Panel Data Analyses using Generalized Moments' Estimators and Empirical Likelihood Estimator

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***Abstract* — Generalized method of moments (GMM) estimation has been popular as a major tool for eliciting inference from different sets of data in econometrics in the last two decades. It encompasses most of the common estimation methods, such as maximum likelihood, ordinary least squares, instrumental variables, and two-stage least squares. The GMM approach is applicable to economic theory where orthogonality conditions that can serve as such moment functions occur as a result of optimization. Recent developments in empirical likelihood (EL) estimators are also discussed and applied to the analyses of econometric panel data for the purpose of comparison with the GMM estimators. The criteria used for comparison are the Mean Square Error (MSE), the Mean Absolute Error (MAE) and the Median Absolute Error (MedAE). Finally, the results from the simulated data showed that the EL estimators are more efficient when error term of the applied model is non-normal and the model is basically non-linear.**

Keywords - *Generalized Method of Moments(GMM), Empirical Likelihood (EL) Estimators, Panel Data, Optimization.*

I. INTRODUCTION

Efficient estimation of regression model is a crucial stage in model building. If the parameters of a regression model are efficiently computed, the inferences drawn from such model would generally be reliable. However, methods to adopt to estimate the parameters of regression models largely depend on the structure of the data at hand. While the method of Least Squares (LS) might be desirable it the data are cross-sectional, this method might be grossly inefficient if the panel or longitudinal data are involved especially when some of the assumptions that govern efficient use of LS method is violated by the data.

In an attempt to determine the goodness of some of the estimators of regression model, this study examines the performances of Empirical Likelihood (EL) and Generalized Method of Moments (GMM) for estimating regression model with panel data.

II. EMPIRICAL LIKELIHOOD (EL) AND GENERALIZED METHOD OF MOMENTS (GMM) ESTIMATORS

EL estimator can be thought of as the minimizer of the "likelihood" distance between the empirical distribution and the distribution supported on the sample, satisfying a given constraint. The empirical likelihood approach (EL) suggested by Owens et al (1988) and Owens (1991), Qin and Lawless (1994), and Mittelhammer et al. (2000) provides another way to estimating the unknown parameters in a moment equation. The moment equations can be interpreted as representing the expectation of the *M* dimensional unbiased vector estimating function

$$\mathbf{h}(p_t, p_{t+1}, S_t, \mathbf{z}_t, \alpha, c) = \left\{ \left[p_t - \frac{c}{S_t} - \alpha(a - 2bS_t)\left[p_{t+1} - \frac{c}{(aS_t - bS_t^2)} \right] \right] \mathbf{z}_t \right\}$$

(1)

the information was combined in the unbiased estimating functions with the concept of empirical likelihood to define an empirical likelihood function for (α, c). Maximizing the empirical likelihood function yields maximum empirical likelihood (MEL) estimates. The first-order asymptotic sampling properties of the MEL estimator are similar to those for parametric likelihood methods. The exposition follows Mittelhammer et al, (2000).

According to Mittelhammer et al (2000), the concept of empirical likelihood begins with the joint empirical probability distribution $\prod_{t=1}^{T} v_t$ that is supported on the sample data. The parameter v_t denotes the probability of observing the *t*th sample outcome, $\{p_t, p_{t+1}, S_t, \mathbf{z}_t\}$. To define the value of the empirical likelihood function for (α, c), the v_t are chosen to maximize $\prod_{t=1}^{T} v_t$, subject to the constraints defined by the moment conditions . Since the v_t's represent a probability distribution, the maximization problem is subject to the additional constraints $\sum_{t=1}^{T} v_t = 1$ and $v_t > 0 \ \forall t$. The maximization procedure assigns the maximum probability possible to the sample outcome actually observed, subject to the information represented by the moment equations. The moment equations link the data, the population distribution, and the parameters.

Using the empirical probabilities v_t, the moment equations can be represented empirically as the $(M \times 1)$ vector equation

$$\sum_{t=1}^{T} v_t \boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) = \sum_{t=1}^{T} v_t \left\{ \left[p_t - \frac{c}{S_t} - \alpha\left(a - 2bS_t\right)\left[p_{t+1} - \frac{c}{\left(aS_t - bS_t^2\right)} \right]\right] z_t \right\} = \mathbf{0},$$

(2)

with the observations ranging from 1 to *T*. Using a logarithmic transformation of $\prod_{t=1}^{T} v_t$ and scaling by $1/T$, the constrained maximization problem can then be defined as

$$\frac{1}{T}\ln\left(L_{EL}(\alpha, c; \boldsymbol{p}, \boldsymbol{p}_{+1}, \boldsymbol{S}, \boldsymbol{Z})\right) \equiv \max_{v}\left[\frac{1}{T}\sum_{t=1}^{T}\ln(v_t) \text{ s.t. } \sum_{t=1}^{T} v_t \boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) = 0 \text{ and } \sum_{t=1}^{T} v_t = 1\right].$$

(3)

The Lagrange function associated with the constrained maximization problem can be represented as

$$L(\mathbf{v}, \eta, \boldsymbol{\lambda}) \equiv \left[\frac{1}{T}\sum_{t=1}^{T}\ln(v_t) - \eta\left(\sum_{t=1}^{T} v_t - 1\right) - \boldsymbol{\lambda}'\sum_{t=1}^{T} v_t \mathbf{h}(p_t, p_{t+1}, S_t, \mathbf{z}_t, \alpha, c)\right]$$

(4)

To solve for (α, c), a specific functional form for the log-empirical likelihood must be discovered.

The first order conditions with respect to the v_t's are

$$\frac{\partial L(\mathbf{v}, \eta, \lambda)}{\partial v_t} = \frac{1}{T}\frac{1}{v_t} - \eta - \sum_{m=1}^{M} \lambda_m h_m(p_t, p_{t+1}, S_t, z_{mt}, \alpha, c) = 0, \quad \forall t.$$

(5)

Therefore,

$$\sum_{t=1}^{T} v_t \frac{\partial L(\mathbf{v}, \eta, \lambda)}{\partial v_t} = \frac{1}{T}T - \eta = 0. \quad (6)$$

With $\eta = 1$ and solving for the v_t from the first order conditions $\partial L / \partial v_t = 0$ yields the optimal weights v_t as a function of α, c and $\boldsymbol{\lambda}$:

$$v_t(\alpha, c, \lambda) = \left[T\left(\sum_{m=1}^{M} \lambda_m h_m(p_t, p_{t+1}, S_t, z_{mt}, \alpha, c) + 1\right)\right]^{-1}.$$

(7)

Therefore,

$$\sum_{t=1}^{T} v_t \boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) = \sum_{t=1}^{T} T^{-1}\left(\sum_{m=1}^{M} \lambda_m h_m(p_t, p_{t+1}, S_t, z_{mt}, \alpha, c) + 1\right)^{-1} \boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) = 0.$$

(8)

From (2.48), the multipliers $\boldsymbol{\lambda}$ are defined as a solution to an implicit function of (α, c),

$$\boldsymbol{\lambda}(\alpha, c) = \arg_{\lambda}\left[\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{1 + \boldsymbol{\lambda}'\boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c)}\right)\boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) = \mathbf{0}\right]$$

(9)

Substituting $\boldsymbol{\lambda}(\alpha, c)$ into (2.52) defines the optimal empirical probabilities evaluated at (α, c) as

$$v_t(\alpha, c, \lambda(\alpha, c)) = \left[T\left(\sum_{m=1}^{M} \lambda_m(\alpha, c) h_m(p_t, p_{t+1}, S_t, z_{mt}, \alpha, c) + 1\right)\right]^{-1}$$

.(10)

Finally, substitution of the optimal empirical probabilities into the objective function $\sum_{t=1}^{T}\ln(v_t)$ yields the expression for the log-empirical likelihood function evaluated at (α, c):

$$\ln\left(L_{EL}(\alpha, c, p_t, p_{t+1}, S_t, z_t)\right) = -\sum_{t=1}^{T}\ln\left[T\left(\boldsymbol{\lambda}(\alpha, c)'\boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c) + 1\right)\right]. (11)$$

The maximum empirical likelihood (MEL) estimator of (α, c) is defined by choosing the value of (α, c) that maximizes the log-empirical likelihood function.

Qin and Lawless (1994) and Mittelhammer *et al*. (2000) noted two principal ways in which the empirical likelihood solution may be computed. First, the optimal parameters (α, c) and the Lagrange multipliers $\boldsymbol{\lambda}$ may be simultaneously selected to maximize the empirical likelihood function. This problem is defined as:

$$L(\mathbf{v}, \eta, \lambda) \equiv -\frac{1}{T}\sum_{t=1}^{T} \ln\left[T\left(\lambda' \mathbf{h}\left(p_t, p_{t+1}, S_t, \mathbf{z}_t, \alpha, c\right) + 1\right)\right] \quad (12)$$

Qin and Lawless (1994) showed that the MEL estimator is consistent and asymptotically normal under general regularity conditions. The present example satisfies the conditions of the twice continuous differentiability of $\boldsymbol{h}(p_t, p_{t+1}, S_t, z_t, \alpha, c)$ with respect to (α, c) and the boundedness of $\boldsymbol{h}$ and its first and second derivatives, both in the neighborhood of the true parameter vector $(\alpha, c)_0$, and the requirement that the row rank of $E\left[\partial \mathbf{h}\left(p_t, p_{t+1}, S_t, \mathbf{z}_t, \alpha, c\right) / \partial(\alpha, c)\big|_{(\alpha, c)}\right]$ equal the number of parameters to be estimated. The covariance matrix $\sum$ of the limiting normal distribution had been consistently estimated as:

$$\hat{\Sigma} = \left[\left[\sum_{t=1}^{T} \hat{v}_t \frac{\partial \boldsymbol{h}(p_t, p_{t+1}, S_t, \boldsymbol{z}_t, \alpha, c)}{\partial(\alpha, c)}\Big|_{(\alpha,c)_{\mathrm{EL}}}\right]\left[\sum_{t=1}^{T} \hat{v}_t \boldsymbol{h}(p_t, p_{t+1}, S_t, \boldsymbol{z}_t, \alpha, c)\boldsymbol{h}(p_t, p_{t+1}, S_t, \boldsymbol{z}_t, \alpha, c)'\right]^{-1} \times \left[\sum_{t=1}^{T} \hat{v}_t \frac{\partial \boldsymbol{h}(p_t, p_{t+1}, S_t, \boldsymbol{z}_t, \alpha, c)}{\partial(\alpha, c)'}\Big|_{(\alpha,c)_{\mathrm{EL}}}\right]\right]^{-1}, \quad (13)$$

where the $\hat{v}_t$'s are the MEL estimates of the empirical probability distribution $\boldsymbol{\nu}$, using $\hat{\alpha}_{EL}, \hat{c}_{EL}$ and $\hat{\boldsymbol{\lambda}}_{EL} = \boldsymbol{\lambda}(\hat{\alpha}_{EL}, \hat{c}_{EL})$.

In recent years, one step estimators called "generalized empirical likelihood" (GEL) estimators (Smith, 1999) began to gain attention as theoretically attractive alternatives to GMM. These estimators are based on information theoretical considerations and include the empirical likelihood (Owen, 1991; Qin and Lawless, 1994) and exponential tilting (Kitamura and Kermanshah, 1983) estimators, together with an entire class of minimizers of certain divergence criteria, continuous updating CU (Hansen et al, 1996), and other members.

It has been established that the first order asymptotic properties of GEL estimators are identical to those of GMM estimators (Smith, 1999). Moreover, it turns out that GEL estimators have certain advantages related to second order asymptotic properties and thus are expected to have better finite sample behaviour. In particular, Bryan and Whitney (2000) found that in a cross sectional context, the GEL estimators do not have some components of the second order bias that are characteristic of GMM estimators resulting from estimating the optimal linear combination of moment conditions at the preliminary step. The empirical likelihood (EL) estimator is the most distinctive in this respect in that its bias is the smallest, and moreover, its bias corrected version is second order asymptotically efficient. This property makes the class of GEL estimators especially efficient in numerous stationary time series models typically estimated by GMM, with wide possibilities of selecting instruments.

This research work focuses on semi-parametric non-linear modelling of panel data when the normality assumption of the error term is violated. Multicollinearity among the predictors and the unobserved heterogeneity variable are also incorporated into the proposed model. Three estimators of semi-parametric models namely; Continuously Updating (CU), Empirical Likelihood (EL) and Exponential Tilting (ET) were employed using some smoothing kernel parameter values and compared with the Ordinary Least Square (OLS) and Generalised Method of Moments (GMM) estimators using the Mean Square Error (MSE), Mean Absolute Error (MAE) and Median Absolute Error (MedAE) criteria.

III. Methodology

A semi-parametric non-linear (SPNL) model that is applicable to fitting panel data was used with the incorporation of multicollinearity among the predictors and the latent variable under the violation of an error assumption structure. The error term of the model is non-normal. The model is given as equation (14)

$$y_{it} = \beta_0 e^{\beta_1\rho_1 X_{1it} + \beta_2\rho_2 X_{2it} + \beta_3\alpha_i + U_{it}}; \quad i = 1, \dots, n; \quad t = 1, \dots, T. \quad (14)$$

Thus,

$$log(y_{it}) = log\beta_0 + \beta_1\rho_1 X_{1it} + \beta_2\rho_2 X_{2it} + \beta_3\alpha_i + U_{it} \quad (15)$$

where,

y_{it} is the response variable,

X_{1it} and X_{2it} are the predictors,

β_0 is the intercept,
β_1 is the slope
U_{it} is the idiosyncratic error term,
α_i is the unobserved heterogeneity variable on U_{it},

$$\rho_1 = cor(X_{1it}, \alpha_i) \quad (16)$$

where ρ_1 is the correlation between the predictor, X_{1it} and the unobserved heterogeneity variable, α_i and

$$\rho_2 = cor(X_{2it}, \alpha_i) \quad (17)$$

where ρ_2 is the correlation between the predictor, X_{2it} and the unobserved heterogeneity variable, α_i.

The linearized model can be re-written as:

$$log(y_{it}) = log\beta_0 + \beta_1 X_{1it}^* + \beta_2 X_{2it}^* + \beta_3 \alpha_i + U_{it} \quad (18)$$

where, for matrix Z_1^* defined by

$$Z_1^* = [\underline{\alpha}, X_1] \otimes \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix},$$

X_1^* is the second column of Z_1^*.
Similarly, for matrix Z_2^* defined by

$$Z_2^* = [\underline{\alpha}, X_2] \otimes \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

X_2^* is the second column of Z_2^* and
$\otimes$ represents the kronecker product of the matrices.

3.3 Simulation Scheme

In order to simulate data for use in this thesis, the following schemes were designed for the generation of the panel data used for parameter estimation from the proposed model.

$$U_{it} \sim \exp(\theta) \quad (19)$$

$$X_{1it} \sim \exp(\theta) \quad (20)$$

$$X_{2it} \sim \exp(\theta) \quad (21)$$

$\alpha_i = 1$, if there is there exists the unobserved attribute

$\alpha_i = 0$, if the unobserved attribute is not present.

The following values were used for the Monte Carlo Simulation:

The Sample sizes and time points investigated are:
n = 20, n = 50, n = 100, n = 200 and n = 300; T = 5, T = 15, and T = 30 (21)

with the following values of the correlations among X_{1it}, X_{2it} and α_i

ρ = 0.1 and ρ = 0.8. (22)

Parameter estimations were replicated at 1000.

IV. Results of the Simulations

The following results were obtained from the simulated data.

Table 1: Mean Square Error (MSE) of the Results when T = 5

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.276334	0.988102	0.819613	0.608762	0.417197
	GMM	1.276444	0.988302	0.819613	0.608672	0.417197
	CUE	**1.204235**	0.998265	0.832657	0.615811	0.409761
	EL	1.276506	0.988926	0.820532	0.608356	0.404376
	ET	1.276394	**0.988042**	**0.81903**	**0.60816**	**0.40426**
0.8	LSE	5.81662	2.476	1.943	1.836	**1.406**
	GMM	5.711252	2.479197	**1.645623**	1.660778	1.41544
	CUE	5.436212	**1.462**	1.844952	1.666882	1.413772
	EL	5.301173	2.945781	1.847553	1.660835	1.416301
	ET	**5.298872**	2.57924	1.762599	**1.660739**	1.41358

Table2: Mean Absolute Error (MAE) of the Results when T = 5

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	**1.430703**	1.380909	**1.171252**	1.097486	**1.045386**
	GMM	**1.430703**	1.380909	**1.171252**	1.097486	1.088786
	CUE	1.463916	1.413074	1.190009	1.241396	1.215096
	EL	1.430878	1.380941	1.171329	1.097364	1.075464
	ET	1.43087	**1.380882**	1.171552	**1.097265**	1.081165
0.8	LSE	2.125321	1.695	1.488	1.416	1.284
	GMM	2.131742	1.671212	1.458731	1.418325	**1.164623**
	CUE	2.018099	1.613935	1.509351	1.430378	1.177663
	EL	**1.89901**	**1.609929**	**1.398725**	**1.368636**	1.314873
	ET	1.966659	1.671185	1.490888	1.418308	1.364813

Table 3: Median Absolute Error (MedAE) of the Results when T = 5

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	**1.395035**	1.319298	1.13636	1.028548	0.958548
	GMM	**1.395035**	1.319298	1.13636	1.028548	0.953348
	CUE	1.403067	1.325391	1.139646	1.035193	0.949458
	EL	1.395475	**1.318841**	**1.135171**	1.029078	**0.94469**
	ET	1.395064	1.319611	1.136032	**1.028462**	0.945222
0.8	LSE	2.582	2.241	1.323	**1.2627**	**1.1321**
	GMM	2.582178	2.239344	1.320135	1.265296	1.132853
	CUE	2.823572	2.532447	1.32416	1.27242	1.132917
	EL	**2.581663**	**2.226809**	1.321547	1.265369	1.132586
	ET	2.579863	2.238825	**1.317005**	1.264994	1.13296

Table 4: Mean Square Error (MSE) of the Results when T = 15

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.261144	**1.146923**	1.061055	1.021912	1.005927
	GMM	1.261144	**1.146923**	1.061055	1.021927	1.005927
	CUE	1.29889	1.179589	1.161566	1.138896	1.131896
	EL	**1.261137**	1.180427	1.060998	**1.021864**	**1.005864**
	ET	1.258371	1.146939	**1.060966**	1.021873	1.005873
0.8	LSE	5.49692	2.485	1.957	1.802	1.372
	GMM	5.410512	2.496197	1.658713	1.637878	**1.01544**
	CUE	5.084882	**1.466**	**1.627962**	**1.518982**	1.104472
	EL	5.070623	2.952551	1.861643	1.638835	1.026301
	ET	**5.068262**	2.5913	1.763609	1.657939	1.02358

Table 5: Mean Absolute Error (MAE) of the Results when T = 15

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.484829	1.334989	1.248486	0.830063	0.600063
	GMM	1.484829	1.334989	1.248486	0.830063	0.600063
	CUE	5.404786	1.362442	1.298517	0.857282	0.627282
	EL	1.484763	1.335027	**1.248405**	**0.830049**	**0.600049**
	ET	**1.484524**	**1.334912**	1.248645	0.830368	0.600368
0.8	LSE	2.159521	1.784	1.514	1.463	1.321
	GMM	2.212642	1.717912	1.469791	1.437176	**1.230023**
	CUE	2.102099	1.735635	1.520411	1.433381	1.242863
	EL	**1.95361**	**1.691229**	**1.412735**	**1.380366**	1.380213
	ET	2.024669	1.755235	1.496488	1.431328	1.429973

Table 6: Median Absolute Error (MedAE) of the Results when T =15

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.324502	**1.269871**	1.186344	1.038224	**0.838224**
	GMM	1.324502	**1.269871**	1.186344	1.008224	**0.838224**
	CUE	1.331551	1.278116	1.194936	0.994418	0.854418
	EL	**1.324113**	1.270042	1.186419	1.038393	0.838393
	ET	1.325229	1.270616	**1.185578**	**0.979509**	0.839509
0.8	LSE	**2.73**	**2.412**	1.435	**1.3189**	**1.1871**
	GMM	2.776578	2.424344	1.424935	1.322366	1.208393
	CUE	2.890272	2.574447	1.42316	1.32977	1.207947
	EL	2.776163	2.413809	1.426147	1.321439	1.207656
	ET	2.752363	2.423825	**1.415905**	1.322054	1.20796

Table 7: Mean Square Error (MSE) of the Results when T = 30

ρ	Estimator	Sample Size				
		20	50	100	200	300
0.1	LSE	**1.435222**	1.125783	**0.993589**	0.898495	0.875495
	GMM	**1.435222**	1.125783	**0.993589**	0.898495	0.875495
	CUE	1.437934	**1.115541**	1.014183	0.909147	0.896147
	EL	1.445748	1.125851	0.993674	0.898581	0.875581
	ET	1.44526	1.131275	0.993672	**0.898166**	**0.875166**
0.8	LSE	1.3947	**1.248**	**1.1511**	**1.1033**	**1.0531**
	GMM	1.394632	1.311094	1.203674	1.128333	1.083115
	CUE	1.513319	1.30012	1.203063	1.134654	1.083272
	EL	1.394486	1.301852	1.203632	1.128379	1.083272
	ET	**1.394369**	1.314909	1.203485	1.128296	1.083078

Table 8: Mean Absolute Error (MAE) of the Results when T = 30

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.627827	1.242042	**1.168945**	1.119331	1.090331
	GMM	1.627827	1.242042	**1.168945**	1.119331	1.090331
	CUE	1.62964	**1.240004**	1.186122	**1.107139**	**1.057139**
	EL	**1.627692**	1.242526	1.169065	1.119513	1.090513
	ET	1.630139	1.244678	1.169017	1.119783	1.090783
0.8	LSE	**1.772**	1.675	1.418	**1.316**	1.301
	GMM	1.772322	1.651212	1.418731	1.318325	**1.288623**
	CUE	1.818199	1.713935	**1.409351**	1.332378	1.301663
	EL	1.77209	**1.594929**	1.418725	1.318636	1.288873
	ET	1.773497	1.651185	1.416888	1.318308	1.288713

Table 9: Median Absolute Error (MedAE) of the Results when T = 30

ρ	Estimator	Sample Sizes				
		20	50	100	200	300
0.1	LSE	1.521861	1.178828	1.13636	0.930424	0.910424
	GMM	1.521861	1.178828	1.13636	**0.924236**	0.910424
	CUE	**1.511465**	**1.164758**	1.139646	0.931232	0.911232
	EL	1.521539	1.178627	**1.13517**	0.925544	**0.909544**
	ET	1.530193	1.179961	1.136032	0.927336	0.911336
0.8	LSE	1.682	1.621	1.323	**1.2627**	**1.1321**
	GMM	1.682178	1.609344	1.320135	1.265296	1.132853
	CUE	1.683572	1.655447	1.31716	1.27142	1.140217
	EL	1.681663	1.625819	1.321547	1.265369	1.132586
	ET	**1.679863**	**1.608825**	**1.317005**	1.264994	1.13296

V. Discussion of the Results

Tables 1 to table 9 showed that the Empirical Likelihood Estimators have the least errors of estimation more often than the Generalized Methods of Moments Estimators.

VI. Conclusion

The Empirical Likelihood Estimators performed better than the Generalized Methods of Moments Estimators in the estimation of parameters using a semi-parametric model from simulated sets of panel data.

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