960	1.9590	2.4774	1.9796	8.1608

Table 6: MLEs with standard errors in parenthesis for Dataset II

No.	Distribution	λ	α	θ	β
1	TE-IE	0.9761 (0.1361)	0.4248 (0.0833)	−0.8580 (0.0887)	–
2	GIE	–	0.5226 (0.0923)	–	0.7905 (0.1251)
3	IE	–	0.6248 (0.0717)	-	–

Table 7: Goodness-of-Fit Statistics for Dataset II

No.	Distribution	$-ll$	AIC	BIC	$CAIC$	$HQIC$
1	TE-IE	151.0667	308.1333	315.1255	308.4666	310.9277
2	GIE	161.9691	327.9382	332.5997	328.1026	329.8011
3	IE	163.1015	329.1344	330.5337	328.257	329.1344

Table 8: The A*, W*, K-S Statistic for Dataset II

No.	Distribution	w^*	A^*	KS
1	TE-IE	0.9132	5.3004	0.2021
2	GIE	1.2376	7.0137	0.2707

3	IE	1.2059	6.8515	0.2900

Table 1 and Table 5 provide the descriptive statistics for the dataset I and II respectively while in Table 2 and Table 6, we provided the estimates of the parameters for the TE-IED and the competing distributions for the dataset I and II respectively. The results of Table 3 and Table 7 show that the Transmuted Exponential Inverse Exponential (TE-IE) distribution has the smallest values of the goodness-of-fit statistics and Table 4 and Table 8 show that the Transmuted Exponential Inverse Exponential (TE-IE) distribution has the smallest values of the W*, A*, and K-S values. Therefore, the TE-IE model is considered as the best among the competing distributions for the dataset I and II respectively.

IV. Concluding Remarks

Here, we proposed a new distribution, the so-called Transmuted Exponential Inverse Exponential (TE-IE) distribution which extends the Inverse Exponential distribution in the analysis of real datasets. An evident reason for generalizing a classical distribution is that the generalized form provides larger flexibility in modeling real data. The study provides some statistical properties of the distribution such as survival function, moment, moment generating function, quantile function, and order statistic. The estimation of parameters is approached by the method of maximum likelihood. We prove empirically by using two real-life datasets that the new model provides adequate fits as compared to other competing models in modeling asymmetric datasets.

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