A Hybridized Linear-Ratio Estimator for Successive Sampling on Two Occasions

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Abstract **— This study proposed efficient linear ratio estimation technique in successive sampling schemes. The proposed estimator was obtained through mathematical expectation and statistical assumptions to derived an unbiased estimate of mean (μ), minimum variance (σ^2) and Relative efficiency comparison (REC). Real life datasets from National Population Commission (NPC) on national censuses conducted in Nigeria in 1991 and 2016 were employed for analysis. The study concluded that the Proposed hybridized linear-ratio estimator is more efficient than the conventional ones in term of precision. The proposed estimator is therefore recommended for use in successive sampling scheme.**

Keywords: *Successive Sampling, Estimators, Hybridization, Population census.*

I. Introduction

Successive sampling is used extensively in applied sciences, sociology and economic researches. Many survey these days are repetitive in character. Government agencies like the National Bureau of Statistics and other research based institutes collect information regularly on the same population to estimate some population parameters for current occasion. When the same population is sampled repeatedly, it is said that a first sample has been taken (on one occasion) from a population of N units and a second sample is to be taken (on another occasion) on the same population, there is thus an opportunity of making use of the information contained in the first sample. Therefore, the problem is determine how best to learn from the past experience and use it for improving precision of future estimates. Estimates can be made not only for the existing time period (current estimates) but also of the change that has taken place since the previous occasion and of the average over a given period.

The problem of sampling on Two Successive Occasions with a partial replacement of sampling units was first considered by [1] in a survey of farm data.
The approach that was used for estimation of current population mean, further extended to develop general theory of estimation in repeated surveys for the current population variance. For the first time in successive sampling, [2] considered the problem of estimating the population variance on current occasion. [3] proposed an estimator by using a linear combination of available sample variances for estimating the current population variance based on matched and unmatched portions of the samples at both occasions and most recently, [4] suggested a class of estimators for estimation of finite population variance on current occasion. [5] use unistage sampling over two occasion using SRSWOR and regression estimator was applied in obtaining current estimates with one auxiliary variable

The aim of the study therefore, is to propose efficient estimator techniques in successive sampling schemes.

II. Methodology

This study focused on measure of precision, which is a function of the variance for the four estimators under the repetitive survey. It also centered on checking precision on estimate and percentage gain in relative efficiency, over each occasion under study.

A. Notations and Meaning

N - is the population size
n - is the sample size taken on the first occasion
m - is the number of matched or retained units from the first occasion and used as part of second occasion
u - is the number of unmatched or a fresh unit on the second occasion from the remaining unit of the population
ρ - is the correlation coefficient between the matched units of x and y
λ - is the proportion of matched or retained units

θ - is the proportion of unmatched or new units
σ^2- is the pooled variance of S_x^2 and S_z^2
S_x^2 - is the sample variance of units (x) on first occasion
$\bar{x}_{1u}$- is the sample mean of unmatched units of x from first occasion.
$\bar{x}_{1m}$ - is the sample mean of matched units of x from first occasion.
$\bar{y}_{2u}$ - is the sample mean of unmatched units of y from second occasion.
$\bar{y}_{2m}$- is the sample mean of matched units of y from second occasion.
$\bar{z}_{1u}$ is the sample mean of unmatched units of z from first occasion.
$\bar{z}_{2m}$- is the sample mean of matched units of z from second occasion.
$\bar{z}_{2u}$ - is the sample mean of unmatched units of z from second occasion.
$\bar{z}_{1m}$ - is the sample mean of matched units of z from first occasion.
$\bar{X}$ - is the population mean of x units.
$\bar{Y}$ - is the population mean of y units.
$\bar{Z}$ - is the population mean of z units.

B. **Derivation of Estimate, Variance and Relative Efficiency of the Sampling Scheme**

This section discusses the derivation of the formula for estimate, variance and relative efficiency of the estimator.

1. Simple Estimator (SE_{st})
   a) The unbiased Simple Estimator of mean for first and second occasion respectively
      i) $\mu_{1S\ st} = \bar{X}$
      $$= \left(\frac{u\bar{x}_u + m\bar{x}_m}{n}\right) \quad (1)$$
      ii) $\mu_{2SE_{st}} = \bar{Y}$
      $$= \left(\frac{u\bar{y}_u + m\bar{y}_m}{n}\right) \quad (2)$$
   b) i) The estimate of this change, $\hat{\Delta}_{(SE_{st})}$, is $\hat{\Delta}_{(SE_{st})} = \bar{Y} - \bar{X}$
      $$\hat{\Delta}_{(SE_{st})} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u) \quad (3)$$
      ii) Variance of Estimate of change
      $$\hat{\Delta}_{SE_{st}} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u)$$
      $$V(\hat{\Delta}_{SE_{st}}) = 2(1 - \lambda\rho)\,{\sigma^2}/{n} \quad (4)$$
   c) i) The Estimate of Sum
      $$\hat{\Sigma}_{SE_{st}} = \bar{X} + \bar{Y}$$
      $$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u) \quad (5)$$
      ii) Variance of the Sum
      $$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u)$$
      $$v(\hat{\Sigma}_{SE_{st}}) = 2(1 + \lambda\rho)\,{\sigma^2}/{n} \quad (6)$$
2) Linear Estimator (LE_{st})
   a) The linear estimator for $\hat{\mu}_{1L}$ and $\hat{\mu}_{2L}$ can also be sought from the form.
      i) $\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1 - d)\bar{x}_{1u}$
      Where *b* and *d* are constant
      $$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{y}_{2u} - \bar{y}_{2m}) + \lambda\bar{x}_{1m} + \theta(1 - \rho^2\theta)\bar{x}_{1u}\} \quad (7)$$
      ii) $\hat{\mu}_{2LE_{st}} = a\,(\overline{x_{1u}} - \overline{x_{1m}}) + C\,\overline{y_{2m}} + (1 - C)\,\overline{y_{2u}}$.
      Where *a* and *c* are constant
      $$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{x}_{1u} - \bar{x}_{1m}) + \lambda\bar{y}_{2m} + \theta(1 - \rho^2\theta)\bar{y}_{2u}\} \quad (8)$$
   b) i) The estimate of change
      $$\hat{\Delta}_{LE_{st}} = \hat{\mu}_{2L} - \hat{\mu}_{1L}$$
      $$= \frac{1}{(1-\rho\theta)}[\theta(1 - \rho)(\bar{y}_{2u} - \bar{x}_{1u}) + \lambda(\bar{y}_{2m} - \bar{x}_{1m})] \quad (9)$$
      ii) Variance of Estimate of change
      $$V(\hat{\Delta}_{LE_{st}}) = \frac{2(1-\rho)}{(1-\rho\theta)}\,{\sigma^2}/{n} \quad (10)$$
   c) i) The estimate of Sum
      $$\hat{\Sigma}_{LE_{st}} = \hat{\mu}_{2L} + \hat{\mu}_{1L}$$
      $$= \frac{1}{(1+\rho\theta)}[\lambda(\bar{x}_{1m} + \bar{y}_{2m}) + \theta(1 + \rho)(\bar{x}_{1u} + \bar{y}_{2u})] \quad (11)$$
      iii) Variance of Sum
      $$V(\hat{\Sigma}_{TSO_L}) = \frac{2(1+\rho)}{(1+\rho\theta)}\,{\sigma^2}/{n} \quad (12)$$

C. **Criteria for Selection**

Consider a population containing of N unit. Let a character under study on first (second) occasion be denoted by *x and y*. It is assumed that the information on auxiliary variable z is available on the first as well as on the second occasion. We consider the population to be large enough and the sample size is constant on each occasion. Using sample random without replacement (SRSWOR), we select a sample of size n on the first occasion of these n units, a sub-sample of size $m = n\lambda$ is retained on the second occasion. This sub sample is supplemented by selecting of $u = (n - m) = n\mu$ units afresh from the units that were not selected on the first occasion.

Following [16], [17] and [21] methods, we use

$$\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1 - d)\bar{x}_{1u} \quad (13)$$
$$\mu_{2_{LE_{st}}} = a(\bar{x}_{1u} - \bar{x}_{1m}) + c\bar{y}_{2m} + (1 - c)\bar{y}_{2u} \quad (14)$$

We Proposed a ratio estimator $\bar{z}$ on the both occasion which is based on a sample of size m common to both the occasion and is given by

$$\mu_{1_{PRATE_{st}}} = b\left(\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + d\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + (1 - d)\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} \quad (15)$$

$$\mu_{2PRATE_{st}} = a\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + c\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + (1-c)\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} \quad (16)$$

where a, b, c and d are constants

- ✓ To determine the value of constants a, b, c and d.
- ✓ Find the variance of $\mu_{1PRATE_{st}}$ and $\mu_{2PRATE_{st}}$ then take the derivatives with respect to constants a, b, c and d, therefore equate the resulting equations to zero to obtain a, b, c and d.

Hence,

a) The Proposed Ratio Estimator of the means are:

i) $\mu_{1PRATE_{st}} = \frac{cH-M}{G}\left(\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + \frac{GF-ME}{GD-HE}\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + \left(\frac{GD+ME-HE-GF}{GD-HE}\right)\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}$

ii) $\mu_{2PRATE_{st}} = \frac{CH-M}{G}\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + \frac{GF-ME}{GD-HE}\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \left(\frac{GD+ME-HE-G}{GD-HE}\right)\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z}$

where

$$C = \frac{GF-ME}{GD-HE}$$
$$D = \frac{A_3A_4}{\theta\lambda} - \frac{A_0k^2B_3}{n}$$
$$E = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda}$$
$$F = \frac{A_5A_6}{\theta\lambda}$$
$$G = \frac{A_1A_2}{\theta\lambda} - \frac{A_0k^2B_1}{n}$$
$$H = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda}$$
$$M = \frac{A_9A_{10}}{\theta\lambda}$$

Also, where

$$A_0 = \frac{1}{\lambda^2} - \frac{1}{\theta\lambda} + \frac{1}{\theta^2}$$
$$A_1 = 4 - R_1^{-2} - 2R_1^{-1}\rho$$
$$A_2 = 1 - 2R_1^{-2} - 2R_1^{-1}\rho$$
$$A_3 = 1 + 2R_2^{-2} - 3R_2^{-1}\rho$$
$$A_4 = 1 - 2R_2^{-2}$$
$$A_5 = 1 + 3R_2^{-2} - 3R_2^{-1}\rho$$
$$A_6 = R_2^{-2} + R_2^{-1}\rho$$
$$A_7 = R_1^{-1}R_2^{-1} - R_2^{-1}\rho - \rho$$
$$A_8 = 2R_1^{-1}R_2^{-1} - 2R_2^{-1}\rho - R_1^{-1}\rho$$
$$A_9 = 2R_2^{-2} - 2R_2^{-1}\rho - R_1^{-1}\rho + \rho$$
$$A_{10} = R_2^{-2} - R_2^{-1}\rho$$

where

$$B_1 = R_1^{-2} - R_1^{-1}\rho + \rho^2$$
$$B_2 = R_1^{-1}\rho - R_1^{-1}R_2^{-1} + R_2^{-1}\rho - \rho^2$$
$$B_3 = R_2^{-2} - 2R_2^{-1}\rho + \rho^2$$

Where

$$R_1 = \frac{\bar{z}}{\bar{x}}$$
$$R_2 = \frac{\bar{z}}{\bar{y}}$$
$$k^2 = \frac{S^2x}{\bar{z}^2}$$
$$\rho = \rho xy = \rho xz = \rho yz$$

b) Estimate of change:

i) The estimate required $\hat{\Delta}_{PRATE_{st}} = \mu_{2PRATE_{st}} - \mu_{1PRATE_{st}}$

Hence

$$\Delta_{PRATE_{st}} = \frac{1}{G(GD-HE)}\left\{\left[(G^2F + GFH - GME - MGD)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[\left((G^2D - G^2F + GME + MGD - GHE - GFH\right)\right)\left(\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} - \frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}\right)\right]\right\} \quad (17)$$

ii) Variance of Estimate of change

$$V(\Delta_{PRATE_{st}}) = \frac{2}{G(GD-HE)}\left\{(G^2D - GHE) - (G^2F + GFH - GME - MGD)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \quad (18)$$

c) Estimate of Sum:

i) The estimate required $\Sigma_{PRATE_{st}} = \hat{\mu}_{2PR\quad st} + \hat{\mu}_{1PRATE_{st}}$

$$\Sigma_{PRATE_{st}} = \frac{1}{G(GD-H\ \ )}\left\{\left[(G^2F + MGD - GME - GFH)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[(G^2D - G^2F + GFH + GME - MED - GHE)\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} + \frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z}\right)\right]\right\} \quad (19)$$

ii) Variance of Estimate of Sum

$$V(\Sigma_{PRATE_{st}}) = \frac{2}{G(GD-H\ \ )}\left\{(G^2D - MED) - (G^2F - GME)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \quad (20)$$

4) Relative Gain in Precision

a) Relative gain in precision of change in Linear Estimator(LE_{st}) over Simple Estimator(SE_{st})

$$R\left(\frac{\Delta_{SE_{st}}}{\hat{\Delta}_{LE_{st}}}\right) = \frac{v(\hat{\Delta}_{SE_{st}}) - v(\hat{\Delta}_{LE_{st}})}{v(\hat{\Delta}_{LE_{st}})}$$

Hence,

$$R\left(\frac{\Delta_{SE_{st}}}{\hat{\Delta}_{LE_{st}}}\right) = \frac{\rho^2\lambda\theta}{1-\rho} \quad (21)$$

b) Relative gain in precision of sum in Linear Estimator(LE_{st}) over Simple Estimator(SE_{st})

$$R\left(\frac{\hat{\Sigma}_{SE_{st}}}{\hat{\Sigma}_{LE\,st}}\right) = \frac{v(\hat{\Sigma}_{SE_{st}}) - v(\hat{\Sigma}_{SE\,st})}{v(\hat{\Sigma}_{LE\,ST})}$$

Hence,

$$R\left(\frac{\widehat{\Sigma}_{SE_{st}}}{\widehat{\Sigma}_{LE_{st}}}\right)=\frac{\rho^{2}\lambda\theta}{1+\rho} \qquad (22)$$

c) Relative gain in precision of change in Proposed Ratio Estimator ($PRATE_{st}$) over Simple Estimator (SE_{st})

$$R\left(\frac{\Delta_{SE_{st}}}{\Delta_{PRATE_{st}}}\right)=\frac{V(\Delta_{SE_{st}})-V(\Delta_{PRATE_{st}})}{v(\Delta_{PRATE_{st}})}\times 100\%$$

Hence,

$$=\frac{G(1-\lambda\rho)(GD-H\ \ )}{(G^{2}D-GHE)-(G^{2}F+GFH-GME-MGD)}\left(\frac{\rho}{\lambda}\right) \qquad (23)$$

d) Relative gain in precision of sum in Proposed Ratio Estimator ($PRATE_{st}$) over Simple Estimator (SE_{st})

$$R\left(\frac{\Sigma_{SE_{st}}}{\Sigma_{PRATE_{st}}}\right)=\frac{v(\Sigma_{SE_{st}})-v(\Sigma_{PRATE_{st}})}{v(\Sigma_{PRATE_{st}})}\times 100\%$$

Hence,

$$=\frac{G(1+\lambda\rho)(GD-HE)}{(G^{2}D-MED)-(G^{2}F-GMF)}\left(\frac{\rho}{\lambda}\right) \qquad (24)$$

III. Analysis and Results

Empirical Study I

The data from census conducted in Nigeria in [6] (1st occasion) and [7] (2nd occasion) was considered. We define the variables X and Y as the population of males and female in each state and Z is defined as the auxiliary variable which is the total number of households in each state and T is the total number of both sex. This information is oresented in Table 1 for simplicity.

Table 1: Discriptions of the Variables

NOTATION	DESCRIPTION OF VARIABLES
1st Occasion	Population census report in 1991
2nd Occasion	Population census report in 2006
X_1	Population of male in 1st occasion (M_1)
Y_1	Population of female in 1st occasion (F_1)
Z_1	No of household in 1st occasion (H_1)
X_2	Population of male in 2nd occasion (M_2)
Y_2	Population of female in 2nd occasion (F_2)
Z_2	No of household in 2nd occasion (H_2)
T_1	Population of both sexes in 1st occasion (B_1)
T_2	Population of both sexes in 2nd occasion (B_2)

Table 2: Selection Procedure

		Parameters					
S/N	**All States**	**N**	**n**	**u**	**m**	**θ**	**λ**
1	36 states & FCT	37	25	13	12	0.52	0.48

Table 3: Estimate of Mean by Country

			Previous Estimate				Current Estimate			
S/N	**COUNTRY**	**ESTIMATOR**	**H_1**	**M_1**	**F_1**	**B_1**	**H_2**	**M_2**	**F_2**	**B_2**
1.	Nigeria	SEst	514,569.24	1,290,049.76	1,284,269.56	2,573,927.5	689,922	1,731,831	1,695,244.44	3,433,199.16
		LEst	537,66.21	1,765,011.32	1,659,34.5	2,957,811.6	765,325	2,361869.2	2,791,349.5	3,980,981.1
		PRATEst	685,231.11	2,310,789.6	2,012,176.7	3,557,918.6	1,067,387.6	2,989,120.3	2,621,520.7	4,759,240.5

Table 4: Estimate of Change and its Variance

			Change				Variances			
S/N	**COUNTRY**	**ESTIMATOR**	**H**	**M**	**F**	**B**	**H**	**M**	**F**	**B**
1.	Nigeria	SEst	175,352.8	441,781.24	410,974.9	859,271.64	186,321.2	165,321.3	432,32.5	876,532.5
		LEst	227,658.8	396,825.6	532,028	1,023,169.5	165,321.3	452,123.5	420,165.7	786.625.2
		PRATEst	382,156.5	678,321.7	609,344	1,201,321.8	95,321.4	311,211.7	300,184.6	605,321.9

Table 5: Estimate of Sum and its Variance

			Sum				Variances			
S/N	**COUNTRY**	**ESTIMATOR**	**H**	**M**	**F**	**B**	**H**	**M**	**F**	**B**
1.	Nigeria	SE_{st}	1,204,491.24	3,021,880.76	2,979,514	6,007,126.7	382,625.6	941,652.6	894,644	1,753,069
		LE_{st}	1,302,991.2	4,126,880.5	3,640,671	6,938,792.7	330,648.8	924,267.8	851,323.4	1,584,415.6
		$PRATE_{st}$	1,752,618.7	5,299,918.9	5,001,297	8,317,159.2	191,642.8	621,423.4	600,469.5	1,221,645.9

Table 6: Relative Gain in Precision (%)

			Change				Sum			
S/N	**COUNTRY**	**ESTIMATOR**	**H**	**M**	**F**	**B**	**H**	**M**	**F**	**B**
1.	**Nigeria**	**LE_{st}**	**12.7**	**9.5**	**2.9**	**11.4**	**15.7**	**1.9**	**5.1**	**10.6**
		$PRATE_{st}$	**95.5**	**49.5**	**44.0**	**44.80**	**99.7**	**51.5**	**49.0**	**43.5**

Table 7: Estimate of Change (Δ) with varying λ's and ρ's

ρ	$\lambda = 0.2$				$\lambda = 0.5$				$\lambda = 0.8$			
	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	159,346.7	166,112.3	177,811.7	182,321.6	186,723.6	193,321.1	265,391.2	478,321.7	196,321	211,321.6	345,391.6	503,316
0.15	135,111.7	141,211.8	156,212.5	166,378.5	175,352.8	178,521.5	227,658.8	382,156.5	186,371.6	191,211.9	285,345.7	485,368.8
0.25	121,321.8	139,321.1	149,316.7	158,379.6	169,321.9	172,321.4	216,381.9	365,491.6	179,321.5	186,841.9	281,496.7	467,496.3
0.35	120,116.8	132,233.7	138,321.6	149,891.7	163,867.1	169,341.6	202,341.7	342,369.9	172,411.5	179,382.1	278,327.9	455,351.8
0.45	119,321.5	129,321.1	133,678.9	138,321.9	159,321.1	162,381.7	199,381.8	335,419.7	166,321.7	175,841.6	269,329.4	441,867.2
0.55	117,381.6	126,481.8	130,781.6	136,891.7	154,311.8	161,321.6	190,384.6	321,321.6	159,311.9	171,321.9	256,794.5	436,321.4
0.65	115,678.6	123,321.7	128,321.9	132,799.6	151,869.1	159,785.8	186,321.5	302,311.5	156,281.6	169,847.6	248,991.2	429,321.8
0.75	110,321.6	120,785.6	125,321.6	130,311.7	149,321.9	152,314.6	179,311.6	295,314.6	147,321.9	163,321.5	236,321.6	427,321.1
0.85	108,567.1	111,321.8	121,321.9	128,869.5	138,891.2	149,385.7	170,311.6	281,345.7	138,319.6	160,876.9	229,316.9	420,691.7
0.95	102,321.7	109,887.8	112,369.8	120,221.6	132,776.8	142,328.9	168,321.7	276,388.8	136,311.5	156,331.7	220,321.6	401,321.9
1.00	97,321.6	102,321.6	109,369.8	112,321.5	129,321	138,326.4	156,321.9	251,321.9	130,816.6	149,321	199,371.9	398,321.6

Table 8: Estimate of Variance of Change (Δ) with varying λ's and ρ's

$N = 37\ n = 25$

ρ	$\lambda = 0.2$				$\lambda = 0.5$				$\lambda = 0.8$			
	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	197,491.1	176,491.2	171,991.7	107,419.6	204,311.8	188,794.5	185,321.9	110,341.9	216,317.9	199,381.7	190,421.4	120,416.1
0.15	186,321.5	165,321.9	160,381.61	95,411.9	197,345.3	176,471.9	171,691.4	107,381.9	201,341.6	192,321.9	185,461.2	110,341.1
0.25	162,321.5	159,321.1	146,321.1	80,317.6	186,821.3	165,321.3	160,111.3	95,321.4	196,321.6	185,411.7	171,321.9	107,321.6
0.35	179,334.6	162,99.7	142,311.8	76,915.4	184,321.4	163,661.2	165,998.4	97,481.4	191,411.7	179,497.4	169,391.6	101,321.1
0.45	177,637.1	159.441.7	140,917.9	74,321.5	179,841.6	162,591.2	162,111.9	92,441.9	186,321.5	168,559.7	164,321.9	97,381.7
0.55	174.391.4	157,816.9	138,416.7	72,491.6	177,736.1	159,331.6	158,321.7	85,441.7	180,416.7	164,321.7	160,321.7	92,321.1
0.65	171,321.5	150,881.9	132,781.6	70,321.6	174,934.6	157,778.4	153,811.7	79,991.9	176,161.8	162,871.8	158,389.8	85,321.9
0.75	163,311.9	147,991.9	130,617.9	67,911.8	171,896.7	150,311.9	149,661.8	64,331.9	170,321.7	159,321.6	153,894.5	79,891.6
0.85	159,341.6	143,678.8	126,321.5	63,511.6	163,481.9	147,921.7	145,811.9	60,349.8	165,497.9	155,321.7	149,381.7	64,321.8
0.95	152,319.7	138,391.6	121,341.9	61,348.9	159,221.7	143,621.1	140,321.1	54,995.5	160,321.5	150,001.1	145,321.1	60,321.7
1.00	149,321	130,117.9	119,345.5	59,811.6	152,378.3	138,321.9	135,321.9	51,321.7	154,949.5	149,321.9	140,783.1	54,321.1

Table 9: Table 10: Estimate of Sum (Σ) with varying λ's and ρ's

$N = 37\ n = 2$

Table 10: Estimate of Sum (Σ) with varying λ's and ρ's
$N = 37\ n = 25$

ρ	$\lambda = 0.2$				$\lambda = 0.5$				$\lambda = 0.8$			
	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	1,711,611.5	1,511,341.6	1,301,321.9	1,138,417.8	1,045,321.9	1,034,167.5	1,145,917.8	996,851	851,321	765,811	653,211	541,321
0.15	1,811,911.6	1,617,811.2	1,491,112.7	1,236,321.5	1,165,186.5	1,141,321.5	1,291,314.8	1,001,351.7	985,321	854,341	789,411	658,321
0.25	1,965,914.2	1,732,651.6	1,514,164.2	1,381,341.6	1,204,491.24	1,219,188.5	1,302,991.2	1,216,314.8	1,012,314	996,841	876,321	745,321
0.35	2,001,111.6	1,813,711.7	1,621,311.9	1,411,416.7	1,311,341.8	1,301,144.8	1,411,414.3	1,311,614.9	1,121,321.4	1,001,112.4	954,321	895,811
0.45	2,114,334.6	1,932,165.6	1,731,211.1	1,586,316.9	1,421,617.5	1,411,321.6	1,564,321.6	1,432,116.8	1,224,381.5	1,134,114.5	1,003,321.4	955,873
0.55	2,214,314.6	2,016,321.7	1,821,441.1	1,669,319.4	1,531,811.6	1,501,481.9	1,665,891.7	1,531,716.5	1,321,341.7	1,241,681.5	1,112,321.5	1,012,321.5
0.65	2,319,314.5	2,116,381.9	1,911,321.6	1,781,375.9	1,658,321.7	1,624,881.5	1,795,321.6	1,632,381.9	1,416,312.8	1,321,471.6	1,321,395.6	1,112,381.7
0.75	2,418,314.6	2,217,381.6	2,033,391.7	1,896,321.2	1,765,321.6	1,741,660.3	1,865,811.8	1,711,816.5	1,567,817.9	1,443,211.8	1,313,496.8	1,212,391.8
0.85	2,521,816.7	2,318,321.9	2,144,391.8	1,997,321.9	1,845,321.8	1,832,117.9	1,965,327.7	1,811,178.8	1,651,321.6	1,511,614.9	1,461,321.5	1,321,416.2
0.95	2,616,317.7	2,417,416.8	2,279,316.7	2,011,319.8	1,985,445.7	1,932,441.4	2,014,321.9	1,961,821.5	1,781,414.9	1,653,416.7	1,532,416.7	1,432,119.6
1.00	2,717,318.6	2,597,845	2,345,381.9	2,179,341.6	2,001,416.5	2,000,116.8	2,001,216.7	2,000,121.4	1,814,321.7	1,791,341.6	1,652,321.9	1,515,117.8

Table 11: Estimate of Variance of Sum (Σ) with varying λ's and ρ's
$N = 37\ n = 25$

ρ	$\lambda = 0.2$				$\lambda = 0.5$				$\lambda = 0.8$			
	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$	SE_{st}	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	461,321.3	365,361.8	296,791.7	165,321.8	365,321.8	311,414.6	295,821.6	171,321.9	354,351.8	305,311.8	299,861.8	159,341.9
0.15	470,321.3	377,311.7	305,311.6	174,811.4	378,311.6	320,116.9	305,178.5	181,179.6	365,517.3	316,411.6	306,179.8	164,311.8
0.25	481,341.6	387,321.6	318,771.9	186,791.4	382,625.6	330,648.8	310,569.6	191,642.8	376,519.6	321,411.5	317,411.9	175,821.6
0.35	492,481.8	397,321.0	324,111.6	198,321.5	394,411.8	341,117.9	324,511.1	200,119.4	386,317.7	331,481.7	328,817.6	183,411.9
0.45	513,118.9	401,831.6	331,416.5	200,116.7	400,114.8	352,311.4	335,891.6	210,219.5	395,321.8	345,381.6	337,661.6	194,399.8
0.55	521,321.3	412,871.8	342,117.8	211,321.8	412,321.9	366,516.6	341,417.9	221,321.7	400,119.9	358,916.5	348,118.7	204,311.9
0.65	532,417.6	423,314.9	354,161.2	221,416.9	422,391.6	379,811.7	356,819.9	235,311.8	410,121.6	366,811.8	357,311.8	217,821.9
0.75	544,311.6	434.816.8	365,381.6	234,314.6	439,381.7	381,656.9	366,314.5	246,321.8	420,321.7	377,411.3	366,381.6	225,119.6
0.85	552,311.8	445,317.1	378,114.8	245,981.7	447,821.6	398,778.1	375,391.7	257,451.9	436,311.8	389,111.4	375,781.7	238,124.5
0.95	563,217.9	456,811.7	388,818.9	255,817.9	458,011.7	405,811.8	384,951.6	267,811.6	447,381.9	398,118.5	384,891.6	249,321.6
1.00	571,321.8	476,812.8	391,811.8	268,111.8	469,812.9	410,911.9	391,652.8	278,618.7	458,312.0	400,321.7	39,816.7	251,321.8

Table 12: Relative Gain in precision of Sum (Σ) with varying λ's and ρ's

ρ	$\lambda = 0.2$			$\lambda = 0.5$			$\lambda = 0.8$		
	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	26.3	55.4	179.4	47.3	83.5	193.2	56.1	118.2	222.4
0.15	25.1	54.8	165.6	46.8	82.7	181.3	55.1	117.8	214.7
0.25	24.8	53.3	155.6	45.2	81.6	176.4	54.3	116.9	201.6
0.35	23.4	52.9	146.7	44.9	80.4	165.4	53.7	115.3	199.6
0.45	22.1	51.7	137.8	43.2	79.6	155.8	52.9	114.6	186.8
0.55	21.9	50.2	126.7	42.6	78.5	145.6	51.2	113.2	175.9
0.65	20.6	49.6	116.8	41.8	77.6	136.7	50.1	112.6	165.1
0.75	19.1	47.6	107.6	40.1	76.5	126.7	49.6	103.7	155.8
0.85	18.9	46.3	98.6	39.8	75.5	116.8	48.9	98.4	146.5
0.95	17.6	45.9	88.6	38.2	74.9	98.7	47.2	88.5	136.7
1.00	16.8	44.9	76.5	37.6	73.4	86.8	46.5	76.7	128.9

$N = 37\ n = 25$

IV. Discussion

The result of analysis presented in Table 1 – 12 can be summarized as follow:

i. Generally we have undertaken an investigation covering some estimators techniques in the context of devising efficient sampling strategies for successive sampling schemes with a view to derive the unbiased estimator of mean (μ) on two occasion (previous and current) using Simple Estimator (SE_{st}), Linear Estimator (LE_{st}), Ratio Estimator (RE_{st}), and Proposed Hybridized Linear-Ratio Estimator ($PHLRE_{st}$) and also to examine the Estimates of change (Δ) and Sum (Σ) respectively and establish minimum variance of the four estimators and to achieve the relative gain in precision of the estimators.

ii. Considering the census figure of the 36 states and FCT of Nigeria in [6] and [7] as population, details of populations and variables description is given in Table 1. The necessary parameters of populations for computing the estimators are given in Table 2. (using Random Number table as a selection procedure).

iii. From Table 3, it was observed that both previous and current Estimate sustained the expected population mean under the three variable of interest ($M_{1\alpha 2} F_{1\alpha 2}$ and $B_{1\alpha 2}$) and auxiliary variable ($H_{1\alpha 2}$) using Simple Estimator (SE_{st}), Linear Estimator (LE_{st}),Ratio Estimator (RE_{st}), and Proposed Hybridized Linear-Ratio Estimator ($PHLRE_{st}$) compare to actual population mean.

iv. From Table 4, it was shown that Estimate of change (Δ) in variables of interest (M, F, and B) and auxiliary variable H over the time interval of the census are unique and consistent considered the actual population value and expected population value. In the same vain it was observed that Proposed Hybridized Linear-Ratio Estimator ($PHLRE_{st}$) has least variance than other three estimators.

v. From Table 5, it was observed that Estimate of sum (Σ) in both variables of interest (M, F and B) and auxiliary variable (H) over a given time interval (1991 - 2006) were also unique and justifiable considered the actual value and expected value among the three Estimators.

vi. From Table 6, it was revealed that gain in efficiency exist in using Proposed Hybridized Linear-Ratio Estimator ($PHLRE_{st}$) than usual Ratio Estimator (RE_{st}) and Linear Estimator (LE_{st}) both at Estimate of change (Δ) and sum (Σ) most especially for auxiliary variable (H) with high reasonable gain in precision.

vii. From Table 7, it was observed that the correlation coefficient (p) increases as the estimate of change (Δ) in estimator decreases and proportion matched (λ) increase as the estimate of change (Δ) in estimator increasing with different varying λ's and ρ's.

viii. From Table 8, it was observed that for varying λ's and ρ's minimum variance as maximum precision is achieved as $\rho \to 1$ as well as $\lambda \to 0$. This implies that there is perfect positive relationship between first and second samplings occasions.

ix. From Table 9, it was observed that the relative or gain in precision was achieved when efficiency $\rho's \to 1$ and $\lambda's \to 0$ under estimate of change with subtended gain in $PHLRE_{st}$.

x. From Table 10, it was observed that the correlation coefficient (ρ) increases in each estimators increasing and proportion matched (λ) increases as the estimators decreases with different λ's and ρ's under the estimate of the sum(Σ).

xi. From Table 11, it was observed that for varying λ's and ρ's minimum variance as maximum precision is achieved as $\rho's \to 0$ and $\lambda's \to 1$ under the estimate of sum of variance.

xii. From Table 12, it was observed that the relative efficiency or gain in precision was achieved when $\rho's \to 0$ and $\lambda's \to 1$ under estimate of sum, with substantial gain in $PHLRE_{st}$.

V. CONCLUSION

In view of the above result, based on the available information, we concluded as follows.

1) For current estimates replacement of part of the sample on the current occasion will go a long way in improving the efficiency of the estimates required.
2) Attempt should be made to retain a greater portion of the samples on the current occasion to enhance the estimate of change.
3) Fresh sample should be taken on each occasion to facilities efficiency when estimating average over time.
4) Percentage gain in precision does not depend on sample size n (i.e. independent of sample size n).
5) Federal government should give priority to population census as stipulated in the constitution for interval of 10 years. Thus it is clear that our country has a long way to go and establish a tradition of conducting periodic census every ten years and allow it to be an administrative and

technical exercise devoid of politics of numbers, region or ethnicity. The overall acceptance of the census therefore should be viewed by all stakeholders in light of these historical facts with a view to strengthen the process of conducting census regularly and at definite intervals.

6) Both theoretical and empirical results of the study are sound, encouraging and of considerable practical importance.

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