A None Explosive Jump Process and Its Application

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Abstract — **Jump processes are stochastic processes whose trajectories have discontinuities called jumps that can occur at random times. None Explosive Jump Process otherwise known as pure jump process is a type of stochastic process that has discrete movements called jumps with random arrival times, other than continuous movement typically modeled as a simple or compound Poisson. In a non-explosive process, there are finite number of jumps at a finite interval. This work examines and compare if jump process model could be used to model sales data before, during and after Covid-19 pandemic of a gas plant in Enugu (NNPC DAAD Gas Station). The number of cylinders filled per month for year 2019 (before Covid-19), 2020 (during Covid-19) and 2021 (after Covid-19) were recorded in kilogram. Results from the analysis showed that the arrival pattern follow Poisson process since the mean and the variance of the arrival data are equal. The sales volumes are also analyzed and found out that it can be modelled with Log-normal distribution. This was repeated with the three periods under study. Though, there are little variations in the outcome of the analysis of the three periods but ultimately the conclusion is still the same. It is then concluded that jump process can be used to analyze and model the data before, during and after covid19 periods.**

Keywords: *Jump process, None Explosive, Poisson process, Log-normal, Covid-19.*

I. Introduction

None Explosive Jump Process otherwise known as pure jump process is a type of stochastic process that has discrete movements called jumps with random arrival times, other than continuous movement typically modeled as a simple or compound Poisson. In a non-explosive process, there are finite number of jumps in a finite interval. Non explosive (pure jump) processes are therefore the processes that change only by jumps. Poisson processes are generalizations of the Poisson distribution which are often used to describe the random behavior of some counting random quantities such as the number of arrivals to a queue, the number of hits to a webpage, number of telephone calls within a time space and likes.

Jump process is one of the building blocks of Jump-diffusion model. The ability to disintegrate jumps from volatility is the essence of risk management which should focus on controlling large risk leaving aside day to day Brownian fluctuation. Volatility simply means uncertainty. Every Jump process ultimately has two components: the arrival pattern which follow Poisson distribution and the size (jump) that is contained in each arrival that follow either Log-normal or F-distribution. The combination of these two components gives rise to jump process. In this work we use Log-normal distribution for the size (jump). The arrival part can be model as simple or compound Poisson. The arrival of aircrafts at an airport follows a Poisson process while size (e.g. weight, height etc. of the passengers in the aircraft) follows Log-normal distribution. Another example is the arrival of different families at a restaurant on a particular day follows Poisson while size (e.g. money spent by the families) follows Log-normal distribution.

None explosive (pure jump) is an extension of jump modeling in that small jumps can eliminate the need for a continuous martingale. There are many different classes of non-explosive jump processes. The class of non-explosive jump models is extremely wide and include but not limited to the following: Normal Inverse Guassian [Rydberg

(1997), Bardorff-Nielsen (1997, 1998)], the variance gamma [Madan, Carr and Chang (1998)], the CGMY model of Carr et al, (2002), Daal and Madan (2005), Carr and Wu (2007)].

1.1 Statement of Problem

The data consist of the arrival and the volume in kilograms of gas sold to customers before, during and after Covid-19 pandemic period. The study examines generally the understanding of the concept of jump and its applications with respect to a gas company in Enugu. The specific objectives of the research are to:

a. examine if there is existence of jumps.
b. examine the distribution of the jumps.
c. examine the distribution of the arrival of the jumps.
d. examine the pattern of the jumps.

1.2 Literature Review

Before now, there was no serious evidence for acceptance of presence of jumps in the asset price processes but now it is widely accepted that asset price contains jumps due to many empirical evidences as heavy tails in the asset returns Carr et al (2002), Cont and Tankov (2004), elimination of the need for continuous martingale and many others. Non-explosive jump is gradually replacing the classical model which has continuous martingale component in recent years in price process Todorov and Tauchen (2010). The concept and class of non-explosive jump models is wide and it includes: Normal Inverse Gaussian Rydberg (1997), Barndorff-Nelson (1997, 1998), the variance gamma (Madan, Carr and Chang (1998)), the CGMY model of Carr et al. (2002), the time-changed Levy models of Carr et al. (2003), the COGARCH model of Kloppenburg, Lindner and Maller (2004) for the financial prices as well as the non-Gaussian Ornstein-Uhlenvbeck-based models of Barndorff-Nielsen and Shephard (2001) and many others.

The Black–Scholes model assumes that the market consists of at least one risky asset, usually called the stock, and one riskless asset, usually called the money market, cash, or bond. Now we make assumptions on the assets which explain their names: (riskless rate). The rate of return on the riskless asset is constant and thus called the risk-free interest rate.

II. METHODOLOGY

2.1 The Jump Process

Jump processes are stochastic processes whose trajectories have discontinuities called jumps that can occur at random times. Dataset from gas station is used in this work. Various stochastic models are used to model the price movements of financial instruments; for example the Black–Scholes model for pricing options assumes that the underlying instrument follows a traditional diffusion process, with continuous, random movements at all scales, no matter how small.

2.2 Poisson Process

A Poisson Process is a model for a series of discrete event where the average time between events is known, but the exact timing of events is random.

$$\sum_{i=1}^{n} \frac{\ell^{-\lambda} \lambda^{x}}{x!} \quad \text{where } x = 1,2,\ldots,n$$

2.3 Compound Poisson Process

The Poisson process itself appears to be too limited to develop realistic price models as its jumps are of constant size. Therefore there is interest in considering jump processes that can have random jump sizes.

$$Ji = \sum_{i=1}^{Nt} Yi$$

where Ji represents a compound Poisson in which the jump sizes Y_i are independently and identically distributed with distribution F and the number of jumps Nt is a Poisson process with the jump intensity λ. Compound Poisson process is a family of Poisson process. A compound Poisson distribution, in which the summands have an exponential distribution, was used by Revfeim in 1984:

$$\sum_{i=1}^{N_t} \sum_{i=1}^{n} \frac{\ell^{-\lambda} \lambda^{x}}{x!}$$

Revfeim, K.J.A. (1984).
Where x=1,2,…,n and Nt is a family of Poisson

2.4 Normal Distribution

Normal distribution, also called Gaussian distribution is the most common distribution function for independent, randomly generated variables.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

2.5 Log-Normal

Log-normal is the same as logarithm of normal distribution. In probability theory, lognormal (log-normal) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Then there is need to understand normal distribution so as to have better understanding of Lognormal. Thus, if the random variable X is log-normally distributed, then Y = ln(X) has a normal distribution.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}$$

The probability density function for the log-normal is defined by the two parameters **μ** and **σ**, where x > 0.

III. APPLICATIONS

The data is for three consecutive years before (2019), during (2020) and after (2021) covid19 pandemic. It shows the number of cylinders filled and the total volume in kilogram on a monthly basis for the three years from a depot of Nigerian National Petroleum Company (NNPC), Enugu branch.

Table 1: NNPC DEPOT EMENE ENUGU BEFORE COVID-19
MONTHLY SALES OF GAS 2019

S/N	MONTH	CYLINDERS	KG	PRICE P/KG
1	Jan	-	-	
2	Febuary	-	-	
3	March	-	-	
4	April	40	321.5	
5	May	459	6682	
6	June	2694	54699	
7	July	2799	14737.7	
8	August	2324	17267.8	
9	September	4528	31074.3	
10	October	2042	13175.1	
11	November	169	1191.2	
12	December	-	-	

SOURCE:NNPC DEPOT ENUGU 2019

Table 2: MONTHLY SALES OF GAS 2020 DURING COVID -19

S/N	MONTH	CYLINDERS	KG	PRICE P/KG
1	Jan	1256	7594	
2	Febuary	NIL	NILL	
3	March	570	3463	
4	April	642	3750.3	
5	May	119	698	
6	June	NILL	NIL	
7	July	122	4187.3	
8	August	184	10149.4	
9	September	NILL	NIL	
10	October	NILL	NIL	
11	November	NILL	NIL	
12	December	NILL	NIL	

SOURCE: NNPC DEPOT ENUGU 2020

Table 3: MONTH Y SALES OF GAS 2021 AFTER COVID -19

S/N	MONTH	CYLINDERS	KG	PRICE P/KG
1	Jan	1487	9628.5	
2	Febuary	1887	12511.2	
3	March	2287	15380.4	
4	April	2340	11542.2	
5	May	1456	9678.3	
6	June	1977	12787.7	
7	July	1686	10495.3	
8	August	1848	12576.6	
9	September	2099	13969.5	
10	October	1575	10076.7	
11	November	1623	9987.3	
12	December	2281	15702.3	

SOURCE: NNPC DEPOT ENUGU

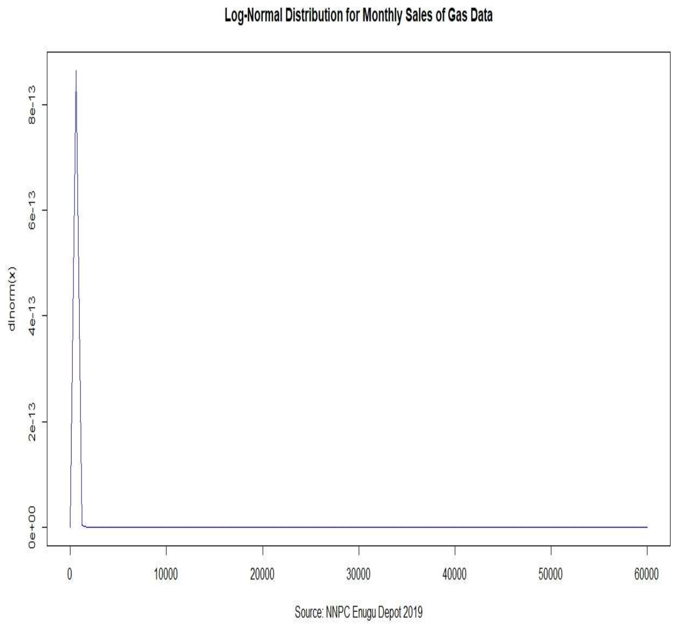
3.1 Analysis

Mean and Variance of Poisson Distribution with Monthly Sales of Gas data 2019.

```
id<-
c(40,459,2694,2799,2324,4528,2042,169)
lambda<-mean(id)
lambda
[1] 1881.875
var<-lambda
var
[1] 1881.875
```

Plot of Log-Normal Distribution with Monthly Sales of Gas data 2019

```
x=c(321.5,6682,54699,14737.7,17267.8,31
074.3,13175.1,1191.2)
curve(dlnorm(x),from=0,to=60000,xlab="S
ource: NNPC Enugu Depot
2019",main="Log-Normal Distribution for
Monthly Sales of Gas Data",col="blue")
```

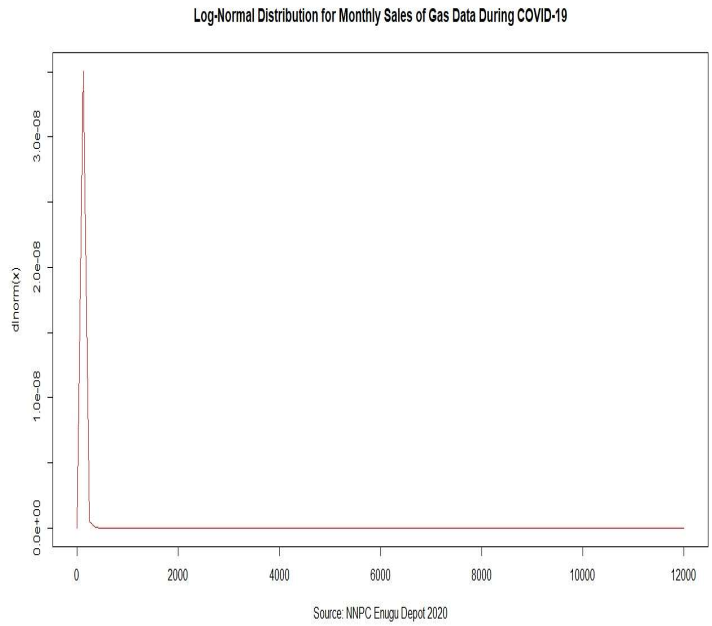


Mean and Variance of Poisson Distribution with Monthly Sales of Gas data 2020 During COVID-19

```
w<-
c(1256,0,570,642,119,0,122,184,0,0,0,0)
lambda<-mean(w)
lambda
[1] 241.0833
var<-lambda
var
[1] 241.0833
```

Plot of Log-Normal Distribution with Monthly Sales of Gas data 2020 During COVID-19

```
x=c(7594,0,3463,3750.3,698,0,4187.3,101
49.4,0,0,0,0)
curve(dlnorm(x),from=0,to=11000,xlab="S
ource: NNPC Enugu Depot
2020",main="Log-Normal Distribution for
Monthly Sales of Gas Data During COVID-
19",col="red")
```

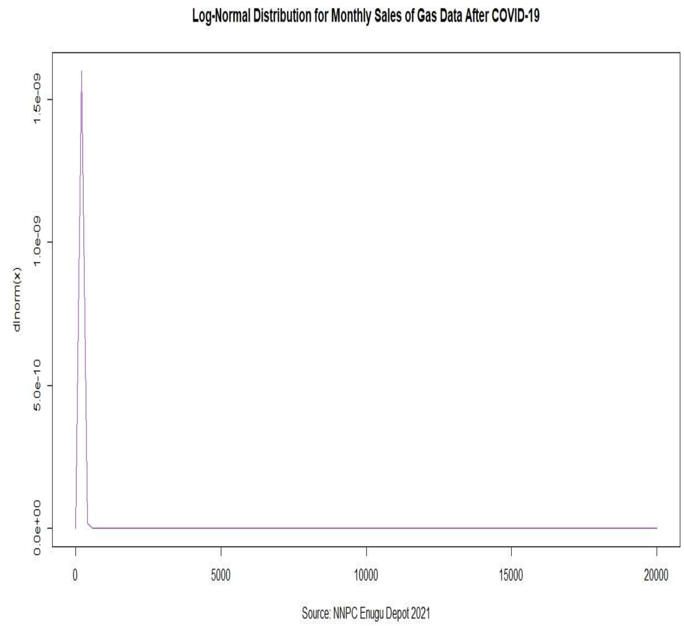


Mean and Variance of Poisson Distribution with Monthly Sales of Gas data 2021 After COVID-19

```
w1<-
c(1487,1887,2287,2340,1456,1977,1686,18
48,2099,1575,1623,2281)
lambda<-mean(w1)
lambda
[1] 1878.833
var<-lambda
var
[1] 1878.833
```

Plot of Log-Normal Distribution with Monthly Sales of Gas data 2021 After COVID-19

```
x=c(9628.5,12511.2,15380.4,11542.2,9678
.3,12787.7,10495.3,12576.6,13969.5,1007
6.7,9987.3,15702.3)
curve(dlnorm(x),from=0,to=16000,xlab="S
ource: NNPC Enugu Depot
2021",main="Log-Normal Distribution for
Monthly Sales of Gas Data After COVID-
19",col="purple")
```



IV. CONCLUSION

The aim of the study was to examine the understanding of the concept of jumps and its applications with a case study of a gas station in Enugu. The specific objectives of the research were to examine the existence of jumps by observing the distribution of arrival and volume of gas filled examine the distribution of the jumps.

The summary of the findings is presented as follows:

i. The existence of jumps were noticed in the data since the arrival (number of gas cylinders brought for filling) follows Poisson process while the volume follow log-normal distribution.

ii. From the plotted graph of the jump size in volume and in value, the graph is skewed to the right and the shape clearly show log-normal graph.

iii.. From the result of the arrival pattern of the distribution showed, each mean equal corresponding variance which conclude that the arrival follow Poisson

i. Existence of jumps was noticed in the data.

ii. From the plotted graph of the jump size in volume and in value, the graph is skewed to the right which show that the jump size follows log-normal distribution.

iii. From the result of the arrival pattern of the distribution, the mean of each of the jump correspondingly equal the variance which show that arrival pattern follows Poisson distribution.

4.1 Recommendations

Based on the findings of this study, it is recommended that the data of gas station can be modeled with explosive jump process.

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